Mutual Fund Families and Performance Evaluation

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Abstract

We develop a continuous-time Bayesian learning model to evaluate the composite skill of a mutual fund manager and a fund family. Our model estimates the composite skill of each fund as a function of its own performance and family performance. We show two competing effects of family performance on the evaluation of a member fund: a positive common-skill effect, and a negative common-noise effect. The overall effect increases with the correlation of unobservable skills, and decreases with the correlation of unobservable noise in fund returns. This pattern is stronger in larger families. Consistent with our assumptions, we find empirically that funds within the same family show higher correlations of estimated alphas and of residual returns. We also find that the effect of family performance on flows to a member fund exhibits strong cross-sectional patterns that are consistent with our model predictions.

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Investment outcomes are driven by skill and by luck. A fundamental issue in delegated portfolio management is performance evaluation; that is, to distinguish skill from luck. This distinction is crucial for appropriate selection of funds and compensation of fund managers. Most methods of performance evaluation focus on the records of individual funds in isolation, apart from any relevant information contained in other funds in the same family. Our objectives are to provide a theoretical framework of performance evaluation for mutual funds within families, and to examine empirically how investors incorporate both fund and family performance information when they allocate money across funds.

Most mutual funds belong to a family where individual managers share common resources. One example of a shared resource is an information system that provides managers tools for portfolio analysis, risk management, and performance measurement. Another is a legal department that counsels portfolio managers. Legal expertise is particularly important for understanding patents; for evaluation of companies facing litigation or engaged in bankruptcy or corporate transactions; and for understanding contractual terms of bond indentures. Fund managers may also have access to the same set of outside experts, such as reports from particular sell-side firms. Finally, family pools of security analysts and traders provide investment ideas and transaction services to portfolio managers.

As a result of family membership, a fund's risk-adjusted performance (its alpha) is determined by the quality of common resources in addition to the expertise of the manager. This is supported by empirical evidence. Baks (2003), for example, attributes the majority of funds' abnormal returns to family membership rather than to individual managers. With this in mind, we develop a model in which the risk-adjusted performance of one fund is driven by its composite skill, which is a summary measure of the quality of family resources and its manager's expertise. Our model recognizes that returns of other funds in the family contain information about the family component of the composite skill. Therefore, the conditional estimate of a fund's composite skill is based on both its own performance and the performance of other funds in the family.

Our model also recognizes that because of reliance on common family resources, the returns of funds in a family contains positively correlated noise.[1] As a consequence, family

[1]Elton, Gruber, and Green (2007) find that mutual fund returns are more closely correlated within families

performance helps to filter out such noise in a member fund's returns. A positive shock leads to good performance by many funds in a family. By comparing one fund's performance with that of the rest of the family, we estimate the fund's composite skill more precisely. This provides another reason to incorporate family performance information into the evaluation of a member fund.

In our model, the key distinction between the skill and the noise is that the skill determines the mean of the risk adjusted returns (i.e., the alpha) while the noise, represented by unobservable non-systematic shocks, leads to temporary fluctuations around the mean. In general, we expect both alphas and noise to be more closely correlated within a family than across families. Family resources induce member funds to tilt their portfolios in similar directions, meaning that funds tend to over- or underweight the same securities relative to their benchmarks. For example, a single idea from the analyst pool can lead several managers to simultaneously increase or decrease positions in a security. As a result, both alphas and short-term fluctuations in returns are highly correlated within the family. However, this is not necessarily always the case. Some family resources affect the alphas of all member funds, but have little impact on the correlations of short-term fluctuations in their returns. Examples are the trading desks retained to execute trades, the risk management process, and the screening mechanisms that the family uses to hire analysts. By contrast, one can also think of situations in which alphas are uncorrelated across funds in the family, but short-term fluctuations are positively correlated. For example, a family may have a focus on certain geographic areas or industries, or securities with certain characteristics, but within these categories individual managers are responsible for selecting stocks independently.

We model learning and investors' optimal responses to fund performance and family performance. Investors observe the performance of funds in a family. Each fund manager's skill is an unknown latent variable. The quality of the common resources of the family (the family skill) is also unknown. A fund's alpha increases with its composite skill, and decreases with fund size. Fund returns are subject to correlated non-systematic shocks. Investors estimate funds' composite skills, conditional on returns, and allocate wealth across the funds, generating flows into and out of funds.

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than across families.

This model of Bayesian learning is based on a well-established theory of continuous-time filtering, and it is an extension of the work of Dangl, Wu, and Zechner (2008). The enviroment mimics that of Berk and Green (2004). There is perfect capital mobility, decreasing returns to scale, and competitive capital provision. In this setting, mutual fund flows directly reflect innovations in investors' beliefs about a fund's composite skills.

We characterize the optimal updating of beliefs about funds' composite skills. Not surprisingly, the estimate of a funds' composite skill is positively related to its own unexpected risk-adjusted return. Good performance indicates either a skilled fund manager or high quality of family resources. The more interesting question concerns the effect of family performance (measured by the average performance of other funds in the family) on the estimated skill of a member fund. Our model highlights two competing effects: a positive common-skill effect and a negative common-noise effect. The positive effect arises because family performance partially reveals the quality of family resources, while the negative effect arises because family performance also partially reveals the common shocks to the returns of all funds in the family. The overall effect is either positive or negative, depending on two correlations. The estimate of a fund's composite skill increases with family performance when the unobservable composite skills in the family are highly correlated, but the unobservable non-systematic shocks are relatively independent of each other. Alternatively, the estimate decreases with family performance when non-systematic shocks are highly correlated, but the correlations of composite skills are low. By varying the number of funds in the family, we further find that this pattern is stronger in bigger families.

The empirical analysis has two objectives. First, we provide evidence of the family's influence on fund performance. Using the standard three- and four-factor models as benchmarks, we estimate alphas and residuals for all actively managed U.S. equity funds in the database of the Center for Research in Security Prices (CRSP). We calculate the correlations of residuals, and the correlation of innovations in the time-varying alphas, for all pairs of funds. We demonstrate that each of these correlations is significantly greater within a family than across families. This evidence supports the premise underlying our theory. Reliance on common resources leads to a family effect in funds' returns.

The second objective is to determine if investors allocate money across mutual funds in a manner that is consistent with our model of optimal learning. We measure the sensitivities of funds' asset flows to their own performance and to the performance of other funds in the family. For a median fund, we find a positive spillover effect. That is, a fund receives higher flows when other funds in the same family perform well. This suggests that the common-skill effect dominates the common-noise effect. More important, the spillover within families increases with the correlation in measured alphas, and decreases with the correlation in residual returns. We also find this pattern is stronger in families with a larger number of funds. These patterns are predicted by our model. Therefore, mutual fund investors appear to incorporate family performance into fund selection in a manner consistent with optimal learning.

Our work contributes to the literature in several ways. First, from a theoretical point of view, we study a dynamic model of multivariate learning. Our main results are relevant for performance evaluation in general, beyond the application to a family of mutual funds. The idea that peer performance can be used to filter out common shocks to multiple agents is well-recognized in the literature on relative performance evaluation (see, for example, Holmstrom (1982) and Gibbons and Murphy (1990)). This consideration generally leads to a negative relation between the optimal compensation of an agent and the performance of his peers. Our model allows for common components in both shocks and skills. As a result, peer performance can have a positive impact on beliefs about an agent's skills, provided that the common-skill effect dominates the common-noise effect.

Second, we present new empirical evidence of a family effect in fund returns, and new empirical descriptions of investor use of family performance information in capital allocation decisions. We show that family membership affects both a fund's alpha and its residual returns, and that mutual fund flows respond to fund performance and family performance in a manner that is consistent with optimal learning about funds' skills.

Third, from a practical point of view, our results suggests that an accurate evaluation of a fund's composite skill should incorporate the performance of all funds within a family. For a fund family whose managers rely little on family resources and whose funds' returns exhibit a high correlation in temporary fluctuations, an estimate of a fund's skill should negatively weight the family performance. By contrast, for a family in which the common resources are the main driver of fund alphas, and there is little correlation in the temporary fluctuations in fund returns, the estimate should positively weight family performance.

There is a large body of literature on mutual fund performance evaluation. Aragon and Ferson (2006) provide an extensive review. Most methods of evaluation rely solely on a fund's own return or portfolio holding information. Several recent papers propose methods incorporating additional information. For example, Pastor and Stambaugh (2002) estimate the alpha of an actively managed fund using the returns on "seemingly unrelated" non-benchmark passive assets. Cohen, Coval, and Pastor (2005) judge a fund manager's skill by the extent to which his or her investment decisions resemble those of managers with distinguished track records. Jones and Shanken (2005) measure performance using the distribution of other funds' alphas in additon to the information in a fund's own return history. Our performance measure is in the spirit of this literature, but differs from it in two respects. First, we exploit the information embedded in the performance of a fund's family. Second, we derive our measure from a model of optimal learning.

Our work is closely related to studies of mutual fund flows. Many authors find that mutual funds with good past performance attract more fund flows (see, for example, Sirri and Tufano (1998). More relevant to our paper, Nanda, Wang, and Zheng (2004) find that the stellar performance of one fund has a positive spillover onto the inflows to other funds in the same family, and Sialm and Tham (2011) find that the prior stock price performance of the management company affects the money flows of the affiliated funds. Berk and Green (2004) develop a learning model that can explain the positive response of fund flows to past performance, even though performance is not persistent. Dangl, Wu, and Zechner (2008) model simultaneously mutual fund flows and termination of fund managers in response to past performance. Other learning-based models for the flow-performance relation include those by Lynch and Musto (2003) and Huang, Wei, and Yan (2007). All these models are silent about spillovers within fund families. The current paper extends the continuous-time model of Dangl, Wu, and Zechner (2008) to account for multiple funds within a family. We derive the optimal response of mutual fund flows to fund and family performances in an economy with rational investors, and find strong empirical patterns of spillovers within families that are consistent with our model.[2]

The paper is organized as follows. Section 1 describes the structure of our model of a mutual fund family. In section 2 we derive the general optimal learning rule. In section 3 we examine the sensitivities of investor beliefs about a fund's composite skill to its own performance and the performance of other funds in the family. We analyze the model in two steps. We first use a simple two-fund family, and then study a family of $n > 2$ homogeneous funds. Section 4 derives the dynamics of fund size. Section 5 presents the empirical evidence for the key assumptions and predictions of our model. Section 6 concludes. The proofs of all propositions are in the Appendix.

1 A Family of Mutual Funds

We model n actively managed mutual funds within a family. The quality of management is an unobservable factor governing the success or failure of a fund. Quality varies through time, and is a linear combination of two components, which together form the composite skill $\widehat{\theta}$ of a fund. One part of $\widehat{\theta}$ is the skill of the fund manager. The second part is the quality of the common resources available to fund managers within the family. A fund's alpha and its expected return are increasing functions of $\widehat{\theta}$, and a fund's realized return is a signal of this unknown quantity. We calculate a conditional distribution of $\widehat{\theta}$ for all funds in the family using fund returns as a continuous signal.

Generalizing Dangl, Wu, and Zechner (2008), we assume that the funds' rates of return, net of fees, are given by

$$\frac{d\mathbf{G}_t}{\mathbf{G}_t} = \left(r_t \mathbf{1}_n + \eta \boldsymbol{\sigma}_m + \boldsymbol{\alpha}_t - \mathbf{f}_t\right) dt + \boldsymbol{\sigma}_m dW_{mt} + \boldsymbol{\sigma}_t \mathbf{B} d\mathbf{W}_t, \tag{1}$$

where $\mathbf{G}_t$ is the $n \times 1$ vector of net asset values per share with dividends reinvested; r_t is the risk-free rate process; $\mathbf{1}_n$ is an $n \times 1$ vector of ones; η is the market price of risk; and $\boldsymbol{\sigma}_m$ is

[2]The recent literature on mutual funds has shown a growing interest in fund families. See for example, Mamaysky and Spiegel (2002), Massa (2003), Gervais, Lynch, and Musto (2005), Massa, Gaspar, and Matos (2006), Ruenzi and Kempf (2008), Pomorski (2009), Bhattacharya, Lee, and Pool (2010), Warner and Wu (2011), and Khorana and Servaes (2011).

an $n \times 1$ vector of exposures to the market risk factor of the funds' portfolios.[3] Together, these determine the expected rates of returns in the absence of portfolio management skills. The $n \times 1$ vector $\boldsymbol{\alpha}$ captures the contribution of the composite skill; an element α_i is the abnormal expected rate of return of fund i generated by active management of the fund. We refer to α_i simply as the alpha of fund i. The $n \times 1$ vector $\mathbf{f}$ is the instantaneous rate of management fees. In total, the drift in equation (1) is the vector of expected rates of return net of fees.

Innovations in fund returns have two components. One is the systematic component $\boldsymbol{\sigma}_m dW_m$, where W_m is a scalar Brownian motion. The second is the non-systematic component $\boldsymbol{\sigma}\mathbf{B}d\mathbf{W}$, where $\mathbf{W}$ is a vector of standard Brownian motions that are pairwise independent, and are independent of W_m. The $n \times n$ diagonal matrix $\boldsymbol{\sigma}$ has elements σ_i along the main diagonal, representing the volatility of non-systematic returns. Matrix $\mathbf{BB}'$ is symmetric and nonsingular, with ones along the main diagonal and off-diagonal elements ρ_{ij}, which are the correlations of non-systematic shocks.[4]

A fund's non-systematic risk σ_i is governed by the scale of the manager's portfolio tilt, which is the difference between the fund's weights in individual securities and the weights of a benchmark portfolio with only systematic risks. A fund with no tilt has $\sigma_i = 0$. As the manager increases the scale of a tilt, with the expectation of increasing fund alpha, σ_i increases.

If managers of two funds i and j follow independent strategies and have orthogonal tilts, the non-systematic returns are uncorrelated, and $\rho_{ij} = 0$. For various reasons noted above, however, we expect fund managers within a family to follow positively correlated strategies. Our empirical estimates of correlations of mutual funds' abnormal returns are positive and are significantly greater than the average correlations of funds across families. Therefore, we assume $\rho_{ij} \geq 0$ (under the constraint that $\mathbf{BB}'$ is nonsingular).

Fund alphas follow the process

$$\boldsymbol{\alpha}_t = \boldsymbol{\sigma}_t \widehat{\boldsymbol{\theta}}_t - \gamma \boldsymbol{\sigma}_t \boldsymbol{\sigma}_t \mathbf{A}_t, \tag{2}$$

[3]Our model can be easily generalized to allow for multiple systematic risk factors.

[4]Matrix $\mathbf{B}$ is the Cholesky decomposition of the correlation matrix.

where

$$\widehat{\boldsymbol{\theta}}_t \overset{def}{=} \mathbf{b}\boldsymbol{\theta}_t + \boldsymbol{\beta}\theta_F;\ \boldsymbol{\theta}_t \overset{def}{=} \begin{pmatrix} \theta_{1t} & ... & \theta_{nt} \end{pmatrix}';\ \boldsymbol{\beta} \overset{def}{=} \begin{pmatrix} \beta_1 & ... & \beta_n \end{pmatrix}'; \tag{3}$$

and $\mathbf{b}$ is a diagonal matrix with vector $\mathbf{1}_n - \boldsymbol{\beta}$ along the diagonal. The vector $\boldsymbol{\sigma}\widehat{\boldsymbol{\theta}}$ has elements $\sigma_i [(1-\beta_i)\theta_i + \beta_i\theta_F]$, and it represents the effect of active management on the expected returns. Here, θ_i is the skill of the fund manager i; θ_F is the quality of the common resources of the family; β_i is the degree to which a manager uses the common resources; and $\widehat{\theta}_i = (1-\beta_i)\theta_i + \beta_i\theta_F$ is the *composite skill* of fund i. For a manager with no individual skill, $\theta_i = 0$. A fund with a pool of excellent analysts has large θ_F. For a manager working independently of the analyst pool, $\beta_i = 0$. We expect managers to rely on the pool for investment ideas, i.e., $\beta_i > 0$. In this case, the fund's alpha increases directly with both θ_i and θ_F.

Fund assets A_i are elements of vector $\mathbf{A}$. The parameter $\gamma > 0$ captures the decreasing returns to scale in active portfolio management. The i-th element of the final term in equation (2) is $-\gamma\sigma_i^2 A_i$. Thus, the alpha of fund i decreases with its own size, and it decreases more if the fund is not well-diversified, i.e., when σ_i is high. Funds with concentrated stock positions suffer most from the price impact of large portfolio transactions. Equation (3) also implies that the marginal return from taking idiosyncratic risk decreases, especially for large funds. This deters funds from taking unlimited idiosyncratic risk.

Mutual funds operate in a rapidly changing business environment. Past success or experience is no guarantee of future performance. To capture this characteristic of the portfolio management business, we assume the unobservable composite skills follow a stationary stochastic process:

$$d\widehat{\boldsymbol{\theta}}_t = k\left(\overline{\boldsymbol{\theta}} - \widehat{\boldsymbol{\theta}}_t\right) dt + \boldsymbol{\Omega} d\mathbf{w}_t, \tag{4}$$

where the constant k governs the speed at which $\widehat{\boldsymbol{\theta}}_t$ reverts to the long-run mean $\overline{\boldsymbol{\theta}}$. If $k = 0$, the composite skill of each fund follows a random walk. Volatility coefficients are in matrix $\boldsymbol{\Omega}$, which has the form

$$\boldsymbol{\Omega} = \left[\mathbf{b}\boldsymbol{\omega} \vdots \boldsymbol{\beta}\omega_F\right], \tag{5}$$

where $\boldsymbol{\omega}$ is a diagonal matrix with coefficients ω_i along the main diagonal. In equation (4), $\mathbf{w}_t$ is a vector of $n+1$ pairwise independent standard Brownian motions, each independent of W_{mt} and $\mathbf{W}_t$. The individual skill of manager i has the instantaneous volatility $\omega_i \geq 0$, while the volatility rate of the common resources of the family is $\omega_F \geq 0$. Thus the stochastic component of an element $\widehat{\theta}_i$ in (4) is $(1-\beta_i)\omega_i dw_i + \beta_i \omega_F dw_{n+1}$, and the instantaneous correlation of the skills for a pair of funds i and j is

$$\lambda_{ij} \stackrel{def}{=} \frac{\beta_i \beta_j \omega_F^2}{\sqrt{(1-\beta_i)^2 \omega_i^2 + \beta_i^2 \omega_F^2}\sqrt{\left(1-\beta_j\right)^2 \omega_j^2 + \beta_j^2 \omega_F^2}}. \tag{6}$$

This is a measure of the importance of variation in the quality of common resources of the family as a driver of composite skills, relative to the importance of variation in managers' skills. It is easy to see that λ_{ij} increases with β_i and β_j, and decreases with the ratios of ω_i and ω_j to ω_F. A value $\lambda_{ij} = 0$ indicates either that the quality of the common resources is constant ($\omega_F = 0$), or that one or both of the managers works independently of those resources $(\beta_i = 0)$. A value $\lambda_{ij} = 1$ indicates instead that the skills of the individual managers are fixed ($\omega_i = \omega_j = 0$), or that the managers act in concert and rely entirely on the common resources for alpha generation $\left(\beta_i = \beta_j = 1\right)$.

2 The Evaluation of Fund Performance

We assume information is symmetric but incomplete. All variables in equation (1) are observable, except the composite skill $\widehat{\boldsymbol{\theta}}_t$ and the non-systematic shocks $d\mathbf{W}_t$. We use a Kalman-Bucy filter to calculate the conditional distribution of $\widehat{\boldsymbol{\theta}}_t$ using fund returns as a continuous signal. We then study the sensitivity of the mean estimate of $\widehat{\theta}_i$ for one fund to the performance of all funds within the family.

Using equation (1) we obtain

$$\begin{aligned} d\boldsymbol{\xi}_t &\stackrel{def}{=} \boldsymbol{\sigma}_t^{-1}\left(\frac{d\mathbf{G}_t}{\mathbf{G}_t} - \left(r_t \mathbf{1}_n - \gamma \boldsymbol{\sigma}_t \boldsymbol{\sigma}_t \mathbf{A}_t + \eta \boldsymbol{\sigma}_m - \mathbf{f}_t\right) dt - \boldsymbol{\sigma}_m dW_{mt}\right) \\ &= \widehat{\boldsymbol{\theta}}_t dt + \mathbf{B} d\mathbf{W}_t. \end{aligned} \tag{7}$$

The first equation defines $d\boldsymbol{\xi}$ as the difference between the vector of fund returns and the observable components of the return, normalized by non-systematic volatilities. The second equality demonstrates that $d\boldsymbol{\xi}$ is a signal of composite skills, where the vector of non-sytematic shocks $d\mathbf{W}$ creates noise in the signal.

At any time, information is the history of fund returns represented by the filtration $\mathcal{F}_t \stackrel{def}{=} \sigma\left\{\boldsymbol{\xi}_s\right\}_{s=0}^{t}$. Given a multivariate normal prior distribution with mean vector $\mathbf{m}_0$ and covariance matrix $\mathbf{V}_0$, the conditional distribution of the composite skills is also multivariate normal.[5]

Proposition 1. *The conditional mean vector* $\mathbf{m}_t \stackrel{def}{=} E(\widehat{\boldsymbol{\theta}}_{\mathbf{t}}|\mathcal{F}_t)$ *and the conditional covariance matrix* $\mathbf{V}_t \stackrel{def}{=} Var(\widehat{\boldsymbol{\theta}}_{\mathbf{t}}|\mathcal{F}_t)$ *for* $t \geq 0$ *follow the processes:*

$$d\mathbf{m}_t = k\left(\overline{\boldsymbol{\theta}} - \mathbf{m}_t\right)dt + \mathbf{S}_t d\mathbf{W}_t^{\mathcal{F}}, \tag{8}$$

$$\frac{d\mathbf{V}_t}{dt} = \boldsymbol{\Omega}\boldsymbol{\Omega}' - 2k\mathbf{V}_t - \mathbf{V}_t\left(\mathbf{B}\mathbf{B}'\right)^{-1}\mathbf{V}_t, \tag{9}$$

where

$$\mathbf{S}_t \stackrel{def}{=} \mathbf{V}_t\left(\mathbf{B}\mathbf{B}'\right)^{-1}, \tag{10}$$

$$d\mathbf{W}_t^{\mathcal{F}} \stackrel{def}{=} \left(d\boldsymbol{\xi}_t - \mathbf{m}_t dt\right). \tag{11}$$

When $k = 0$, *equation (9) has the analytic solution:*

$$\mathbf{V_t} = \mathbf{BD}(\mathbf{Q_t^{-1}} + \boldsymbol{\Pi}^{\mathbf{1/2}})\mathbf{D}'\mathbf{B}', \tag{12}$$

where the matrices $\mathbf{D}, \boldsymbol{\Pi}$, *and* $\mathbf{Q_t}$ *are as defined in Appendix A.1.*

Proof. See Appendix A.1.

Note that the conditional mean $\mathbf{m}_t$ follows a multi-variate Orsten-Uhlenbeck process with long-run mean $\overline{\boldsymbol{\theta}}$. The vector $d\boldsymbol{\xi}_t$ is adjusted by the conditional means $\mathbf{m}_t dt$ to obtain the vector $d\mathbf{W}_t^{\mathcal{F}}$, which is a Brownian motion under filtration $\mathcal{F}_t$. The vector $d\mathbf{W}_t^{\mathcal{F}}$ has zero mean, unit variance, and correlation matrix $\mathbf{B}\mathbf{B}'$. This is a vector of *normalized unexpected non-systematic fund returns.*

[5]Investor beliefs are conditional distributions for $\widehat{\boldsymbol{\theta}}_t$, which has the same dimension as the observation equation. A conditional distribution can be calculated numerically for individual components θ_i and θ_F. Learning about the components is important for the hiring and firing decisions in fund families, and investor responses to these decisions, but it is not necessary for investors to form an expectation of fund alphas in our model.

The elements of matrix $\mathbf{S}_t$ defined in equation (10) are the sensitivities of mean beliefs to these unexpected returns. They increase with uncertainty about composite skills, which is in matrix $\mathbf{V}_t$. Elements on and off the main diagonal of $\mathbf{V}_t$ are conditional variances and covariances, respectively. Generally, if skills are estimated precisely, elements of $\mathbf{V}_t$ and $\mathbf{S}_t$are small, and the mean $\mathbf{m}_t$ is insensitive to unexpected returns. If instead little is known about skills, elements of these matrices are large, and unexpected returns are important signals of skill. Therefore, estimates of future performance are most sensitive to past performance when investors are least confident in their knowledge about either the skills of managers or the quality of common resources, or both.

An element on the main diagonal of $\mathbf{S}_t$, say, s_{nt}, is the sensitivity of the mean belief about a fund's composite skill to its own unexpected return. This coefficient should be positive; good performance increases the estimate of the composite skill of a fund's manager. An off-diagonal element $\overline{s}_{nt}$ is the sensitivity of mean belief about the composite skill of one fund to the unexpected return of a second fund. The sign of this cross-coefficient can be different in different situations.

Suppose first the non-systematic shocks in the returns are uncorrelated, and that the quality of the common resources of the family is volatile.[6] Each fund's return is a signal of the quality of its particular manager and simultaneously a signal of the quality of family resources. Although returns are uncorrelated, each fund tends to perform well when θ_F is high, because θ_F is a common component in fund alphas. Hence, m_i is adjusted upward when fund j has a positive unexpected return, and similarly m_j rises with the unexpected return of fund i. In this case, $\overline{s}_{nt} > 0$, which is the result of the common-skill effect.

Now suppose that fund managers work independently, i.e., $\hat{\theta}_i = \theta_i$, and that the non-systematic shocks are positively correlated because of the similar tilts of the fund managers, meaning that the off-diagonal elements of $\mathbf{BB}'$ are large. If fund j has unexpectedly good performance, fund i should perform similarly, given the funds' similar tilts. The return of fund i is a noisy signal of the manager's skill θ_i, but some of this noise can be removed. A linear combination of returns with positive weight on the return of fund i and negative

[6]These assumptions imply, respectively, that $\mathbf{BB}'$ is the identity matrix, and that ω_F and therefore the off-diagonal elements of $\mathbf{\Omega\Omega}'$ are large.

weight on that of fund j has less noise and is a better signal of θ_i than the return on fund i alone. This is the common-noise effect, and it suggests that the cross-coefficient is negative, i.e., $\bar{s}_{nt} < 0$.

A thorough understanding of the coefficients in $\mathbf{S}_t$ requires investigation of the relative size of the two effects. In turn, this requires a study of the covariance matrix $\mathbf{V}_t$, which is the solution to the Ricatti equation (9). $\mathbf{V}_t$ is a function of family age, family size, instantaneous volatility of true skills, and prior beliefs.

The analytic solution for $\mathbf{V}_t$ given in equation (12) obtains when $k = 0$, i.e., when the skills follow a random walk. Alternatively, in the case of mean reversion, we can easily calculate numerical solutions to equation (9). Elements of $\mathbf{V}_t$ may either increase or decrease with time, depending on the level of initial uncertainty $\mathbf{V}_0$ and the instantaneous volatilities of skills. For reasonable parameter values, however, we expect that $\mathbf{V}_t$ decreases over time, so that investors know the skill of an older fund more precisely than the skill of a young fund.

The steady-state covariance matrix, $\mathbf{V}^*$, is the solution to equation (9) with $\frac{d\mathbf{V}_t}{dt} = \mathbf{0}_{n\times n}$. In comparison to $\mathbf{V}_t$, $\mathbf{V}^*$ is relatively simple because it is time-independent. It is also independent of prior beliefs, and has the simple form given in equation (A.2) in the case of a random-walk. Furthermore, $\mathbf{V}^*$ as the limiting value is a good approximation to $\mathbf{V}_t$ after the passage of enough time. For these reasons, we study the steady-state covariance matrix $\mathbf{V}^*$ and sensitivity matrix $\mathbf{S}^*$ in detail. The latter is given by:

$$\mathbf{S}^* = \mathbf{V}^* \left(\mathbf{B}\mathbf{B}'\right)^{-1}. \tag{13}$$

The following section characterizes investors' learning of funds' composite skills. It demonstrates how investors' reactions to past performance depend on the correlation of the non-systematic shocks (ρ), the correlation of alpha-generating skills (λ), and the size of the fund family.

3 Sensitivities of Beliefs to Performance

Investor beliefs about composite skills are functions of historical fund performance. The sensitivity of beliefs is represented by the elements of the matrix $\mathbf{S}_t$ as demonstrated by equation (8). Here we study $\mathbf{S}_t$.

For simplicity, we assume from now on that our mutual fund family is homogeneous, with $\beta_i = \beta \in [0,1]$, $\rho_{ij} = \rho \in [0,1)$, $\lambda_i = \lambda \in [0,1]$, $\omega_i = \omega$, and $\overline{\theta}_i = \overline{\theta}$, for all funds $i = 1, ..., n$.[7] These assumptions imply that the matrices $\mathbf{V}_t$ and $\mathbf{S}_t$ are homogeneous, meaning that elements along the main diagonal are equal, and similarly all off-diagonal elements are equal. As a result, equation (8) simplifies, and the estimate of skill for fund i, i.e., the conditional mean of $\widehat{\theta}_i$ follows the process:

$$dm_{it} = s_{nt} dW_{it}^{\mathcal{F}} + \overline{s}_{nt} (n-1) \, dW_{-i,t}^{\mathcal{F}}, \tag{14}$$

where s_{nt} and $\overline{s}_{nt}$ are the diagonal and off-diagonal elements, respectively, of $\mathbf{S}_t$, $dW_{it}^{\mathcal{F}}$ is the unexpected non-systematic return of the fund, and $dW_{-i,t}^{\mathcal{F}}$ is the average of unexpected non-systematic returns of the other $n-1$ funds in the family. We refer to $dW_{-i,t}^{\mathcal{F}}$ as the unexpected family performance. Equation (14) has an obvious advantage over equation (8) in that the performance of all the other funds is summarized in the single statistic $dW_{-i,t}^{\mathcal{F}}$.

Our primary interest is in the fund coefficient s_{nt} and the family coefficient $\overline{s}_{nt} (n-1)$, which govern how investor beliefs react to the performance of funds in a family. We ask: What determines the sign of $\overline{s}_{nt}$? How large is $\overline{s}_{nt} (n-1)$ relative to s_{nt}, or, in other words, how important are other funds in estimating the composite skills of a given fund? What factors govern the distribution of the coefficients across families? For example, how do s_{nt} and $\overline{s}_{nt}$ vary with the number of funds in a family and the age of a family?

In Sections 3.1 and 3.2 below, we focus on the steady state in which uncertainty is characterized by $\mathbf{V}^*$ in equation (A.2), and the sensitivity matrix is $\mathbf{S}^*$ in equation (13). We assume a random walk in composite skills, which has the advantage of having analytical solutions, and is a good approximation of the stationary case, provided that the rate of mean reversion is low. Section 3.1 starts with the case of two funds, while Section 3.2 deals with

[7] We exclude the case $\rho = 1$ to avoid a singular variance-covariance matrix of funds' returns $\mathbf{BB}'$.

$n \geq 2$ funds. We analyze numerically the general case with mean-reverting skills in Section 3.3.

3.1 The Two-Fund Family

Consider a family with two funds. We find:

Proposition 2. *When composite skills follow a random walk ($k = 0$), the conditional covariance matrix of composite skills in the steady state of a two-fund family is:*

$$\mathbf{V}^* = \begin{bmatrix} v_2 & \overline{v}_2 \\ \overline{v}_2 & v_2 \end{bmatrix}, \tag{15}$$

where

$$v_2 = \frac{1}{2}a\left(\sqrt{1+\rho}\sqrt{1+\lambda} + \sqrt{1-\rho}\sqrt{1-\lambda}\right) \geq 0, \tag{16}$$

$$\overline{v}_2 = \frac{1}{2}a\left(\sqrt{1+\rho}\sqrt{1+\lambda} - \sqrt{1-\rho}\sqrt{1-\lambda}\right) \geq 0, \tag{17}$$

$$a \overset{def}{=} \sqrt{\omega^2\left(1-\beta\right)^2 + \beta^2\omega_F^2}. \tag{18}$$

Proof. See Appendix A.2.

For a given level of volatility of the composite skills (the coefficient a defined in equation (18)), the conditional variance in a large family v is a symmetric function of ρ and λ.[8] The conditional variances and covariances of composite skills are v_2 and $\overline{v}_2$, respectively. Each of these values is a symmetric function of ρ and λ. For each fund, v_2 is a concave function of ρ. It increases with ρ for $\rho \in [0, \lambda)$, decreases with it for $\rho \in (\lambda, 1]$, and reaches a maximum at $\rho = \lambda$. Therefore, investors are least confident about funds' composite skills when the correlations of unexpected returns ρ and the correlations of composite skills λ are equal. As we see below, when $\rho = \lambda$, learning about a fund's composite skill is entirely based on its own performance. The uncertainty is therefore highest because of the lack of a second source of information.

[8] Using equations (18) and (6), we see that a is constant for any $\lambda \in [0, 1]$ when we set $\omega_F = \omega$.

The uncertainty about skills in the steady state declines as ρ deviates from λ. Investors are most certain about composite skills when either (i) noise in fund returns is highly correlated, and fund alphas depend solely on the skills of individual managers, or (ii) family resources are the dominant factor driving fund alphas, and noise in fund returns has low correlation. In the first case, an accurate estimate of one fund's composite skill is derived from it's own return together with the returns of the other fund, which are used to remove the noise in the first fund's returns. In the second case, the returns of both funds combined together provide a more precise estimate of the quality of the common resources of the family.

The conditional correlation of composite skills of the two funds is $\overline{v}_2/v_2$, which is a strictly increasing function of ρ and λ within the unit square. It equals ρ when $\lambda = \rho$, and approaches 1 when either ρ or λ approaches 1. When λ is close to 1.0, composite skills consist mainly of common family resources. Therefore investors' estimates of those skills are highly correlated. Alternatively when ρ is close to 1.0, the funds' non-systematic returns are highly correlated. The returns of one fund contain nearly the same information as those of the other fund. Again, estimates of composite skills are highly correlated.

Proposition 3. *When composite skills follow a random walk ($k = 0$), the matrix of sensitivity coefficients in the steady state of a two-fund family is:*

$$\mathbf{S}^* = \begin{bmatrix} s_2 & \overline{s}_2 \\ \overline{s}_2 & s_2 \end{bmatrix}, \tag{19}$$

where

$$s_2 = \frac{1}{2}a\left(\sqrt{\frac{\lambda+1}{\rho+1}} + \sqrt{\frac{1-\lambda}{1-\rho}}\right) \geq 0, \tag{20}$$

$$\overline{s}_2 = \frac{1}{2}a\left(\sqrt{\frac{\lambda+1}{\rho+1}} - \sqrt{\frac{1-\lambda}{1-\rho}}\right), \tag{21}$$

and a is defined in equation (18).

Proof. See Appendix A.3.

Coefficient s_2 is the sensitivity of the conditional mean of either fund's composite skill to its own unexpected return, and $\overline{s}_2$ is the sensitivity to the performance of the other fund. The direct sensitivity s_2 is positive as long as composite skills are stochastic (i.e., the coefficient a defined in equation (18) is not 0), which means that unexpectedly good performance by a fund raises beliefs about its composite skill. Furthermore, s_2 decreases with λ and increases with ρ, so that investors put more weight on a fund's own performance when there are high correlations of non-systematic returns within a family, and low correlation of true skills.

The cross-coefficient $\overline{s}_2$ is either positive, negative, or zero, depending on the relative sizes of ρ and λ. It decreases with ρ, and increases with λ. When $\rho = 0$, we obtain a measure of the pure common-skill effect, $\overline{s}_2 = \frac{1}{2}a\left(\sqrt{1+\lambda} - \sqrt{1-\lambda}\right)$, which is positive as long as $\lambda > 0$. Similarly, when $\lambda = 0$, we have $\overline{s}_2 = \frac{1}{2}a(\frac{1}{\sqrt{1+\rho}} - \frac{1}{\sqrt{1-\rho}})$, which measures the pure common-noise effect, and is negative as long as $\rho > 0$.[9] Between these two extreme cases, there is generally a mixture of both effects. $\overline{s}_2$ is positive when $\lambda > \rho$, suggesting that in the face of high correlation of skills and low correlation of non-systematic shocks to returns, the optimal estimate of the composite skill of one fund puts a positive weight on the performance of the second fund. This is evidence that the common-skill effect dominates the common-noise effect. Alternatively, when $\lambda < \rho$, the common-noise effect is dominant. Finally, when $\lambda = \rho$, the two effects completely offset each other, and the evaluation of a fund's skill is entirely based on its own performance.

The ratio $\overline{s}_2/s_2$ represents the sensitivity to the performance of the second fund relative to the sensitivity to the performance of the fund in question. It depends only on the correlation of true composite skills λ and the correlation of non-systematic shocks ρ, and is independent of other parameters.[10] Not surprisingly, this ratio increases with λ, and decreases with ρ, and is zero when $\lambda = \rho$. Furthermore, it is always smaller than one in absolute value, indicating less of a reaction to the second fund's performance than the reaction to own performance.

We summarize these results in the following corollary:

Corollary 1. *In the steady state of a two-fund family, we have: (1) whenever there is uncertainty about the composite skill, the direct sensitivity of the conditional mean belief to*

[9] Recall that $\lambda = 0$ occurs either if managers work independently, or if the quality of family resources stays constant while managers' skills vary over time.

[10] The other parameters $(\beta, \omega, \omega_F)$ in our model have no direct impact on the ratio $\overline{s}_2/s_2$, other than their impact through λ. This nice feature simplifies empirical tests of our model.

a fund's own unexpected performance is positive; (2) the ratio of the cross sensitivity to the direct sensitivity depends only on the instantaneous correlation of true composite skills and the correlation of noise in returns; (3) this ratio increases with the correlation of composite skills, and decreases with the correlation of noise in returns, and is zero when these two correlations are equal; and (4) this ratio is never greater than one in absolute value.

3.2 The Family with Many Funds

We now analyze the family with $n \geq 2$ funds. We first present two propositions for the limiting case in which $n \to \infty$. These results are simpler than those for finite families, but at the same time descriptive of large families. We then describe the impact of family size using the results for finite n. The general results for multi-fund families are fully derived in Appendices A.4 and A.5, where the matrices $\mathbf{V}^*$ and $\mathbf{S}^*$ are calculated analytically.

Proposition 4. *When composite skills follow a random walk ($k = 0$), the conditional variance of composite skills in the steady state of a n-fund family, v_n, has the limiting value:*

$$v \stackrel{def}{=} \lim_{n\to\infty} v_n = a\left(\sqrt{\lambda\rho} + \sqrt{(1-\lambda)(1-\rho)}\right). \tag{22}$$

the conditional covariance of skills of any pair of funds is $\overline{v}_n = v_n\phi_n$, where a is defined in equation (18), and ϕ is the limiting value of the conditional correlation given by

$$\phi \stackrel{def}{=} \lim_{n\to\infty} \phi_n = \begin{cases} 1, & \rho = 0, \quad \lambda = 1; \\ \frac{1}{1+\sqrt{\left(\frac{1}{\lambda}-1\right)\left(\frac{1}{\rho}-1\right)}} \in [0,1], & \textit{otherwise}. \end{cases} \tag{23}$$

Proof. See Appendix A.4.

As in the two-fund case, the conditional variance of composite skills for a large family, v, is a symmetric function of ρ and λ. The variance is greatest when $\rho = \lambda$, in which case $\phi = \rho = \lambda$, and $v = a$, and it declines as ρ and λ diverge. When $\rho = \lambda$, the common skill effect is offset by the common noise effect. Therefore, family performance does not improve the learning about the composite skill of an individual fund. As a result, investor uncertainty is highest. The conditional correlation in a large family, ϕ, also exhibits similar patterns as in the two-fund case, and is a symmetric function of ρ and λ. It increases with each parameter, and approaches one (zero) as either parameter goes to one (zero, respectively).

Equation (22) presents two interesting special cases in which skills are perfectly revealed. It shows that when $\rho = 0$ and $\lambda = 1$, or alternatively, $\lambda = 0$ and $\rho \to 1$, the uncertainty v is zero. The intuition is as follows. In the first case, alpha is entirely driven by family resources, and the skills of individual managers are irrelevant. Because the noise in fund returns are uncorrelated, the law of large numbers guarantees that the quality of family resources, although varying over time, is revealed perfectly as the number of funds goes to infinity. In the second case, managers work independently of family resources, but shocks to their fund returns are perfectly correlated. By assumption, manager skills vary independently, and by the law of large numbers, the average skill of all managers converges to the population mean, which is a known constant. As a result, the average return of funds in the family reveals perfectly the shocks, which are common to all funds. For any fund, the difference between it's returns and the average return reveals perfectly the skills of it's manager.

Proposition 5. *When composite skills follow a random walk ($k = 0$), in the steady state of an n-fund family, the sensitivity of the conditional mean m_i to a fund's own unexpected performance has the limiting value:*

$$s \stackrel{def}{=} \lim_{n\to\infty} s_n = v\frac{1-\phi}{1-\rho} \geq 0, \tag{24}$$

and the sensitivity of m_i to the unexpected family performance has the limiting value:

$$\overline{s} \stackrel{def}{=} \lim_{n\to\infty} \overline{s}_n\,(n-1) = \begin{cases} 0, & \rho = 0, \lambda = 0; \\ \infty, & \rho = 0, \lambda \neq 0; \\ v\frac{\phi-\rho}{\rho(1-\rho)}, & \text{otherwise.} \end{cases} \tag{25}$$

Proof. See Appendix A.5..

It is immediately clear that investors react strongly to past performance when they know little about composite skills. The direct coefficient s and the cross-coefficient $\overline{s}$ are proportional to the conditional variance v and therefore increase with uncertainty. The direct coefficient s is nonnegative. It differs from v only to the extent that ϕ differs from ρ. When

$\phi = \rho$, $s = v$. For a given v, the cross-coefficient $\bar{s}$ increases with the difference between ϕ and ρ, and it is zero if these values are equal.

The difference between ϕ and ρ depends on the spread between ρ and λ. For example, when $\lambda = \rho$, equation (23) gives $\phi = \rho = \lambda$, and equation (22) gives $v = a$. The conditional variance is at its largest possible value. The coefficient s is large, so investors put considerable weight on the past performance of a fund to judge its future performance. Yet $\bar{s} = 0$, and the average performance of the other funds, or the statistic $dW^{\mathcal{F}}_{-i,t}$ in equation (14), is irrelevant in judging the composite skill of a given fund. Therefore, as in the two-fund case, when $\lambda = \rho$, the common-noise effect completely offsets the common-skill effect.

Equation (25) also shows that if $\rho = 0$ and $\lambda \neq 0$, the cross-coefficient $\bar{s}$ explodes to infinity. This is not surprising, however. Because the noise in fund returns are uncorrelated, and managers' skills vary independently, as the number of funds goes to infinity, both the noise and the variations in individual managers' skills average out, thus allowing perfect learning of the quality of family resources.[11] Because the errors in the beliefs about individual managers' skills tend to cancel out, perfect knowledge about the quality of family resources implies that $dW^{\mathcal{F}}_{-i,t}$ goes to zero as the number of funds goes to infinity. As a result, the variance of $\bar{s}dW^{\mathcal{F}}_{-i,t}$ is finite even though $\bar{s}$ is not.

The general case of a finite family with $n \geq 2$ funds is solved analytically in Lemma 3 in Appendix A.5. Following these results, one learns that the ratio of the cross-sensitivity to direct sensitivity depends on three parameters: the two correlations λ and ρ, and the number of funds in the family n. We plot this ratio as a function of ρ (Panel A) and λ (Panel B) for alternative values of n in Figure 1. The solid line represents the case of a two-fund family. The other lines, each with increasingly steeper slopes, represent fund families with $n = 5$, $n = 20$, and the limiting case described in Proposition 5, respectively.

The figure confirms our analytical results. The ratio $\frac{\bar{s}_n(n-1)}{s}$ increases with λ and decreases with ρ. It is zero when $\lambda = \rho$. When $\lambda > \rho$, the common-skill effect dominates the common-noise effect, and the overall effect is positive. When $\lambda < \rho$, the opposite is true. The figure also provides some important new insights. The positive impact of λ and the negative

[11] However, the uncertainty about the composite skill remains as long as $\lambda < 1$, because idiosyncratic shocks to each fund's returns prevents perfect learning about individual manager's skills. This explains why v is positive when $\lambda < 1$.

impact of ρ on the ratio of sensitivity coefficients are progressively stronger as the size of the family grows, indicating that the optimal learning rule puts more weight on the unexpected performance of the fund family in evaluating a single fund for bigger families. For example, in a family with more than two funds, while the ratio of cross-sensitivity to direct sensitivity is never lower than -1, it can be substantially bigger than 1 when λ is high or ρ is low. This is because the average performance of a large family reveals the quality of the family resources more accurately, especially when ρ is small. In this case, investors would react to the family performance more strongly than to a fund's own performance, if family resources play an important role in the alpha generation, i.e., if λ is high.

We summarize the main properties of the sensitivities in the steady state of n-fund case as follows:

Corollary 2. *In the steady state of an n-fund family, we have: (1) the ratio of sensitivity coefficients, $\frac{\bar{s}_n(n-1)}{s_n}$, is determined by the instantaneous correlation of true composite skills, the correlation of noise in returns, and the number of funds in the family; (2) this ratio increases with the correlation of composite skills, and decreases with the correlation of noise, and is zero when these two correlations are equal; and (3) the impacts of these two correlations on the ratio of sensitivities increase with the number of funds in the family.*

3.3 Non-steady State and Mean-Reverting Skills

When the true composite skills follow a mean-reverting process, i.e., $k > 0$, the Riccati equation (9) does not have an analytical solution. It is a deterministic differential equation, though, so the conditional covariance matrix $\mathbf{V}_t$ and the sensitivity matrix $\mathbf{S}_t$ can be easily calculated numerically.

Panel A of Figure 2 shows the sensitivity of investors' conditional mean beliefs to unexpected fund performance, s_2, as a function of fund (and family) age. We fix the value of the instantaneous correlation of the true skills at $\lambda = 0.2$ by fixing the values of ω, ω_F, and β, and we vary the correlation of non-systematic shocks ρ in the figure. The highest curve corresponds to $\rho = 0.5$, and the lowest one $\rho = 0$. A prominent feature common to these results is that the sensitivity to own performance drops substantially in the first few years and then stabilizes at a steady-state value above zero. This is because high uncertainty about true composite skill leads to strong reaction to fund performance in the early stages of a fund family's life. Comparison of the curves shows that investors' belief respond more strongly to a fund's own unexpected performance as ρ increases, although the effect is almost invisible in some cases.

Panel B of Figure 2 shows the cross-sensitivity, $\bar{s}_2$, of conditional mean beliefs to surprises in the second fund's performance, under the same set of parameter values. In contrast to Panel A, the value of ρ increases successively from the highest curve to the lowest one. While the cross-sensitivities decrease in magnitude over time in all cases, they have different signs. When ρ is low relative to λ (the highest curve), $\bar{s}_2$ starts from a high value and converges to a positive steady-state value. When ρ is high relative to λ (the lowest curve), $\bar{s}_2$ increases over time and converges to a negative value. When $\rho = \lambda = 0.2$, $\bar{s}$ starts from a positive value and converges to zero.[12] Comparison of the curves indicates that at any given time, higher ρ is associated with a lower cross-sensitivity. These properties accord well with our analytical results in Section 3.1

To investigate the impact of λ on the sensitivities, we fix the value of ρ at 0.2, and vary the degree of reliance on family resources, β. For given volatilities of manager and family skills, an increase in β corresponds to a higher λ. The direct sensitivity, s_2, is shown in Panel A of Figure 3, and the cross-sensitivity, $\bar{s}_2$, is shown in Panel B. In Panel A, we see that the direct sensitivity s_2 declines over time, and it declines more when β (and as a result, λ) is high – the lower curves are associated with higher values of λ. In Panel B, we see an opposite pattern in the cross-sensitivity $\bar{s}_2$. At any given time (except the initial point, where the sensitivities are fixed exogenously by the *a priori* covariance matrix V_0 and the correlation ρ), a higher λ is alway associated with a higher $\bar{s}_2$, i.e., investors' mean beliefs respond more positively to the unexpected returns of the second fund.

Varying the mean-reverting rate k allows us to see that a higher speed of reversion reduces the absolute value of sensitivities to both own performance and family performance. This is not surprising, because it reduces the investor uncertainty about composite skill. Our key results that the cross-sensitivity increases with correlation of skills and decreases with

[12] The initial value of $\bar{s}$ depends also on initial covariance matrix $\mathbf{V}_0$.

correlation of noise in returns continue to hold, independent of the mean-reverting rate.

Numerical examples with more than two funds also suggest that both the positive impact of λ and the negative impact of ρ are stronger in larger families. We do not show these examples in the interest of conserving space.

4 Fund Performance and Fund Flows

As in Berk and Green (2004), we assume that investors provide capital to mutual funds competitively and without transaction costs. Active management may generate alpha, but the rents are captured by the mutual fund company. Investors direct assets toward funds with positive expected alpha, net of fees, and pull assets from funds with negative expected net alpha, and their evaluations are based on the information $\mathcal{F}_t$. In equilibrium, the size of fund i satisfies the condition $E(\alpha_{it}|\mathcal{F}_t) = f_{it}$ or, specifically:

$$A_{it} = \frac{1}{\gamma}\left(\frac{m_{it}}{\sigma_{it}} - \frac{f_{it}}{\sigma_{it}^2}\right). \tag{26}$$

A mutual fund family maximizes total income $\mathbf{f}'\mathbf{A}$. Thus, an optimal fee satisfies

$$\frac{f_{it}}{\sigma_{it}} = \frac{1}{2}m_{it}. \tag{27}$$

The ratio on the left-hand side is determined for each fund i in equilibrium, but neither the fee nor the non-systematic risk is unique. A fund may set a high fee, attract a low level of assets, and take large positions in mispriced assets. Or, it may set a low fee, attract a high level of investment, and stick closely to a benchmark portfolio. Provided that the fund's fee and non-systematic risk satisfy equation (27), the total fee income is the same in either case. For this reason, without loss of generality, we follow Dangl, Wu, and Zechner (2008) and set fees equal to the constant vector $\mathbf{f} = (f_i)$. Because $\sigma_{it} \geq 0$, equation (26) implies that a fund is viable, i.e., it has $A_{it} > 0$ and earns a positive fee, only if the expected composite skill m_t is positive. Otherwise, the fund is either reorganized or closed.

Equations (26) and (27) determine the equilibrium size of a fund. For $m_{it} > 0$, we have

$$A_{it} = \frac{m_{it}^2}{4\gamma f_i}.$$

Using Ito's lemma, we derive the asset growth rate of a viable fund as

$$\frac{dA_{it}}{A_{it}} = 2\frac{dm_{it}}{m_{it}} + \frac{(dm_{it})^2}{m_{it}^2}, \tag{28}$$

where m_{it} follows equation (14).[13]

Equation (28) demonstrates that a fund's asset growth rate reflects changes in investor beliefs about the composite skill of a fund. The increment dm_{it} is positive when information arrives that raises investors' estimates of the skill of fund manager i. This implies in turn that the assets of fund i increase. Similarly, all funds in the family grow when information arrives that raises investors' estimates of the quality of the common resources in the family. The second term represents a positive drift in the size of the fund, which is due to the convex relation between the assets and investors' mean beliefs about the composite skill of the fund.

We use fund flows, i.e., money coming into or out of a fund, to study the sensitivity coefficients of equation (14) in the empirical work below. The proportional fund flow equals the asset growth rate minus the realized fund return. We use fund flows instead of asset growth rate to better capture changes in fund size due to the active reallocations of mutual fund investors, and to be consistent with the extensive literature on the flow-performance relation. We regress fund flows on measured fund alphas. According to our theory, the expected alpha of a fund is zero in equilibrium. Therefore, any realized fund alpha represents a surprise that investors use to adjust their beliefs. We study the sensitivities of fund flows to fund and family alphas and describe their relation to the characteristics of funds and fund families. In particular, we provide empirical proxies for the correlations of non-systematic returns ρ and the correlations of composite skills λ. We show that the empirical relations are consistent with the predictions of our theory.

[13] Using equation (14), we calculate $(dm_i)^2 = \left[s_n^2 + \bar{s}_n^2 (n-1) + \rho (n-1) \left(2 s_n \bar{s}_n + (n-2) \bar{s}_n^2\right)\right] dt$.

5 Empirical Analysis

Below we provide empirical evidence on the key assumptions and predictions of our model. We test the assumptions:

A1: There is a family component in a fund's alpha-generating skill.

A2: The returns of funds within a family are subject to positively correlated non-systematic shocks.

We test the predictions:

P1: The spillover effect of the family performance on a member fund's flow increases with the correlation of true skills.

P2: The spillover effect decreases with the correlation of non-systematic shocks.

P3: The impacts of these two correlations on the spillover effect are amplified as the size of the family increases.

5.1 Data

We use the CRSP survivor-bias-free mutual fund database for our empirical tests. Our sample covers the period from January 1999 through December 2009.[14] We use Lipper investment objective codes to identify all equity funds categorized as: Growth; Growth and Income; Small-Cap; Mid-Cap; and Equity Income Funds.[15]

To attract investors with different preferences, a fund typically consists of multiple share classes, tied to the same underlying portfolio but differing in fee structure. We collect the monthly returns, total net asset values, annual expense ratios, and fund inception dates of each share class. We then identify all the share classes belonging to the same fund (i.e., portfolios) using the MFLINKS database provided by Wharton Research Data Services (WRDS). A fund's total net asset (TNA) is the sum of TNAs of all share classes. A

[14] The Lipper fund classification information begins in the year 1999. The database uses different classification systems for years prior to 1999. Furthermore, for most funds, the management company code, a data item we use to identify the fund family, begins in 1999.

[15] These are the five largest categories of actively managed U.S. domestic equity funds in terms of total net assets under management.

fund's return is the weighted average of returns of the share classes, where the weights are proportional to TNAs at the end of the previous month. Similarly, fund expense is the weighted average of expense ratios of all share classes. Fund age is measured in years and is the age of the oldest share class. Family age also is measured in years and is the age of the oldest fund in the family.

We exclude funds with fewer than 36 monthly return observations. The final sample consists of 2316 funds belonging to 653 fund families. The average number of monthly observations across funds is 100.

The fund flow for fund i in month t is the growth rate of TNA minus the rate of return on the fund's net asset:

$$Flow_{it} = \frac{TNA_{it} - TNA_{i,t-1}}{TNA_{i,t-1}} - R_{it}.$$

We winsorize the monthly flow, as well the expense ratio, at the 1st and 99th percentiles, although none of our results changes qualitatively without the winsorization.

Summary statistics are in Table 1.

5.2 Evidence of Family Effect in Fund Returns

A premise of our theory is that there is a family effect in fund performance. We assume the composite skill contains a common component for funds in the same family. We also assume that noise in returns is more highly correlated for two funds in the same family than for two funds chosen at random. Here we gauge the plausibility of these assumptions.

For each fund we estimate the Fama-French three-factor model and the Carhart (1997) four-factor model, and in this manner we estimate skill and luck components of fund returns. The skill component is alpha, which is the intercept of these models, and the residuals are the luck component, i.e., the temporary fluctuations due to random events. We calculate correlations of residuals for all pairs of funds and correlations of estimated alphas. We then analyze the family effect in these correlations using cross-sectional regressions.

For the four-factor model, we use the ordinary least squares method to estimate for each fund:

$$R_{it} - RF_t = \alpha_i + \beta_{iRMF} RMF_t + \beta_{iSMB} SMB_t + \beta_{iHML} HML_t + \beta_{iMOM} MOM + e_{it}. \quad (29)$$

where RF_t is the one-month Treasury bill rate in month t, RMF_t is the monthly equity market return in excess of the Treasury rate, SMB_t, HML_t, and MOM_t are the monthly returns on the mimicking portfolios for the size, book-to-market, and momentum factors, respectively.[16] The correlations of residuals are calculated for each pair of funds with at least 36 common months of observations. The procedure using the three-factor model is identical, except that the momentum factor is not included in the regression.

To calculate the correlations of alphas, we first estimate a time series of alphas for each fund using overlapping 36-month windows. Each time series provides as many as 97 observations of alphas. These are highly persistent because of the overlapping windows, so we fit a pooled first-order autoregressive model, i.e., an AR(1) model, and collect the errors. The correlations of the errors of the AR(1) model are calculated for each pair of funds with at least 36 common monthly observations.

The correlations of the residuals of the factor models for funds i and j, say, Rho_{ij}, represent the correlation of the noise in returns. The correlations of residuals of the AR(1) models for the estimated alphas, say, $Lambda_{ij}$, represent the degree to which fund managers rely on family resources to generate risk-adjusted performance. We ask if there is a family effect in these correlations.

Using the estimated correlations, we run the cross-sectional regressions:

$$Lambda_{ij} = \beta_F Family_{ij} + \beta_{Obj} Objective_{ij} + \beta_{Rho} Rho_{ij} + e_{ij},$$

$$Rho_{ij} = \beta_F Family_{ij} + \beta_{Obj} Objective_{ij} + \beta_{Lambda} Lambda_{ij} + e_{ij}.$$

The dummy variable $Family_{ij}$ is one if the funds are in the same family, and zero otherwise. Similarly, $Objective_{ij}$ is one if funds in a pair have the same investment objective, and zero otherwise.

[16] All the factor returns, as well as the Treasury rates, are obtained from the Ken French website at Dartmouth College.

The null hypothesis in each regression is $\beta_F = 0$, i.e., there is no family effect. The conservative alternative hypothesis is $\beta_F \neq 0$, although in each regression we expect $\beta_F > 0$ when our theory holds. In particular, $\beta_F > 0$ in the first regression indicates that the common resources of families drive average investment performance; this means that good performance by one fund tends to be accompanied by good performance by other funds in the same family. Similarly, $\beta_F > 0$ in the second regression indicates that fund managers in a family tend to tilt their portfolios in similar directions, and they do this with greater effect than two managers chosen at random.

We use Rho_{ij} as a control variable in the first regression, and $Lambda_{ij}$ in the second regression for two reasons. First, observational errors in these correlations are themselves correlated. Second, two funds may use similar investment strategies by chance, or if two funds are in the same family, they may adopt similar tilts due to the influence of family resources. In either case, the population values of Rho_{ij} and $Lambda_{ij}$ are correlated. Each model is estimated using ordinary least squares. We adjust the standard errors for clustering at fund levels.[17]

Table 2 reports our results for the three-factor and four-factor models. Results in both panels provide strong evidence of a family effect in the correlations. The coefficients of Model 1 in Panel A suggest that the correlation of alphas is 0.061 for two funds with different investment objectives and from different families. Two funds with the same investment objective have a correlation that is greater by 0.055. If the two funds are from the same family, the correlation increases further by 0.067. These effects are both statistically and economically significant.

Results for Model 2 demonstrate that $Lambda_{ij}$ is highly correlated with Rho_{ij} in the cross-section of funds. Nonetheless, the coefficient on the family dummy is highly significant, although smaller in size than in Model 1. This is expected; some of the family effect is captured indirectly by the correlations in residuals. The family effect on the Rho_{ij} is estimated in Models 3 and 4, and these results are similar to those of Models 1 and 2. The estimates obtained from the four-factor model reported in Panel B are remarkably similar to those of the three-factor model.

[17] Because a correlation involves two funds, we adjust for the two-dimensional clustering at both funds.

These results extend the findings of Baks (2003) and Elton, Gruber, and Green (2007) on the contribution of a family to fund performance. Pairs of funds in a family exhibit higher correlations in skill and luck components of returns estimated from standard factor models. This evidence provides a solid foundation for a theory in which a latent family component drives fund skill, and in which non-systematic returns are more highly correlated in families.

5.3 Spillovers in Mutual Fund Flows

We now present evidence that supports the predictions of our model. Our theory highlights two possible effects of family performance on the assessed composite skill of a member fund in the family: the positive common-skill effect due to the common resources provided by the family, and the negative common-noise effect due to the correlation of noise in fund returns. Our theory gives specific predictions about the size and direction of the overall effect as a function of family characteristics. We test these predictions by examining the sensitivities of mutual fund flows to a fund's own alpha and its family alpha, i.e., the average alpha of other funds in the family. As we have noted, fund flows are proxies for changes in investors' beliefs, and realized alphas represent signals about composite skill (because expected alphas are zero in equilibrium).

As we do in our theory, we treat funds within a fund family as homogeneous in their correlations of true composite skills and correlations of factor-model residuals. These correlations are family specific rather than fund specific. Correlations of alphas are proxies for the parameter λ, and correlations of residuals are estimates of the parameter ρ. A simple average of these pairwise correlations between funds is calculated for each family. Families that do not manage multiple funds simultaneously are excluded. As a result, our sample for the spillover analysis includes 1954 funds managed by 230 management companies.

We follow the Fama-MacBeth procedure, and run cross-sectional regressions quarter by quarter:

$$\begin{aligned}
Flow_{it} &= \beta_0 + \beta_1 Alpha_{i,t-1} + \beta_2 (Alpha_F)_{i,t-1} + \beta_3 TNA_{i,t-1} \\
&+ \beta_4 Age_{i,t-1} + \beta_5 Expense_{i,t-1} + \beta_6 StarFund_{i,t-1} \\
&+ \beta_7 (Lambda * Alpha)_{i,t-1} + \beta_8 (Lambda * Alpha_F)_{i,t-1} \\
&+ \beta_9 (Rho * Alpha)_{i,t-1} + \beta_{10} (Rho * Alpha_F)_{i,t-1} \\
&+ \beta_{11} (N * Lambda * Alpha)_{i,t-1} + \beta_{12} (N * Lambda * Alpha_F)_{i,t-1} \\
&+ \beta_{13} (N * Rho * Alpha)_{i,t-1} + \beta_{14} (N * Rho * Alpha_F)_{i,t-1} \\
&+ \beta_{15} (Age * Alpha)_{i,t-1} + \beta_{16} (Age * Alpha_F)_{i,t-1} \\
&+ \beta_{17} (Age_F * Alpha)_{i,t-1} + \beta_{18} (Age_F * Alpha_F)_{i,t-1} + e_{it}.
\end{aligned}$$

In this regression, $Flow_{it}$ is the fund flow received by fund i in quarter t; $Alpha$ is a fund's own alpha estimated from the 36 months ending at the end of the previous quarter; $Alpha_F$ is the average alpha of all other funds in the same family at the end of the previous quarter; TNA, Age, and Age_F are the natural logarithms of lagged total net asset value, fund age and family age, respectively; $Expense$ is the lagged expense ratio; $StarFund$ is a dummy equal to one if a fund's alpha is in the top 5 percentiles at the end of the previous quarter; $Lambda$ and Rho are the average correlations of alphas and of residuals for each fund family, respectively; and N is the natural logarithm of the number of funds in a fund family at the end of the previous quarter.

The model includes ten interaction terms of fund and family alphas with various fund and family characteristics, including $Lambda$ and Rho, N, Age, and Age_F. These terms are designed to measure the impact of those characteristics on the sensitivities of fund flows to fund and family performances. They are included to test the predictions **P1** and **P2** of our model. To test whether the impacts of the two correlations λ and ρ on the spillover effect grow with family size (**P3**), we further interact the number of funds N with the two interactions $Lambda * Alpha_F$ and $Rho * Alpha_F$. All the regressors, except the three performance measures ($Alpha$, $Alpha_F$, and $StarFund$) and the interaction terms, are adjusted by the median value of all funds, so that our results represent a median fund in the sample.

There are altogether 32 cross-sectional regressions. The time series averages of the coefficients are reported in Table 3, together with the t-statistics based on the standard error of the time series. The t-statistics are Newey-West-corrected for first-order autocorrelations in the estimated coefficients. The first three columns report the results when alphas and correlations are estimated from the three-factor model, while last three columns correspond to the four-factor model.

P1 and **P2** predict the signs of the coefficients of two interaction terms, $Lambda * Alpha_F$ and $Rho * Alpha_F$, i.e., β_8 and β_{10}, respectively. If investors learn optimally about fund skill using both the fund's performance and the family's performance, then $\beta_8 > 0$ (**P1**), and $\beta_{10} < 0$ (**P2**). The results provide strong support for these predictions. In all the six models, the estimates of β_8 are positive, and significant at least at the 5% level. The estimated values of β_{10} are all negative, however, and significant at least at the 5% level. This suggests that investors use family performance to update their beliefs about a particular fund. Furthermore, they do so in a manner consistent with rational learning: they account for both the common-skill effect and the common-noise effect.

The coefficient of family alpha, β_2, is positive and significant, suggesting that for fund families with median correlations of alphas and residuals, good family performance is a positive signal about the composite skill of a member fund. This is consistent with the spillover effect found in Nanda, Wang, and Zheng (2004). It suggests that for the median fund, the common-skill effect dominates the common-noise effect.

P3 predicts the signs of the coefficients β_{12} and β_{14} of the interaction terms: $N * Lambda * Alpha_F$ and $N * Rho * Alpha_F$, respectively. According to our model, both the positive effect of *Lambda* and the negative effect of *Rho* on the spillover get stronger as the number of funds in the family increases. This implies that the coefficient of the first term, β_{12}, is positive, and the coefficient of the second term, β_{14}, is negative. The results for the four-factor model (columns 5 and 6) strongly support these predictions. The coefficients have the expected signs and are significant at the 5% level. For the three-factor model, the coefficients have the expected signs but are not significant at standard levels. Investors pay more attention to family performance for large families, and they do so in a manner consistent with optimal learning.

The results also confirm some other important predictions of our theory. The coefficient on a fund's own lagged alpha is always positive and highly significant, as the theory suggests. The coefficient of the star fund dummy is also positive and significant, indicating that funds with extremely good performance attract disproportionately more fund flows. This is consistent with the convex flow-performance relation in our model. Furthermore, by examining the coefficients of the interaction terms of the fund's own alpha with the two correlation measures, we find that fund flows become more sensitive to a fund's own performance as the residuals are more correlated, and this sensitivity decreases as alphas are more correlated. This is also consistent with our theory.

Finally, Table 3 shows that the sensitivity of fund flows to a fund's own performance is negatively related to fund age, and positively related to family age (the coefficient β_{15} of $Age{*}Alpha$ is negative and significant, while the coefficient β_{17} of $Age_F{*}Alpha$ is positive and significant). The negative relation between fund age and the sensitivity to own performance is a natural outcome in the non-steady state of the theory. As the uncertainty about a fund's composite skill decreases over time, the sensitivity of the conditional mean belief to own performance also declines.[18]

The positive relation between family age and sensitivity to a fund's own performance seems surprising at first, but it is consistent with our theory. If a fund family is sufficiently well-established and has a long track record, there is less uncertainty about the family component of composite skill. When such a family introduces a new fund, the conditional covariance of the composite skills of the new fund with the rest of the family, i.e., the covariance term in the matrix V_t, is relatively small. As a result, the conditional mean belief is more sensitive to a fund's own performance, and less sensitive to the performance of other funds in the family.

Overall, the empirical results provide strong support for the key predictions of our theory of performance evaluation using family information.

[18]This also explains the negative coefficient of fund age in the table. Equation (28) shows that the expected fund flow increases with the sensitivities to fund and family performance. As those sensitivities decreases over time, the expected fund flows decline.

6 Conclusion

The risk-adjusted performance of a mutual fund depends not only on the expertise of the acting fund manager, but also on the resources of the family to which it belongs. Reliance on family resources produces a common component of the unobservable alpha-generating skill across funds in the same family. It also introduces correlations in the noise in returns of funds in the family. Both of these suggest that it can be helpful to take into account the performance of all funds within a family when we evaluate the composite skill of a single fund. We build on this idea, and develop a model that characterizes the optimal evaluation of a fund's skill based on its own performance and its family performance.

Our theoretical model highlights two potential impacts of one fund's performance on the assessed skill of another fund in the family. When one fund performs well, it indicates the quality of the common resource is high. This is good news about the composite skill of another fund. We call this positive effect the "common-skill effect." When a fund is doing well, this also suggests that it may have had some good luck. Due to the correlation of the noise in returns, we may also attribute a greater portion of another fund's performance to good luck. This is bad news about the composite skill of another fund. We call this negative effect the "common-noise effect." The overall effect depends then on the relative strength of these two opposite effects. Our theory shows that the overall effect increases with the correlation of composite skills, and decreases with the correlation of noise in fund returns, and that this pattern is stronger in larger families.

Our empirical analysis shows a strong family effect in both the estimated skill and luck components of mutual fund returns. Funds within the same family show economically and statistically stronger correlations in time-varying alphas and the residual returns. Using the correlation of the estimated alpha as a proxy for the correlation of composite skills, we find strong patterns in the mutual fund flow-performance relation that are consistent with our model predictions. The spillover effect of family performance on fund flow is positively related to the correlation of the time-varying alphas and negatively related to the correlation of residuals, and this pattern is stronger in larger families. Mutual fund investors appear to update their beliefs about fund skill in a way remarkably consistent with our theory of optimal learning.

We have focused on a Berk and Green (2004) type of equilibrium without any frictions. As a result, there is no predictability in mutual fund returns. Family performance information incorporated in the updating of beliefs is immediately reflected in fund flows. In practice, it is very likely that transaction costs in reallocating money across funds may lead to temporary deviations from such an equilibrium. In this case, family performance information is not only useful in predicting fund flows, but it also has some predictive power for fund returns. This is a fruitful avenue for future research.

A Appendix

A.1 Proof of Proposition 1

Equations (8)-(11) follow from Theorem 12.7 of Lipster and Shiryaev (2001), using (4) and (7) as the state and observation equations, respectively.

For the analytic solution to the Ricatti equation (9) given in equation (12), we assume that composite skills follow a random walk, i.e., $k = 0$ In the steady state, the covariance matrix of skills $\mathbf{V}^*$ is defined by the equation:

$$\mathbf{0}_{n\times n} = \mathbf{\Omega\Omega}' - \mathbf{V}^* \left(\mathbf{BB}'\right)^{-1} \mathbf{V}^*. \tag{A.1}$$

The solution to this equation is

$$\mathbf{V}^*= \mathbf{BD\Pi}^{1/2}\mathbf{D}'\mathbf{B}', \tag{A.2}$$

where $\mathbf{D}$ and $\mathbf{\Pi}$ are, respectively, the $n \times n$ matrix of orthonormal eigenvectors and the diagonal matrix of eigenvalues, say $\pi_i > 0$ of the symmetric, positive-definite matrix

$$\mathbf{D\Pi D}' = \mathbf{B^{-1}\Omega\Omega}'\mathbf{B}'^{-1}. \tag{A.3}$$

We write a general solution to equation (9) as

$$\mathbf{V}_t = \mathbf{B}\mathbf{P}_t\mathbf{B}' + \mathbf{V}^*, \tag{A.4}$$

where $\mathbf{P}_t$ is to be found. Substitution of (A.4) into equation (9) gives a homogeneous equation:

$$\frac{d\mathbf{P}_t}{dt} = -\mathbf{P}_t\mathbf{D}\mathbf{\Pi}^{1/2}\mathbf{D}' - \mathbf{D}\mathbf{\Pi}^{1/2}\mathbf{D}'\mathbf{P}_t - \mathbf{P}_t\mathbf{P}_t. \tag{A.5}$$

We solve for $\mathbf{P}_t$ indirectly. We define the matrix $\mathbf{Q}_t$ by

$$\mathbf{P}_t = \mathbf{D}\mathbf{Q}_t^{-1}\mathbf{D}'. \tag{A.6}$$

For any invertible $\mathbf{P}_t$, $\frac{d\mathbf{P}_t^{-1}}{dt} = -\mathbf{P}_t^{-1}\frac{d\mathbf{P}_t}{dt}\mathbf{P}_t^{-1}$. Thus, equation (A.5) implies that $\mathbf{Q}_t$ solves the linear equation

$$\frac{d\mathbf{Q}_t}{dt} = \mathbf{\Pi}^{1/2}\mathbf{Q}_t + \mathbf{Q}_t\mathbf{\Pi}^{1/2} + \mathbf{I}_{n\times n}. \tag{A.7}$$

Given a solution $\mathbf{Q}_t$, equations (A.2), (A.4), and (A.6) together lead to the solution for $\mathbf{V}_t$ in equation (9) in the text. A solution to equation (A.7) has the individual elements

$$Q_{ij}(t) = \begin{cases} e^{2\sqrt{\pi_i}t}Q_{ij}(0) + \frac{1}{2\sqrt{\pi_i}}\left(e^{2\sqrt{\pi_i}t} - 1\right), & i = j, \\ \\ e^{\left(\sqrt{\pi_i}+\sqrt{\pi_j}\right)t}Q_{ij}(0), & \text{otherwise.} \end{cases} \tag{A.8}$$

The initial values $Q_{ij}(0)$ are calculated as the solution to equation (A.4) for a given $\mathbf{V}_0$. For any funds i and j, $Q_{ij}(t) \underset{t\to\infty}{\longrightarrow} \infty$, provided $Q_{ij}(0) > 0$. Thus, $\mathbf{V}_t \underset{t\to\infty}{\longrightarrow} \mathbf{V}^*$.

A.2 Proof of Proposition 2

In the case of $n = 2$, we obtain $\mathbf{V}^*$ directly from equation (A.2). The matrices within this equation are

$$\mathbf{B} = \begin{bmatrix} 1 & 0 \\ \rho & \sqrt{1-\rho^2} \end{bmatrix}; \quad \mathbf{D} = \frac{1}{\sqrt{2}} \begin{pmatrix} -\sqrt{1-\rho} & \sqrt{1+\rho} \\ \sqrt{1+\rho} & \sqrt{1-\rho} \end{pmatrix};$$

$$\mathbf{\Pi}^{1/2} = a \begin{pmatrix} \sqrt{\frac{1-\lambda}{1-\rho}} & 0 \\ 0 & \sqrt{\frac{1+\lambda}{1+\rho}} \end{pmatrix}.$$

The product of these matrices in (A.2) is given by equations (15)-(17).

A.3 Proof of Proposition 3

The sensitivity coefficients s_2 and $\overline{s}_s$ in equations (19)-(21) are obtained by multiplying $\mathbf{V}^*$ in equation (15) by the matrix $(\mathbf{BB}')^{-1}$ for a two-fund family.

A.4 Proof of Proposition 4

Proposition 4 gives the conditional covariance matrix $\mathbf{V}^*$ of composite skills in the steady state for the limiting case as $n \longrightarrow \infty$. To prove the proposition, we first prove two lemmas for a family with a finite number of funds. Lemma 1 derives the matrix $(\mathbf{BB}')^{-1}$, and Lemma 2 derives the matrix $\mathbf{V}^*$. We then take the limit of these results in the proof of Proposition 4.

Lemma 1. *For all integers $n \geq 2$, the matrix $(\mathbf{BB}')^{-1}$ has homogeneous diagonal elements:*

$$b_n \overset{def}{=} b_{ii} = \frac{1 + (n-2)\,\rho}{1 + (n-2)\,\rho - (n-1)\,\rho^2}, \tag{A.9}$$

and homogeneous off-diagonal elements:

$$\overline{b}_n \overset{def}{=} b_{ij} = -\frac{\rho}{1 + (n-2)\,\rho - (n-1)\,\rho^2}. \tag{A.10}$$

Proof of Lemma 1. The proof proceeds by induction. For $n = 2$, equations (A.9) and (A.10) are checked directly. For $n > 2$, we assume that these equations characterize a family with n funds, and we demonstrate that the equations also hold for a family of $n+1$ funds.

To begin, let $\mathbf{P}$ be an $(n+1) \times (n+1)$ symmetric, non-singular, homogeneous correlation matrix. Partition it as

$$\mathbf{P} = \begin{bmatrix} \mathbf{M}_{n\times n} & \mathbf{r}_{n\times 1} \\ \mathbf{r}' & 1 \end{bmatrix}.$$

Matrices $\mathbf{M}$ and $\mathbf{P}$ represent $\mathbf{BB}'$ for families with n and $n+1$ funds respectively, where $\mathbf{BB}'$ is a matrix with unit elements on the main diagonal, and correlations $0 \leq \rho \leq 1$ off the diagonal. Thus, $\mathbf{r} = \rho \mathbf{1}_n$.

The inverse of $\mathbf{P}$ is

$$\mathbf{P}^{-1} = \begin{bmatrix} p\mathbf{M}^{-1} + \mathbf{M}^{-1}\mathbf{r}\mathbf{r}'\mathbf{M}^{-1} & -\mathbf{M}^{-1}\mathbf{r} \\ -\mathbf{r}'\mathbf{M}^{-1} & 1 \end{bmatrix} p^{-1}, \tag{A.11}$$

where $p = 1 - \mathbf{r}'\mathbf{M}^{-1}\mathbf{r}$. See page 966 in Greene (2008).

Evaluate p:

$$\begin{aligned} p &= 1 - \rho^2 \mathbf{1}_n' \mathbf{M}^{-1} \mathbf{1}_n = 1 - \rho^2 n b_n - \rho^2 n (n-1) \bar{b}_n \\ &= \frac{1 + (n-1)\rho - n\rho^2}{1 + (n-1)\rho}. \end{aligned} \tag{A.12}$$

The final term in (A.12) is b_{n+1}^{-1}, where b_n is defined by equation (A.9). This demonstrates that the last diagonal element of $\mathbf{P}^{-1}$ in equation (A.11) also satisfies equation (A.9). Consider

$$\begin{aligned} -p^{-1}\mathbf{M}^{-1}\mathbf{r} &= -p^{-1}\rho \left(b_n + (n-1)\bar{b}_n\right) \mathbf{1}_n \\ &= -p^{-1} \left(\frac{\rho}{1 + (n-1)\rho}\right) \mathbf{1}_n. \end{aligned} \tag{A.13}$$

The final term is $-\bar{b}_{n+1}\mathbf{1}_n$. Thus, the final column and the final row of $\mathbf{P}^{-1}$ in equation (A.11) satisfy equation (A.10). Finally, consider the upper left matrix in equation (A.11).

Using equations (A.9), (A.12), and (A.13), a diagonal element of the matrix is

$$\begin{aligned} \left[\mathbf{M}^{-1} + p^{-1}\mathbf{M}^{-1}\mathbf{r}\mathbf{r}'\mathbf{M}^{-1}\right]_{ii} &= b_n + p^{-1}\rho^2\left(b_n + (n-1)\,\overline{b}_n\right)^2 \\ &= \frac{1+(n-1)\,\rho}{1+(n-1)\,\rho - n\rho^2}. \end{aligned} \tag{A.14}$$

The final term in (A.14) is b_{n+1}, and this demonsrates that a diagonal element of $\mathbf{P}^{-1}$ satisfies equation (A.9). A similar evaluation completes the proof by demonstrating that off-diagonal elements satisfy equation (A.10). *Q.E.D.*

Lemma 2. *When composite skills follow a random walk (k=0), the steady-state conditional variance matrix* $\mathbf{V}^*$ *for a n-fund family has homogeneous diagonal element* v_n*, and homogeneous off-diagonal element* $v_n\phi_n$ *, where* ϕ_n *and* v_n *are given by:*

$$\phi_n = \frac{1}{a_{1,n}}\left(1 - \sqrt{1 - a_{0,n}a_{1,n}}\right), \tag{A.15}$$

$$v_n = \begin{cases} a\sqrt{\lambda}\sqrt{\frac{(\rho-1)(n\rho-\rho+1)}{\left(\rho\phi_n^2-\phi_n^2\right)n+\left(\rho-2\phi_n+2\phi_n^2-\rho\phi_n^2\right)}}, & \lambda \neq 0, \\ a\sqrt{\frac{(\rho-1)(n\rho-\rho+1)}{\left(\phi_n^2-2\rho\phi_n+\rho\right)n+\left(2\rho\phi_n-\phi_n^2-2\rho+1\right)}}, & \lambda = 0, \end{cases} \tag{A.16}$$

where

$$a_{1,n} \overset{def}{=} \frac{(n-1)(\lambda+\rho)-(n-2)}{(n-1)\,\rho\lambda+1}, \tag{A.17}$$

$$a_{0,n} \overset{def}{=} \frac{\rho+\lambda\left(\rho\,(n-2)+1\right)}{(n-1)\,\rho\lambda+1}, \tag{A.18}$$

and a *is defined in equation (18) of the text.*

Proof of Lemma 2. Consider two representative equations that appear within the matrix equation (A.1). One of these equations defines each element of the main diagonal of (A.1). This is

$$a^2 = v_n^2 b_n\left(1+(n-1)\,\phi_n^2\right) + \overline{b}_n v_n^2\phi_n\,(n-1)\left(2+(n-2)\,\phi_n\right), \tag{A.19}$$

where b_n and $\overline{b}_n$ are given by Lemma 1. Similarly, each off-diagonal element is defined by

$$a^2\lambda = v_n^2\phi_n b_n\left(2+(n-2)\,\phi_n\right) + \overline{b}_n v_n^2\left(\left(1+(n-2)\,\phi_n\right)^2 + (n-1)\,\phi_n^2\right). \tag{A.20}$$

These equations identify v_n and ϕ_n. From equation (A.20) we obtain

$$0 = v_n^2 - a^2 \lambda \frac{(\rho - 1)(n\rho - \rho + 1)}{\left(\rho\phi_n^2 - \phi_n^2\right) n + \left(\rho - 2\phi_n + 2\phi_n^2 - \rho\phi_n^2\right)}, \tag{A.21}$$

if $\lambda \neq 0$, while if $\lambda = 0$ we get from equation (A.19)

$$0 = v_n^2 - a^2 \frac{(\rho - 1)(n\rho - \rho + 1)}{\left(\phi_n^2 - 2\rho\phi_n + \rho\right) n + \left(2\rho\phi_n - \phi_n^2 - 2\rho + 1\right)}. \tag{A.22}$$

These results are equivalent to the two expressions in equation (A.16). (Equations (A.21) and (A.22) lead to equivalent solutions when $\lambda \neq 0$, but when $\lambda = 0$, we must use equation (A.22) to define v_n. However, it is convenient to use equation (A.21) when $\lambda \neq 0$.)

We take the ratios of left- and right-hand sides of the equations (A.19) and (A.20) and set them equal. This eliminates v_n^2, and after some algebra we get

$$0 = a_{1,n}\phi_n^2 - 2\phi_n + a_{0,n}, \tag{A.23}$$

where $a_{0,n}$ and $a_{1,n}$are given by equations (A.17) and (A.18). This equation identifies the correlation ϕ_n. It has two roots, but only the root in (A.15) is sensible. Numerical evaluation of the second root for $n \geq 2$ gives values $\phi_n > 1$, which is inconsistent with the definition of a correlation. *Q.E.D.*

To prove Proposition 4, we study the limits of the equations in Lemma 2 and in its proof as $n \to \infty$. We first demonstrate the limiting results for ϕ_n, taking several cases separately, and then demonstrate the results for v_n, again using several cases.

Suppose $\rho = 0$ and $\lambda = 1$. Then $a_{1,n} = a_{0,n} = 1$, using equations (A.18) and (A.17). The solution to equation (A.23) is $\phi_n = 1$ for all n, and this implies $\phi = 1$. This is the first part of equation (23).

Suppose $\rho = 0$ and $\lambda \neq 1$. Equations (A.18) and (A.17) give $a_{0,n} = \lambda$ for all n, and

$$a_{1,n} = (\lambda - 1) n + (2 - \lambda),$$

respectively. The solution in equation (A.15) gives

$$\phi_n \sqrt{n} = \frac{1/\sqrt{n} - \sqrt{1/n - \lambda(\lambda-1) - \lambda(2-\lambda)/n}}{(\lambda-1) + (2-\lambda)/n} \underset{n\to\infty}{\longrightarrow} \frac{\sqrt{\lambda}}{\sqrt{1-\lambda}}. \tag{A.24}$$

This implies that $\phi_n \underset{n\to\infty}{\longrightarrow} 0$, which is a result given by taking the limit of the second part of equation (23) as $\rho \to 0$. (It is sufficient here to show that $\phi_n \underset{n\to\infty}{\longrightarrow} 0$; however, equation (A.24) is used in the limiting results for v_n below.)

Suppose $\lambda = 0$. Recognizing that $a_{0,n}$ and $a_{1,n}$ are symmetric functions of ρ and λ, we have $\phi_n\sqrt{n} \underset{n\to\infty}{\longrightarrow} \frac{\sqrt{\rho}}{\sqrt{1-\rho}}$ and $\phi_n \underset{n\to\infty}{\longrightarrow} 0$ by same argument used to demonstrate equation (A.24). This result is given by taking the limit of the the second part of equation (23) as $\lambda \to 0$.

The final case is $\lambda \neq 0$ and $\rho \neq 0$. Then $a_{1,n} \underset{n\to\infty}{\longrightarrow} a_1 \overset{def}{=} 1 - \left(\frac{1}{\lambda} - 1\right)\left(\frac{1}{\rho} - 1\right)$ and $a_{0,n} \underset{n\to\infty}{\longrightarrow} 1$, using equations (A.17) and (A.18), respectively. Then the solution in equation (A.15) has the limit

$$\phi_n \underset{n\to\infty}{\longrightarrow} \frac{1 - \sqrt{1-a_1}}{a_1} = \frac{1}{1+\sqrt{1-a_1}},$$

which is equivalent to the second part of equation (23). This completes all cases for the limiting values of ϕ_n.

Now consider the limiting values of v_n. Suppose $\rho = 0$. Equation (A.21) gives

$$v_n = a\sqrt{\lambda}\sqrt{\frac{1}{\phi_n^2 n + 2\phi_n - 2\phi_n^2}} \underset{n\to\infty}{\longrightarrow} a\sqrt{1-\lambda},$$

using the result in (A.24). Suppose $\lambda = 0$. Equation (A.22) gives

$$v_n = a\sqrt{\frac{(\rho-1)(n\rho - \rho + 1)}{\left(\phi_n^2 - 2\rho\phi_n + \rho\right)n + \left(2\rho\phi_n - \phi_n^2 - 2\rho + 1\right)}} \underset{n\to\infty}{\longrightarrow} a\sqrt{1-\rho},$$

using the result that $\phi_n\sqrt{n} = \underset{n\to\infty}{\longrightarrow} \frac{\sqrt{\rho}}{\sqrt{1-\rho}}$ demonstrated above. If $\lambda \neq 0$ and $\rho \neq 0$, the limit of equation (A.21) gives

$$v_n \underset{n\to\infty}{\longrightarrow} a\frac{\sqrt{\lambda\rho}}{\phi},$$

recognizing that $\phi \neq 0$ in this case, where ϕ is given by equation (23). Each of these three cases gives a result equivalent to equation (22). *Q.E.D.*

A.5 Proof of Proposition 5

Proposition 5 gives the steady-state matrix of sensitivity coefficients $\mathbf{S}^*$ in the limiting case as $n \longrightarrow \infty$. To prove this proposition, we first derive the sensitivity matrix $\mathbf{S}^*$ for families with $n \geq 2$ funds in Lemma 3. We then take the limit of the results in the lemma to prove the proposition.

Lemma 3. *When composite skills follow a random walk (k=0), the sensitivity matrix* $\mathbf{S}^*$ *in the steady state of a family with* $n \geq 2$ *funds is homogeneous. Each diagonal element is*

$$s_n \overset{def}{=} s_{ii} = v_n \left(b_n + (n-1) \bar{b}_n \phi_n \right), \tag{A.25}$$

and each off-diagonal element is

$$\bar{s}_n (n-1) \overset{def}{=} s_{ij} (n-1) = v_n \frac{\phi_n - \rho}{1-\rho} \frac{n-1}{n\rho - \rho + 1}, \tag{A.26}$$

where v_n *and* ϕ_n *are given by Lemma 2, and* b_n *and* $\bar{b}_n$ *are given in Lemma 1.*

Proof of Lemma 3. The sensitivity coefficients s_n and $\bar{s}_n (n-1)$ in equations (A.25) and (A.26), respectively, are obtained by calculating individual elements of $\mathbf{S}^* = \mathbf{V}^* (\mathbf{BB}')^{-1}$. The on- and off-diagonal elements of $\mathbf{V}^*$, i.e., v_n and $v_n \phi_n$, respectively, are given by Lemma 2, while the on- and off-diagonal elements of $(\mathbf{BB}')^{-1}$, i.e., b_n and $\bar{b}_n$, respectively, are given by Lemma 1. *Q.E.D.*

To prove Proposition 5, we substitute the values of v_n and ϕ_n from equations (A.16) and (A.15), respectively, and the values of b_n and $\bar{b}_n$ from equations (A.9) and (A.10), into equations (A.25) and (A.26). The limiting result for s_n in equation (24) follows immediately by letting $n \longrightarrow \infty$. For the limiting results $\bar{s}_n (n-1)$ in equation (25) we consider three cases.

Suppose $\rho = 0$ and $\lambda = 0$. Then $\phi_n = 0$ and $v_n = a$ for all n,using equations (A.15) and (A.16), respectively. As a consequence, $\bar{s}_n (n-1) = 0$ for all n, and therefore the first part of equation (25) obtains. Suppose $\rho = 0$ and $\lambda \neq 0$. Then $\phi_n \sqrt{n} \underset{n \to \infty}{\longrightarrow} \frac{\sqrt{\lambda}}{\sqrt{1-\lambda}}$, from equation

(A.24) above, and $v_n \underset{n\to\infty}{\longrightarrow} a\sqrt{1-\lambda}$. Therefore,

$$\overline{s}_n (n-1) \underset{n\to\infty}{\longrightarrow} \infty,$$

which is the second part of equation (25). Finally, suppose $\rho \neq 0$ and $\lambda \neq 0$. Then v_n and ϕ_n take the limiting values given in Proposition 4. Therefore,

$$v_n \frac{\phi_n - \rho}{1-\rho} \frac{n-1}{n\rho - \rho + 1} \underset{n\to\infty}{\longrightarrow} v \frac{\phi - \rho}{\rho (1-\rho)},$$

which is the final part of equation (25).

References

Aragon, George, and Wayne Ferson, 2006, Portfolio Performance Evaluation, *Foundations and Trends in Finance* 2, 83–190.

Baks, Klaas, 2003, On the Performance of Mutual Fund Managers, Working paper, Emory University.

Berk, Jonathan, and Richard Green, 2004, Mutual Fund Flows and Performance in Rational Markets, *Journal of Political Economy* 112, 1269–1295.

Bhattacharya, Utpal, Jung Hoon Lee, and Veronica Krepely Pool, 2010, Conflicting Family Values in Mutual Fund Families, Working paper, Indiana University Bloomington.

Carhart, Mark, 1997, On Persistence in Mutual Fund Performance, *Journal of Finance* 52, 57–82.

Cohen, Randolph, Joshua Coval, and Lubos Pastor, 2005, Judging Fund Managers by the Company they Keep, *Journal of Finance* 60, 1057–1096.

Dangl, Thomas, and Youchang Wu, 2010, Corporate Investment Under Uncertain Business Cycles, Working paper, Vienna University of Technology.

Dangl, Thomas, Youchang Wu, and Josef Zechner, 2008, Market Discipline and Internal Governance in the Mutual Fund Industry, *Review of Financial Studies* 21, 2307–2343.

Elton, Edwin J., Martin J. Gruber, and T. Clifton Green, 2007, The Impact of Mutual Fund Family Membership on Investor Risk, *Journal of Financial and Quantitative Analysis* 42, 257–278.

Gervais, Simon, Anthony Lynch, and David Musto, 2005, Fund Families as Delegated Monitors of Money Managers, *Review of Financial Studies* 18, 1139–1169.

Gibbons, Robert, and Kevin J. Murphy, 1990, Relative Performance Evaluation for Chief Executive Officers, *Industrial and Labor Relations Review* 43, 30S–51S.

Greene, William H., 2008, *Econometric Analysis*. (Pearson Prentice Hall, Upper Saddle River, New Jersey) 6th edn.

Holmstrom, Bengt, 1982, Moral Hazard in Teams, *Bell Journal of Economics* 13, 324–340.

Huang, Jennifer, Kelsey Wei, and Hong Yan, 2007, Participation Costs and the Sensitivity of Fund Flows to Past Performance, *Journal of Finance* 62, 1273–1311.

Jones, Christopher, and Jay Shanken, 2005, Mutual Fund Performance with Learning across Funds, *Journal of Financial Economics* 78, 507–552.

Khorana, Ajay, and Henri Servaes, 2011, What Drives Market Share in the Mutual Fund Industry, *Review of Finance* forthcoming.

Lipster, Robert S., and Albert N. Shiryaev, 2001, *Statistics of Random Process II*. (Springer-Verlag Berlin, Heidelberg, New York).

Lynch, Anthony W., and David K. Musto, 2003, How investors interpret past fund returns, *Journal of Finance* 58, 2033–2058.

Mamaysky, Harry, and Matthew Spiegel, 2002, A Theory of Mutual Funds: Optimal Fund objectives and Industry Organization, Working paper, Yale School of Management.

Massa, Massimo, 2003, How Do Family Strategies Affect Fund Performance? When Performance-Maximization is not the Only Game in Town, *Journal of Financial Economics* 67, 249–304.

Massa, Massimo, Jose-Miguel Gaspar, and Pedro Matos, 2006, Favoritism in Mutual Fund Families? Evidence of Strategic Cross-Fund Subsidization, *Journal of Finance* 61, 73–104.

Nanda, Vikram, Jay Wang, and Lu Zheng, 2004, Family Values and the Star Phenomenon: Strategies of Mutual Fund Families, *Review of Financial Studies* 17, 667–698.

Pastor, Lubos, and Robert Stambaugh, 2002, Mutual Fund Performance and Seemingly Unrelated Assets, *Journal of Financial Economics* 63, 313–349.

Pomorski, Lukasz, 2009, Acting on the Most Valuable Information: "Best Idea" Trades of Mutual Fund Managers, Working paper, University of Toronto.

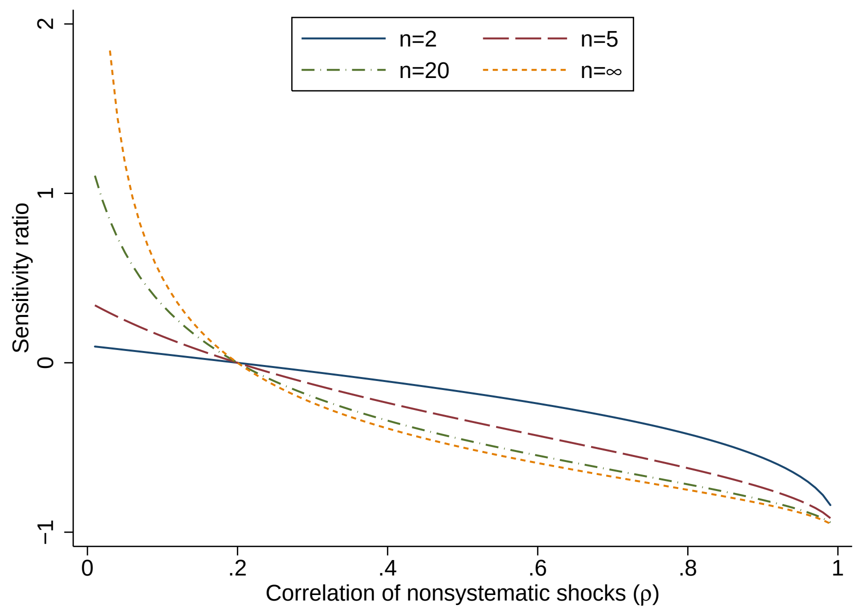
Ruenzi, Stefan, and Alexander Kempf, 2008, Tournaments in Mutual Fund Families, *Review of Financial Studies* 21, 1013–1036.

Sialm, Clemens, and T. Mandy Tham, 2011, Spillover Effects in Mutual Fund Companies, Working paper, University of Texas at Austin.

Sirri, Erik R., and Peter Tufano, 1998, Costly Search and Mutual Fund Flows, *Journal of Finance* 53, 1589–1622.

Warner, Jerold B., and Joanna Shuang Wu, 2011, Why Do Mutual Fund Advisory Contracts Change? Performance, Growth, and Spillover Effects, *Journal of Finance* 66, 271–306.

Panel A. Sensitivity ratio as a function of ρ



Panel B. Sensitivity ratio as a function of λ

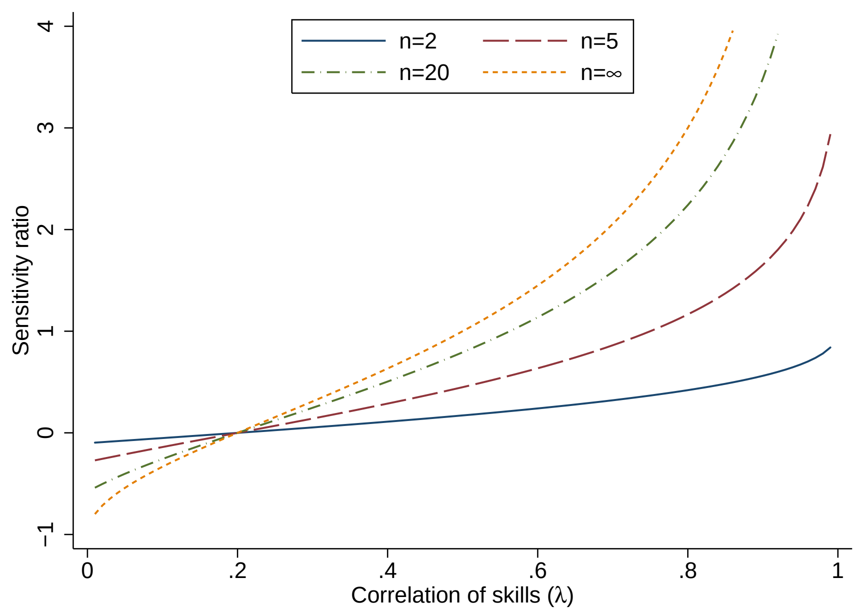
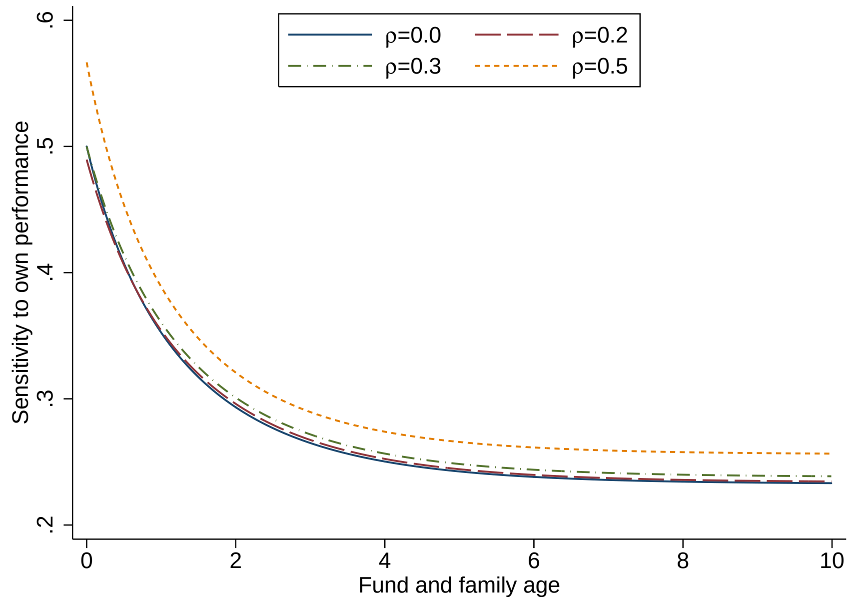


Figure 1: **Ratio of cross-sensitivity to direct sensitivity.** Plotted here is the ratio $\frac{\bar{s}_n(n-1)}{s_n}$, where s_n and $\bar{s}_n(n-1)$ are the sensitivities of the conditional mean belief to a funds own unexpected return and the average unexpected return of other funds in the same family, respectively. n is the number of funds in the family. The ratio is plotted as a function of ρ, which is the correlation of non-systematic shocks in Panel A, and λ, which is the correlation of increments in composite skills in Panel B. The solid line represents a family with two funds, the dashed line represents a family of five funds, the dot-dashed line represents a family of 20 funds, while the dotted line represents the limiting case when the number of funds in a family goes to infinity. $\lambda = 0.2$ in Panel A, $\rho = 0.2$ is Panel B.

Panel A. Sensitivity to own performance (s_2)



Panel B. Sensitivity to family performance ($\bar{s}_2$)

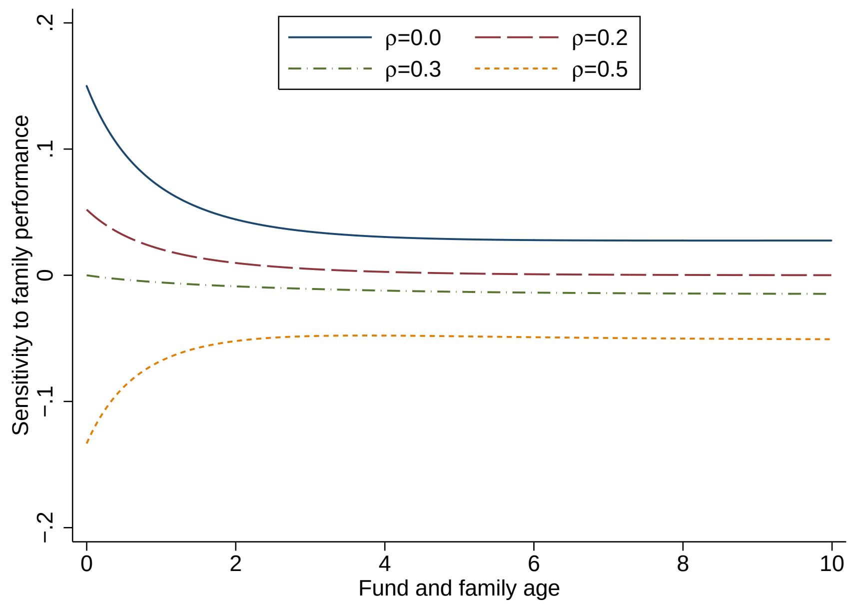
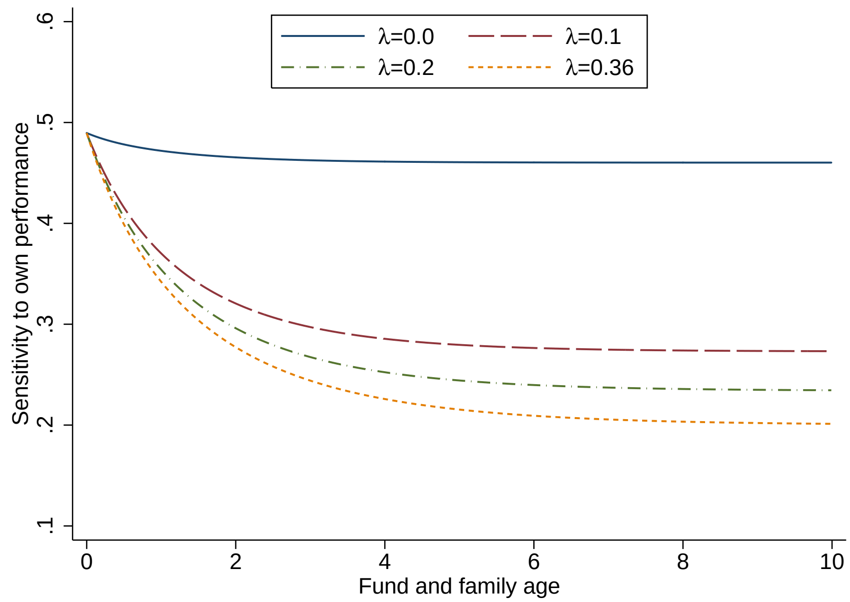


Figure 2: **Sensitivities in the non-steady state: Impact of ρ.** Panel A shows the sensitivity of conditional mean belief to a fund's own performance, s_2, as a function of fund/family age in a two-fund family. Panel B shows the sensitivity to family performance, $\bar{s}_2$. The four curves in each panel represent the cases with $\rho = 0, \rho = 0.2, \rho = 0.3$, and $\rho = 0.5$, respectively. The other parameter values are as follows: $V_{11}(0) = 0.5$, $V_{12}(0) = 0.15$, $k = 0.05$, $\omega = 0.5$, $\omega_F = 0.25$, $\beta = 0.5$. The instantaneous correlation of true composite skills, λ, implied by these values is 0.2.

Panel A. Sensitivity to own performance (s_2)



Panel B. Sensitivity to family performance ($\bar{s}_2$)

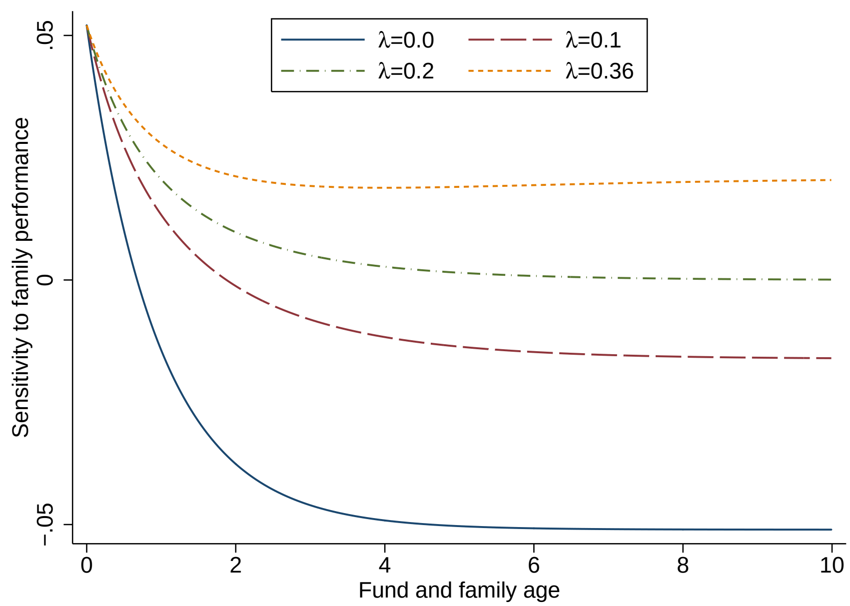


Figure 3: **Sensitivities in the non-steady state: Impact of λ.** Panel A shows the sensitivity of conditional mean belief to a fund's own performance, s_2, as a function of fund/family age in a two-fund family. Panel B shows the sensitivity to family performance, $\bar{s}_2$. The four curves in each panel represent the cases with $\lambda = 0(\beta = 0), \lambda = 0.1(\beta = 0.4), \lambda = 0.2(\beta = 0.5)$, and $\lambda = 0.36(\beta = 0.6)$, respectively. The other parameter values are as follows: $V_{11}(0) = 0.5$, $V_{12}(0) = 0.15$, $k = 0.05$, $\omega = 0.5$, $\omega_F = 0.25$, and $\rho = 0.2$.

Table 1: **Summary statistics**

This table shows the summary statistics of our mutual fund sample over January 1999 through December 2009. We start with all equity fund share classes in the database in five major fund categories according to Lipper investment objective codes: (1) Growth Funds (G); (2) Growth and Income Funds (GI); (3) Equity Income Funds (EI); (4) Small-Cap Fund (SG); (5) Mid-Cap Funds (MC). We collect the monthly returns, total net asset values, annual expense ratios, and fund inception dates of all share classes. We then use the MFLINKS database provided by WRDS to identify all the share classes belonging to the same underlying portfolio (i.e. the fund). A fund's total net asset (TNA) is computed by aggregating across the share classes. Fund return is the weighted average of returns on the share classes, where the weights are determined by the total net asset value at the end of the previous month. Fund expense is the asset-weighted average of expense ratios of all share classes. Fund age (in years) is measured by the age of the oldest share class. Family age (also in years) is measured by the age of the oldest fund in the family. Fund flow is defined as the growth rate of TNA minus the fund's rate of return. Fund flow and expense ratio are winsorized at the 1st and 99th percentiles. We exclude funds that have fewer than 36 monthly return observations. The final sample consists of 2316 funds belonging to 653 fund families. Each fund on average has 100 monthly return observations.

Variable	**Mean**	**Std. Dev.**	**Number of Obs.**
Monthly TNA (million dollar)	1227.223	5058.833	235271
Monthly return (%)	0.346	5.655	232510
Monthly fund flow (%)	0.715	6.022	232510
Annual expense ratio (%)	1.261	0.469	232731
Fund age (year)	12.116	13.1	235271
Family age (year)	30.677	24.109	235271
Number of fund per family	4.827	6.771	653

Table 2: **Family effect in fund returns**

This table shows the family effect in estimated alpha and residual returns. *Lambda* and *Rho* are the pairwise correlations in *alpha* innovations and in residuals, respectively; *Family* is a dummy variable equal to one if both funds in a pair are from the same family, and zero otherwise; *Objective* is a dummy variable equal to one if both funds in a pair have the same investment objective, and zero otherwise. For each pair of funds, *Rho* is calculated using the time series of residuals estimated from a factor model; *Lambda* is calculated using the innovation terms in an AR(1) model of fund alphas. In the latter case, the time series of alphas for each fund are estimated using rolling windows of 36 months. In Panel A, the correlations are estimated based on the Fama-French three factor model, while in Panel B they are estimated using the Carhart four-factor model. The reported coefficients of the cross-sectional correlation regressions are estimated using OLS, and the t-statistics are adjusted for clustering errors at fund levels.

Panel A. Using Fama-French model as benchmark

	(1) *Rho*	(2) *Rho*	(3) *Lambda*	(4) *Lambda*
Family	0.067***	0.016***	0.072***	0.010***
	(16.79)	(6.70)	(15.05)	(3.91)
Objective	0.055***	0.013***	0.062***	0.009***
	(17.87)	(8.52)	(16.51)	(5.75)
Lambda		0.697***		
		(156.69)		
Rho				0.942***
				(132.31)
Constant	0.061***	0.043***	0.042***	-0.026***
	(32.86)	(26.65)	(24.77)	(-17.12)
Observations	2397543	1535752	1535752	1535752
R^2	0.012	0.661	0.012	0.661

Panel B. Using Carhart four-factor model as benchmark

	(1) *Rho*	(2) *Rho*	(3) *Lambda*	(4) *Lambda*
Family	0.065***	0.018***	0.069***	0.017***
	(16.56)	(6.84)	(15.80)	(7.56)
Objective	0.051***	0.014***	0.057***	0.015***
	(17.88)	(8.86)	(17.38)	(10.01)
Lambda		0.663***		
		(123.68)		
Rho				0.814***
				(124.98)
Constant	0.068***	0.045***	0.051***	-0.013***
	(34.63)	(25.81)	(28.50)	(-8.47)
Observations	2397543	1535752	1535752	1535752
R^2	0.013	0.546	0.014	0.545

* p<0.1, ** p<0.05, *** p<0.01

Table 3: **Responses of fund flows to fund and family performances**

This table shows the results of Fama-MacBeth regressions of quarterly fund flows on various explanatory variables. *Alpha* is a fund's own alpha estimated from the 36 months ending at the end of the previous quarter; $Alpha_F$ is the average alpha of all other funds in the same family at the end of the previous quarter; TNA, *Age*, and Age_F are the natural logarithms of lagged total net asset value, fund age, and family age, respectively; *Expense* is the lagged expense ratio; $StarFund$ is a dummy equal to one if a fund's alpha is in the top 5 percentiles at the end of the previous quarter; *Lambda* and *Rho* are the average correlations in alpha innovations and in residuals estimated for each fund family; and N is the natural logarithm of the number of funds in the family at the end of the previous quarter. The model also includes ten interaction terms of fund and family alphas with various fund or family characteristics, including *Lambda* and *Rho*, N, *Age*, and Age_F. All the regressors, except for the three performance measures and the interaction terms, are adjusted by the median value of the sample. The first three columns report the results when alphas and correlations are estimated from the three-factor model, while last three columns report those corresponding to the four-factor model. The t-statistics are Newey-West-corrected for first-order autocorrelation in the estimated coefficients.

	three-factor model			four-factor model		
	(1)	(2)	(3)	(4)	(5)	(6)
Alpha	3.045***	3.063***	3.104***	3.144***	3.149***	3.215***
	(12.40)	(12.37)	(11.93)	(12.73)	(12.75)	(12.73)
$Alpha_F$	0.629***	0.623***	0.651***	0.805***	0.801***	0.854***
	(4.76)	(4.93)	(5.15)	(4.99)	(4.95)	(5.37)
TNA	-0.003***	-0.004***	-0.003***	-0.004***	-0.004***	-0.003***
	(-4.89)	(-4.99)	(-5.00)	(-4.41)	(-4.58)	(-4.60)
Age	-0.017***	-0.017***	-0.019***	-0.017***	-0.017***	-0.019***
	(-10.06)	(-10.02)	(-7.09)	(-9.57)	(-9.67)	(-7.02)
Expense	-0.978***	-1.009***	-1.012***	-0.851***	-0.889***	-0.878***
	(-4.48)	(-4.46)	(-4.49)	(-4.21)	(-4.05)	(-4.08)
StarFund	0.025***	0.024***	0.023***	0.026***	0.026***	0.026***
	(3.13)	(3.06)	(3.00)	(3.84)	(3.79)	(3.86)
$Lambda * Alpha$	-2.305*	-0.733	-1.070	-1.612	-0.768	-1.217
	(-1.91)	(-0.45)	(-0.74)	(-1.13)	(-0.39)	(-0.68)
$Lambda * Alpha_F$	2.064**	4.097**	5.219***	2.538**	9.826***	9.858***
	(2.36)	(2.39)	(2.99)	(2.10)	(2.99)	(3.15)
$Rho * Alpha$	3.009**	0.886	1.252	1.787	0.072	0.742
	(2.37)	(0.53)	(0.76)	(1.22)	(0.04)	(0.44)
$Rho * Alpha_F$	-2.921***	-5.353*	-6.805**	-2.829**	-9.169***	-9.580***
	(-2.76)	(-2.00)	(-2.47)	(-2.40)	(-2.77)	(-3.11)
$N * Lambda * Alpha$		2.531*	2.379		0.889	0.514
		(1.72)	(1.65)		(0.60)	(0.39)
$N * Lambda * Alpha_F$		1.760	2.457		6.232**	5.964**
		(1.12)	(1.54)		(2.71)	(2.70)
$N * Rho * Alpha$		-3.263**	-3.571**		-1.886	-1.702
		(-2.20)	(-2.13)		(-1.32)	(-1.31)
$N * Rho * Alpha_F$		-1.880	-2.597		-5.153**	-4.955**
		(-0.84)	(-1.11)		(-2.36)	(-2.46)
$Age * Alpha$			-0.985***			-0.961***
			(-4.02)			(-4.16)
$Age * Alpha_F$			-0.245			-0.287
			(-1.25)			(-1.47)
$Age_F * Alpha$			0.481**			0.405**
			(2.62)			(2.39)
$Age_F * Alpha_F$			-0.148			-0.183
			(-0.43)			(-0.57)
Constant	0.014***	0.014***	0.014***	0.015***	0.015***	0.015***
	(2.78)	(2.79)	(2.82)	(2.99)	(3.00)	(3.07)
Observations	41109	41109	41109	41109	41109	41109
R^2	0.077	0.078	0.076	0.079	0.079	0.078

* p<0.1, ** p<0.05, *** p<0.01