Financing Investment with Long-Term Debt and Uncertainty Shocks*

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Abstract

We extend the quantitative corporate finance framework of Hennessy and Whited (2005) by introducing long-term defaultable debt and stochastic volatility. These features lead to significantly lower leverage and higher default probabilities, and a stronger negative correlation of investment with credit spreads, consistent with the data.

JEL Codes: G30, G32.

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1 Introduction

According to the neoclassical theory of investment (Abel (1979), Hayashi (1982)), Tobin's Q, the ratio of the firm value to its capital stock, is a sufficient statistic for the firm's optimal investment. However, this model is empirically rejected: Tobin's Q is weakly correlated with investment; moreover, cash flow enters significantly and reduces further the economic significance of Q.

Recent empirical work shows that bond yields, on the other hand, are strongly correlated with investment, both in the cross-section (Gilchrist and Zakrajsek (2008)), and in the time series (Philippon (2009)). This result is surprising, because in a standard investment model, there is no reason why one of the two asset prices - stocks or bonds - should correlate more with Q: a positive shock to profitability increases *both* the stock price and the bond price, i.e. decrease the yield or spread, as well as increase investment. That is true even if the firm never uses the equity market for financing – the equity value, which is the expected present discounted value of dividends, *still* reflects the higher profitability, a point emphasized by Gomes (2001).

In this paper, we propose a model of financing and investment for firms that attempts to explain *both* the weak correlation of Tobin's Q with investment, and the higher correlation of bond yields with investment. Following Gomes (2001), Hennessy and Whited (2005) and Gomes and Schmid (2010), our model augments the standard neoclassical model with financing frictions. Specifically, we assume that firms fund their investment by issuing equity or long-term defaultable debt. The firm-specific probability of default leads to a firm-specific interest rate, which exactly compensates investors for the risk of default. When the only impulses in the model are shocks to profitability, the model generates a high correlation between investment and Q, and between investment and bond prices.

Our proposed mechanism is that firm-level volatility is time-varying (and stochastic). When volatility goes up, the option value of default goes up, leading the equity value to go up while the bond value goes down. At the same time, investment falls because of the higher cost of financing (and possibly because of the higher uncertainty in itself[1]). Hence, investment is correlated with bond prices but not with stock prices. Overall, unconditional correlations are a weighted average of the correlation conditional on profitability shocks, and the correlation conditional on volatility

[1]This is the effect emphasized by the real options literature, e.g. Pindyck (1989), Bloom, Bond and Van Reenen (2007), and others.

shocks. Both shocks lead to a positive correlation of bond prices and investment, hence the model predicts that this correlation is high. But only the profitability shock generates a positive correlation between stock prices and investment, hence the correlation of stock prices and investment is low. Hence, our model can replicate the patterns of correlations in the data.

Our contribution to the literature goes beyond simply introducing stochastic volatility; we also are the first to analyze the investment problem of a firm which finances itself with long-term defaultable debt. This is a technically and computationally challenging extension over the case of short-term debt, which has been the focus of the literature so far.

The assumption that all debt is short-term implies that firms can adjust their leverage very easily, and as a result credit spreads are low. Hence, it is important to depart from this assumption, and explore the more realistic case of long-term debt. There are two major implications of long-term debt. First, firms issue long-term debt because it allows a better match of the duration of assets and liabilities; that is, it reduces rollover risk. Second, long-term debt leads to agency costs, because bondholders cannot prevent equityholders from maximizing equity value, rather than total firm value, in the future. Note that the agency costs include both debt dilution, and risk-shifting (i.e. both choices of future debt and capital will not maximize firm value).

These two effects do not arise with short-term debt. While the extension to long-term debt is hence well motivated, it is technically difficult as we explain below, which is why our paper is, to the best of our knowledge, the first to incorporate long-term debt in a setup with flexible debt and investment and persistent productivity shocks.

While a large empirical literature documents that stock returns are heteroskedastic, there is relatively little structural modeling which incorporates heteroskedasticity into models with financing and investment decisions. We find the idea of time-varying risk intuitively appealing. In reality, the increase in risk may arise because of changes in the market structure (e.g., new regulation or entry of new competitors), or for technological reasons (e.g., if a firm introduces a new product). The idea of time-varying risk has received some attention in recent macroeconomic research, as we discuss in the literature review.

Organization of the Paper

The rest of the introduction reviews the related literature. Section 2 presents our quantitative model. Section 3 calibrates it and studies its implications numerically. Section 4 concludes. The appendix details our numerical method, and studies a simple two period example which helps clarify some key intuitions.

Literature Review

Our paper is directly related to the recent literature on quantitative corporate finance, and extends it significantly by adding long-term defaultable debt and stochastic volatility. While the effect of debt maturity has been characterized in models with exogenous investment (e.g. "structural debt models" as in Leland and Tofts (1996)), we are not aware of any paper that models long-term debt and endogenous investment.

Our paper also speaks to the vast literature on the Q-theory of investment and the cash flow sensitivity (e.g. Fazzari, Hubbard and Petersen (1988), Gilchrist and Himmelberg (1995), Gomes (2001)). This literature documents the empirical significance of cash flows and discusses its interpretation. One conclusion from this literature is that, while there are many potential mechanisms that break the theoretical link between Q and investment, such as fixed costs, financing constraints, or decreasing return to scales, there are few quantitative models which replicate the failure of the investment regressions, and most researchers appeal to measurement error in Q (e.g., Erickson and Whited (2001), Eberly, Rebelo, and Vincent (2008)).

Second, there is a smaller literature on the relation between investment and uncertainty. This literature emphasizes that models with real options or equivalently fixed cost or irreversibility imply a negative effect of uncertainty on investment (see e.g. Bernanke (1983), Dixit and Pindyck (1994), Caballero (1991), Pindyck (1993), Bloom, Bond and Van Reenen (2007)). In our model, the effect of uncertainty occurs through a different mechanism: debt becomes a less efficient means of finance when uncertainty increases. (Our mechanism is hence similar to that in Arellano, Bai and Kehoe (2010), and Gilchrist, Sim and Zakrajek (2010).) Empirical evidence suggests a significant effect of uncertainty on investment (Leahy and Whited (1996), Guioso and Parigi (1999)) but the channel through which the effect operates is not fully established.

Recently several studies have considered the possibility that idiosyncratic risk varies over time (Bloom (2009), Arellano, Bai and Kehoe (2010), Bachmann and Bayer (2009), Gilchrist, Sim and Zakrajek(2010)). These papers all have in common that they consider situations where idiosyncratic volatility rises in all firms at the same time (i.e., the increase in idiosyncratic volatility is an aggregate shock). In contrast, we focus on the case of idiosyncratic shocks to idiosyncratic volatility.

The corporate finance literature has emphasized the endogeneity of volatility, known as the risk-shifting or asset-substitution problem (equity holders may choose to increase volatility). Our model has exogenous increases in the volatility of the technology shocks, but firms respond to these changes by altering capital, hence the overall volatility of profits is endogenous. Most of the quantitative research assumes that debt is short-term (one period), hence the volatility is known by debtholders when they decide to buy the firm's bonds and there is no risk-shifting problem. Because we consider long-term debt, and the debt contract is not contingent on future realizations of shocks, our solution embeds a risk-shifting agency friction. That is, we assume that equity holders at each date maximize equity value, and they cannot commit to maximizing firm value. Bondholders take this behavior into account when pricing and buying new debt issues.

2 Model

This section presents a partial equilibrium dynamic model of investment and financing decisions. Firms are ex-ante identical but become heterogeneous ex-post because of different idiosyncratic realizations of productivity and volatility shocks. Firms decide on investment, dividend payouts, and debt issuance, subject to physical adjustment costs and financing frictions. Firms borrow in the form of a multi-period defaultable debt. Bankruptcy costs are incurred upon default and the interest expense on debt can be deducted from taxable income. Hence, our model is a dynamic trade-off model, that builds on the quantitative corporate finance literature,[2] and more specifically is an extension of the recent study of Gomes and Schmid (2009). There are two main differences with the existing literature. First, we explicitly introduce stochastic volatility shocks in order to quantify the effect of stochastic volatility on investment, credit spreads, and Tobin's Q. Second, we introduce long-term debt financing, which is more realistic, allows a better match of the maturity of assets and liabilities, but creates significant agency problems, as we discuss later.

2.1 Firm problem

We assume that the firm is run by a manager that maximizes the expected discounted present value of dividends. We write this problem directly in recursive formulation. The dynamic programming

[2] Cooley and Quadrini (2001), Gomes (2001), Hennessy and Whited (2005), Livdan, Sapriza and Zhang (2008).

problem for a firm is to choose next period capital k', debt b', and a default policy, taking as given the three exogenous shocks: idiosyncratic productivity shock z_i, aggregate productivity shock z_a, and volatility shock σ (that affects only the volatility of the idiosyncratic shock). We denote by a vector s all these exogenous state variables.

Firms use only physical capital as input to their production function. Operating profits are denoted by $\pi(k, s)$ and are strictly increasing and strictly concave in capital: $\pi_k > 0$ and $\pi_{kk} < 0$. Capital depreciates at rate δ_k, thus investment (capital expenditures) satisfies,

$$i = k' - (1 - \delta_k)k.$$

In addition we assume that firms incur an adjustment cost $\phi(i, k)$ to install i units of capital from the outstanding level k. We assume that adjustment costs take a linear form,

$$\begin{aligned} \phi(i,k) &= \phi_+ \times i \text{ if } i > 0, \\ &= \phi_- \times i \text{ if } i < 0, \end{aligned}$$

as in Abel and Eberly (1996). These costs may represent transaction cost of purchasing or selling capital.

Firms have access to lenders and can borrow in the form of multi-period debt contracts. We use a version of the exponential model introduced in Leland and Toft (1996), and used by Arellano and Ramanarayanan (2008), Chatterjee and Eyigungor (2011), Hackbarth, Miao, and Morellec (2006), and Philippon (2009) among others. In this model, firms continuously issue and repay debt. Firms can issue a new loan every period, denoted by l. The loan sells for a price q, thus firms raise the amount ql. (The price q will be endogenously determined by the probability of default below, and firms understand that changing their debt issuance will affect their debt price.) Firms promise to repay a fixed proportion of the balance on the loan every period. Specifically, a \$1 loan at time t will yield a payment δ_b at time $t + 1$, a payment $\delta_b(1 - \delta_b)$ at time $t + 2$, a payment $\delta_b(1 - \delta_b)^2$ at time $t + 3$, and so on. In other words a \$1 loan issued at time t will require an infinite stream of annuity payments, where the annuity at time $t + s$ is equal to $\delta_b(1 - \delta_b)^{s-1}$. Denote the sum of all outstanding loans by b.

Corporate profits are taxed at a rate $\tau \in [0,1]$ and interest expense can be deducted from taxable income. We assume that the firm can shield the net present value of the interest expenses on the loan at the date of issuance. Therefore the firm will raise $\tilde{q}l$ instead of ql, where $\tilde{q}$ will include the tax subsidy.[3]

The flow of funds constraint for the firm relates its dividends, debt issuance, debt repayment, operating profits, and investment and investment cost:

$$\tilde{d} = (1-\tau)\pi(k,s) + \tilde{q}l - \delta_b b - i - \phi(i,k),$$

where the new loan issued is,

$$l = b' - (1-\delta_b)b.$$

We also make the standard assumption that equity issuance is costly. Following Gomes (2001), equity can be issued at a proportional cost λ, that is dividend net of equity issuance is given by,

$$d = (1 + \lambda \mathbf{1}_{\{\tilde{d}<0\}})\tilde{d},$$

where $\mathbf{1}_{\{\tilde{d}<0\}}$ is an indicator function of strictly negative dividends, i.e. $\mathbf{1}_{\{\tilde{d}<0\}} = 1$ if $\tilde{d} < 0$ (and 0 otherwise).

The firm's manager maximize the present discounted value of dividends, and can file for bankruptcy, and will find it optimal to do so when the equity value reaches zero. The firm does not file for bankruptcy if its equity value V_t is positive. The equity value of the firm, can be expressed as the infinite horizon sum of dividends, up to the period of default,

$$V_0(k_0, b_0, s_0) = \max_{\{k_t,b_t\}_{t=1}^{\infty}} \sum_{t=0}^{\infty} \mathbb{E}_0 \left[M_{0,t}\ d_t \mathbf{1}_{\{V_t>0\}} \right],$$

where $M_{0,t}$ is a stochastic discount factor, and given the initial capital k_0, initial debt b_0, and initial state s_0. Therefore the equity value maximization problem can be written in recursive form

[3]Assuming that tax subsidies are received at issuance rather than over the lifetime of the bond simplifies our analysis because we do not need to include the past issuance of debt as an additional state variable, and should have little effect on our results.

as follows,

$$V(k,b,s) = \max_{k',b'} \quad d + \mathbb{E}\left[M(s,s')\max\left(0, V(k',b',s')\right)\right], \tag{1}$$

where the maximum operator inside the brackets captures the default option.

2.2 Debt pricing

Debt is priced by competitive lenders: the price of any new loan issued l has to equal the expected discounted value of the stream of repayments. Mathematically, at time t the price of the new loan l_t, denoted by q_t, satisfies,

$$q_t l_t = \sum_{s=1}^{\infty} \mathbb{E}_t \left[M_{t,t+s} \left(\begin{array}{c} \delta_b (1-\delta_b)^{s-1} l_t \ \mathbf{1}_{\{V_{t+s}>0\}} \\ +\xi \ \left(k_{t+s} - \phi(-k_{t+s}, k_{t+s})\right) \frac{(1-\delta_b)^{s-1} l_t}{b_{t+s}} \left(1 - \mathbf{1}_{\{V_{t+s}>0\}}\right) \end{array} \right) \right].$$

This expression assumes that, in the event of default, debtholders can appropriate the capital stock, net of adjustment costs, but only recover a fraction ξ of the proceeds. We assume that all debt claims have equal seniority, regardless of when they were issued. The lenders of a new loan have a claim on the default payoff that is decreasing over time, given that this loan is being repaid at rate δ_b over time. Hence the lenders of new loan l_t have a claim equal to $(1-\delta_b)^{s-1} l_t / b_{t+s}$ of the total default payoff at time $t+s$.

Because repayments of loans occurs over multiple periods in the future, lenders have to know the future investment and debt choices of firms. Therefore the recursive formulation of the debt price will depend on the firm's optimal capital and debt polices, denoted by $k' = g_k(k,b,s)$ and $b' = g_b(k,b,s)$, respectively. The recursive formulation for the loan price schedule $q(k',b',s)$ is,

$$q(k',b',s) = \mathbb{E}_{s'|s} \left[M(s,s') \left(\begin{array}{c} \left(\delta_b + (1-\delta_b) q(g_k(k',b',s'), g_b(k',b',s'), s')\right) \mathbf{1}_{\{V(k',b',s')>0\}} \\ +\xi \left(\frac{k' - \phi(-k',k')}{b'}\right) \left(1 - \mathbf{1}_{\{V(k',b',s')>0\}}\right) \end{array} \right) \right]. \tag{2}$$

2.3 Credit Spreads and Duration

The credit spread is the difference between the yield to maturity of the new loan and the risk free rate. The yield to maturity is the constant rate c that equates the price of the loan issue to the discounted payoff, assuming no default. Mathematically, c satisfies the following equation

$$q = \sum_{t=1}^{\infty} \left(\frac{1}{1+c} \right)^t \delta_b (1-\delta_b)^{t-1} = \frac{\delta_b}{c+\delta_b}.$$

Using this relationship, the yield to maturity can be expressed as a function of the price of the loan,

$$c = \delta_b \left(\frac{1}{q} - 1 \right).$$

and the credit spread, denoted by CS, is given by,

$$CS = c - r_f(s) = \delta_b \left(\frac{1}{q} - 1 \right) - \frac{1}{E_{s'|s}\left[M(s,s')\right]}.$$

Because we assume that the firm can shield the net present value of the interest expenses on the loan at the date of issuance, the price of the new loan inclusive of tax subsidy $\tilde{q}$ can be calculated as follows,

$$\tilde{q} = \sum_{t=1}^{\infty} \left(\frac{1}{1+(1-\tau)c(q)} \right)^t \delta_b (1-\delta_b)^{t-1} = \frac{1}{1+(1-\tau)(q^{-1}-1)}.$$

This relation allows to find the price $\tilde{q}(q;\tau)$ inclusive of the tax subsidy, provided that we know the lenders price schedule $q(k', b', s)$.

The duration of the loan is,

$$D(\delta_b) = \frac{1}{q} \sum_{t=1}^{\infty} t \left(\frac{1}{1+c} \right)^t \delta_b (1-\delta_b)^{t-1}.$$

Duration allows us to map an exponentially decaying debt with duration $D(\delta_b)$ to the maturity of a pure discount bond of D periods.

2.4 Recursive Formulation of the Firm Problem

The firm problem is stated formally in the following definition.

Problem 1 (Recursive Formulation of the Firm Problem)**.** *Given the loan price schedule $\tilde{q}(k', b', s)$, firms solve the following program,*

$$V(k,b,s) = \max_{k',b'} \quad d + \ \mathbb{E}_{s'|s}\left[M(s,s')\max\left(0, V(k',b',s')\right)\right], \tag{3}$$

subject to,

$$\begin{aligned} d &= \left(1+\lambda \mathbf{1}_{\{\tilde{d}<0\}}\right)\left((1-\tau)\pi(k,s) + \tilde{q}(k',b',s)l - \delta_b b - i - \phi(i,k)\right), \\ i &= k' - (1-\delta_k)k, \\ l &= b' - (1-\delta_b)b. \end{aligned}$$

Our formulation implies that managers maximize equity value at all dates, and cannot commit to maximize firm value. This creates an agency cost: managers will pick future capital and debt in order to maximize the equity value. This problem is anticipated by bondholders, which require a higher discount to hold the debt. This incentive problem disappears when debt is short-term (as is commonly assumed in the literature), because debtholders know the capital and debt choices of managers when they lend. On the other hand, long term debt allows to better match the maturity of assets and liabilities, i.e. to avoid "rollover risk" (e.g., a large temporary productivity shock may make it impossible to rollover the debt, even though the investment is profitable in the long run.)

An equilibrium for this model is a value function V, and associated policy functions g_k and g_b, and a loan price schedule $\widetilde{q}$, such that V, g_k, g_b solve the problem (3), and $\widetilde{q}$ satisfies (2).[4]

2.5 Numerical solution method

This section describes our approach to solve the model. There are no closed-form expression for the solution, so we must use numerical techniques. First, we discretize the state space for capital

[4] We conjecture that the existence of equilibrium can be proved as in Chatterjee and Eyigungor (2011). However, uniqueness is generally not guaranteed.

and debt.[5] As in many recent studies (e.g. Hennessy and Whited (2005)), our model incorporates two endogenous state variables (physical capacity and outstanding debt obligations), as well as exogenous states reflecting the ongoing uncertainty in which the firm operates. Our model adds an important layer of complication, however, because bond prices are endogenous and must be solved as part of the equilibrium. We use an inner loop/outer loop formulation. In the inner loop, for a given bond price schedule, standard value function iteration techniques can be used to solve the firm problem, i.e. its equity value, and choices of capital, debt and whether to file for bankruptcy. In the outer loop, we update bond prices to reflect the default probabilities. With long-term debt, the bond pricing equation is itself a recursion, that must be updated to convergence. The algorithm iterates between the inner loop and the outer loop until convergence.

This procedure, while standard in the sovereign debt literature, raises two numerical issues. First, the presence of the two loops slows down significantly the computation. Second, this simple algorithm is not guaranteed to converge, and indeed we often encountered non-convergence, even when bond prices are updated slowly (i.e. with significant relaxation). Non-convergence is especially frequent with long-term debt (i.e. when the debt maturity is larger than one period).

A recent important contribution by Chatterjee and Eyigungor (2011) provides an algorithm that performs well in the context of a sovereign default model. We adapt and extend their algorithm to our problem, which has one additional state variable (capital) and is somewhat different. The algorithm relies on the introduction of economically small i.i.d. *continuous* shocks that smooth the default decision. Furthermore, the exact default thresholds must be computed to solve for the optimal policies. Finally, this algorithm requires the per-period objective function to be slightly concave; we hence assume that firms maximize not the present-value of dividends, but of a quadratic function of dividends. The details of the algorithm are in appendix B.

Summarizing, we change our model so that firms dividends are,

$$\hat{d} = d + m,$$

with m iid, distributed according to a truncated normal with small volatility σ_m, and firms maximize the present discounted value of $h(\hat{d})$, with $h(x) = x - \frac{\kappa}{2}x^2$. These two features do not appear to

[5] Our grid includes 96 points for k, 32 points for b/k, 24 points for z_i, 4 points for z_a, and 2 points for σ. (We transform the problem in a choice of leverage, rather than capital, in order to obtain a higher precision.)

affect our results significantly. Although these modifications are introduced for numerical reasons, they could also be motivated economically: transitory shocks to profits correspond to windfalls such as legal settlements, and the function h (i.e. parameter κ) reflects a dividend smoothing motive, as used in many studies (for instance Jermann and Quadrini (2011)).

Once we have found the value functions and optimal policy functions, we simulate the optimal policies for a large panel of firms ($N = 10,000$ firms for $T = 200$ periods). The initial condition for each firm is obtained by assuming that its initial productivity z_0 is drawn from the invariant distribution of z, and the capital k_0 is raised by an initial equity issue. That is, k_0 is picked such that,

$$k_0 \in \arg\max_k \; V(k, 0, z_0) - k(1 + \lambda),$$

where λ is the equity issuance cost. We dismiss the first 5 periods in the simulations and compute statistics.

3 Quantitative Implications

This section discusses the choices of parameters and then examines the key implications of our quantitative model.

3.1 Parametrization

We pick the parameters to roughly match basic moments. We stress that the current calibration is illustrative rather than definitive. The goal is to illustrate the implications of our new framework.

The time period is one year. First, following Gomes (2001) and Hennessy and Whited (2005), the idiosyncratic productivity process z_i is restricted to follow a first order autoregressive process,

$$\log z_i' = \rho_z \log z_i - \frac{\sigma^2}{2} + \sigma\epsilon_i, \tag{4}$$

where ϵ_i are independently and identically distributed shocks drawn from a standard normal distribution, and $\rho_z = 0.75$. The stochastic volatility process is assumed to be a Markov chain with 2 states $\sigma \in \{\sigma_L, \sigma_H\}$ and transition matrix $\Gamma_{\sigma\sigma'}$. We set $\sigma_L = 0.10$, $\sigma_H = 0.25$, and the transition matrix Γ is such that $\Gamma_{LL} = 0.9$ and $\Gamma_{HH} = 0.65$.

The aggregate shock similarly follows an AR(1) process,

$$\log z'_a = \rho_a \log z_a + \sigma_a \epsilon_a,$$

with $\rho_a = 0.85$ and $\sigma_a = 0.02$. The stochastic discount factor is,

$$M(z_a, z'_a) = \beta e^{-\gamma_0 (\log z'_a - \rho_a \log z_a)},$$

which ensures a constant risk-free interest rate, and hence a flat term structure. We set $\beta = 0.98$ and $\gamma_0 = 15$.

Firm profits are a concave function of physical capacity, and we assume that the firm must pay a fixed cost of operation f each period. Profits are assumed to take the following functional form,

$$\pi(k, s) = z_a z_i k^{\alpha} - f, \tag{5}$$

where $\alpha \in (0, 1)$ characterizes the degree of decreasing returns to scale for capital. In line with the literature, we set $\alpha = 0.4$.

Finally, the adjustment cost parameters are set to $\phi_+ = 0.05$, and $\phi_- = -1$. Hence this formulation entails irreversible investment, as in Gomes and Schmid (2009).

The cost of equity issuance λ equals 25%.[6] The average tax rate is 20%, reflecting that corporations typically face a marginal tax rate less than the statutory rate. Bankruptcy costs are assumed to correspond to a dead-weight cost of 50% of current capital stock, hence a recovery rate $\xi = 0.5$. The set of parameters used to solve the model is summarized in Table 1.

3.2 Definition of Variables

In analyzing the cross-sectional implications of firms' optimal investment choice, we are particularly interested in the correlations between investment rates and Tobin's Q and credit spreads CS. The definitions of the variables used in this paper are standard in the investment and capital structure

[6] This figure is higher than the estimates in the literature. For example Hennessy and Whited (2008) estimated these issuance costs to be about 6%.

	Parameter	Mode	Description
	β	0.98	Subjective discount rate
Preference	γ_0	15	Parameter for the pricing kernel
	α	0.40	Production parameter
	ϕ_-	-1	Cost of negative investment
Technology	ϕ_+	0.05	Cost of positive investment
	f	0.97	Fixed cost of operation
	δ_k	0.140	Capital depreciation rate
	δ_b	$\{1, 0.2\}$	Exponential debt rate of decay
	λ	0.25	Linear cost of issuing equity
Institution	ξ	0.50	Recovery rate in event of bankruptcy
	τ	0.20	Average corporate tax rate
	ρ_a	0.85	Autocorrelation of z_a
	σ_a	0.02	Volatility of z_a
	ρ_z	0.75	Autocorrelation of z_i
Uncertainty	σ_L	0.10	Low Volatility of z_i
	σ_H	0.25	High Volatility of z_i
	Γ_{LL}	0.90	Probability of staying in the low volatility state
	Γ_{HH}	0.65	Probability of staying in the high volatility state
	σ_m	0.04	Volatility of continuous shock m
Numerical	κ	0.01	Dividend smoothing motive

Table 1: **Parameter Values.** The model is calibrated at the annual frequency.

literatures,

$$\begin{aligned} \text{Tobin's Q:} \qquad & Q = \frac{V(k,b,s) + b'(k,b,s)}{k'(k,b,s)}, \\ \text{Investment Rate:} \qquad & \frac{i}{k} = \frac{k'(k,b,s) - (1-\delta_k)k}{k}, \\ \text{Profitability:} \qquad & \frac{\pi}{k} = \frac{\pi(k,s)}{k}. \end{aligned}$$

Firms' financing policies are defined as follows,

$$\begin{aligned} \text{Leverage:} \qquad & \frac{b'}{k'} = \frac{b'(k,b,s)}{k'(k,b,s)}, \\ \text{Credit Spreads:} \qquad & CS(k,b,s) = \frac{\delta_b}{q(k'(k,b,s), b'(k,b,s), s)} - \delta_b - \frac{1}{E_{s'|s}\left[M(s,s')\right]}, \\ \text{Default:} \qquad & I^{DF}(k,b,s) = \mathbf{1}_{\{V(k,b,s)\leq 0\}}. \end{aligned}$$

3.3 Strategy

We use six different model specifications in order to isolate the various effects at work. Specifications 1, 3, and 5 have deterministic volatility, i.e. $\sigma = \bar{\sigma}$, whereas specifications 2, 4, and 6 include stochastic volatility in the productivity shocks, i.e. $\sigma \in \{\sigma_L, \sigma_H\}$. Note that $\bar{\sigma}$ is set to the long-run mean volatility level in the stochastic case.

Specifications 1 and 2 have a one period debt, i.e. $\delta_b = 1$. Specifications 3 and 4 extends the duration to 5 periods, i.e. $\delta_b = 0.2$, and specifications 5 and 6 have a 10 period duration debt, i.e. $\delta_b = 0.1$. These pairs will allow us to isolate the effect of stochastic volatility in otherwise identical economies, for debt contracts of different maturity.

Specification 6 –the benchmark– will allow us to contrast an environment with multi-period debts and stochastic volatility with the standard benchmark in the literature: one period debt with the standard productivity process.

3.4 Optimal Policy Rules

Figure 1 depicts the optimal policy rules in the benchmark model. The policies of interest include leverage b'/k', investment rate i/k, credit spreads CS, and Tobin's Q. All variables are plotted versus capital stock k and debt level b, and evaluated at the mean productivity shocks $z_a = 1$ and $z_i = 1$ and at the low volatility level σ_L.

To understand these figures, note that firms in the region with high capital k and low debt b are far away from default, while firms with low capital k and high debt b are in default (and hence their policies are not plotted). The patterns shown in these figures are consistent with intuition. Investment is decreasing in k (firms with high capital have less need for investment) and, weakly, in b (higher outstanding debt increases the cost of investment). Leverage is decreasing in k (higher capital means higher cash flows and less need for investment, hence less debt tomorrow), and increasing in b (higher outstanding debt today implies higher debt to rollover and thus higher debt tomorrow).

Turning to asset prices, the bottom panels depict credit spreads and Tobin's Q. For high k and low b, firms will have a very low (or even zero) probability of default next period. As a result, credit spreads are very low (or even zero). But when k is lower and b higher, firms come closer to

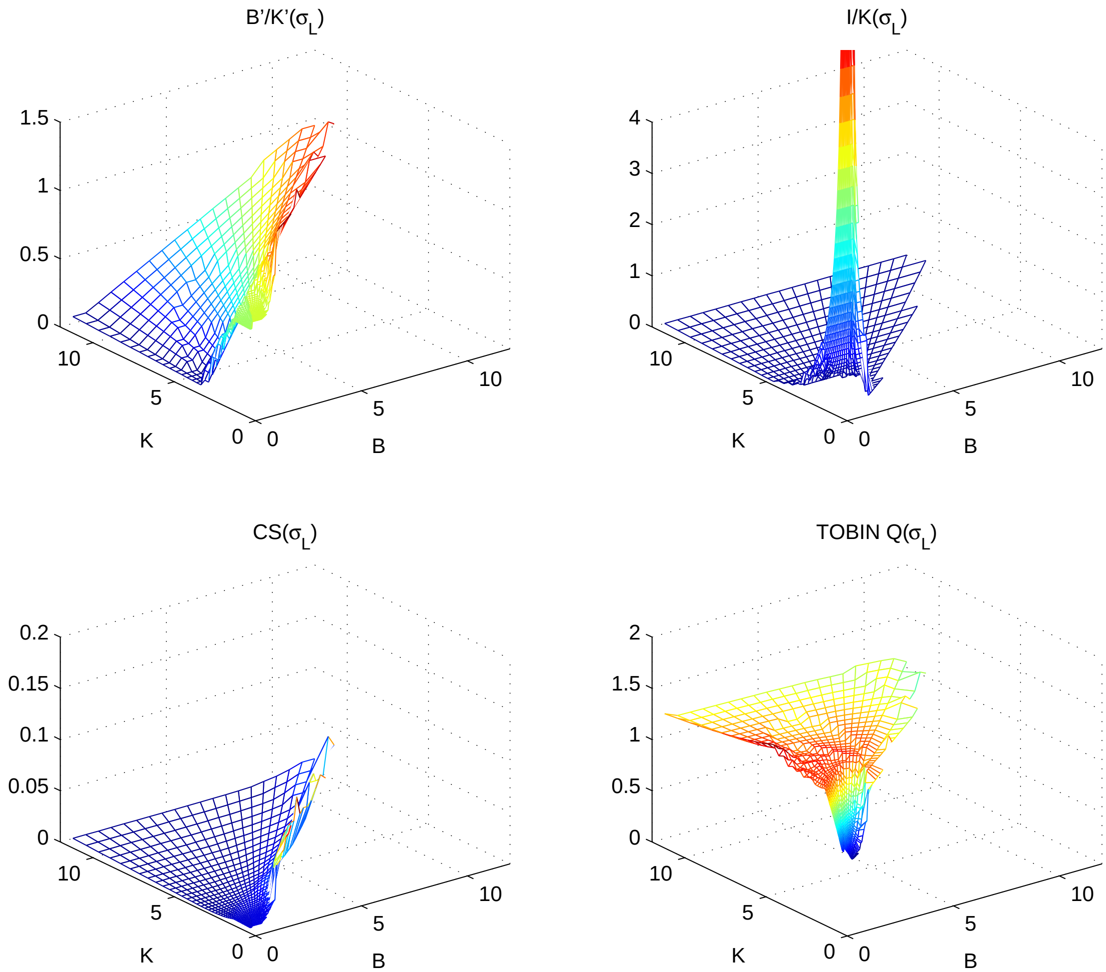

Figure 1: **Optimal Policy Rules.** The figure shows optimal policy rules in the benchmark model: (i) leverage b'/k', (ii) investment rate i/k, (iii) credit spreads CS, and (iv) Tobin's Q. All variables are plotted versus capital stock k and debt level b, and evaluated at the mean productivity shocks $z_a = 1$, $z_i = 1$ and at the low volatility level σ_L.

default and spreads rise in a nonlinear fashion. The introduction of long-term debt is important in this regard: with short-term debt, credit spreads are very small, as firms can readjust their debt every period and avoid becoming close to default. In contrast, with long-term debt and costly disinvestment, firms sometimes find themselves with "excess debt" (above their long-run leverage target) and hence spreads are high. Last, the bottom right panel shows that Tobin's Q falls as firms find themselves close to default: the high likelihood of bankruptcy and the associated costs reduces firm value.

Overall, the figure suggests that there are two regions of interest. Firms are either far from default, or close to default. In the former case, spreads and leverage are low and Tobin's Q is high, while in the latter case spreads and leverage are large and Tobin's Q is low.

Figure 2 plots the difference in policy rules in the benchmark model between the high and low volatility levels, i.e. σ_H versus σ_L. The figure presents the differences in debt levels, investment, credit spreads, and Tobin's Q. All variables are plotted as a function of the current capital stock k and outstanding debt level b, and evaluated at the mean productivity shocks $z_a = 1$ and $z_i = 1$. Plotting the difference between the two different volatility states is helpful in understanding the effect of a shock to volatility in isolation of productivity shocks, in various regions of the state space for capital stock and debt.

Again it is useful to distinguish the two regimes, close to default (bottom right frontier in each panel) and far from default (top left corner). As volatility goes up, both investment and leverage falls for firms that are small and close to default (and the ensuing deadweight loss). This higher default risk also leads credit spreads to rise, and this effect is especially strong close to default. Tobin's Q falls for small firms that are far to default, and rises for small firms that are close to default. This result appears to be quantitative rather than qualitative: on the one hand, firms close to default benefit most from the upside potential of higher volatility, i.e. the default option of equity is more valuable is a more risky environment. On the other hand, the expected deadweight cost of default is higher if the firm is close to default.

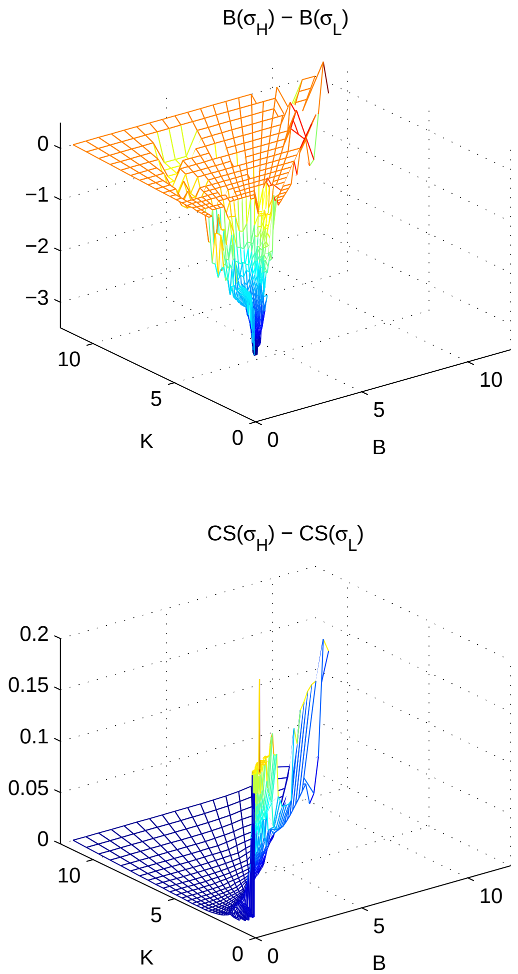

Figure 2: **Difference in Optimal Policy Rules.** The figure shows the difference in policy rules in the benchmark model between the high and low volatility levels, i.e. σ_H versus σ_L: (i) debt $b'(\sigma_H) - b'(\sigma_L)$, (ii) investment $k'(\sigma_H) - k'(\sigma_L)$, (iii) credit spreads $CS(\sigma_H) - CS(\sigma_L)$, and (iv) Tobin's $Q(\sigma_H)$ - Tobin's $Q(\sigma_L)$. All variables are plotted versus capital stock k and debt level b, and evaluated at the mean productivity shocks $z_a = 1$, $z_i = 1$.

3.5 Effect of stochastic volatility and long-term debt on means and volatilities of real and financial variables

We report the real side data moments from Eberly, Rebelo, and Vincent (2009), who use a subsample from Compustat in the period 1981-2003. Their sample is the top quartile of firms sorted by size of the capital stock in 1981. The leverage, spread and default statistics are taken from Chen, Collin-Dufresne, and Goldstein (2008). Summary statistics of the firms' policies along with the data counterpart are given in Table 2. All moments are reported at the annual frequency.

Model Specification		Data	(1)	(2)	(3)	(4)	(5)	(6)
δ_b			1	1	0.2	0.2	0.1	0.1
Volatility			D	S	D	S	D	S
Real Policies:								
Tobin's Q	$\mathrm{E}(Q)$	1.298	1.327	1.391	1.291	1.372	1.308	1.389
	$\sigma(Q)$	0.625	0.225	0.224	0.238	0.224	0.237	0.226
Investment Rate	$\mathrm{E}(i/k)$	0.150	0.153	0.153	0.151	0.151	0.150	0.150
	$\sigma(i/k)$	0.055	0.172	0.190	0.179	0.183	0.180	0.184
Profitability	$\mathrm{E}(\pi/k)$	0.169	0.171	0.174	0.173	0.176	0.173	0.176
	$\sigma(\pi/k)$	0.078	0.097	0.101	0.099	0.102	0.099	0.102
Financing Policies:								
Leverage	$\mathrm{E}(b/k)$	0.350	0.355	0.375	0.247	0.266	0.255	0.279
	$\sigma(b/k)$	0.090	0.164	0.172	0.170	0.196	0.189	0.214
Credit Spreads (%)	$\mathrm{E}(c - R^f)$	1.090	0.028	0.179	0.356	0.408	0.525	0.520
	$\sigma(c - R^f)$	0.410	0.408	1.588	1.231	1.372	1.379	1.323
Default (%)	$\mathrm{E}(I^{DF})$	0.400	0.032	0.172	0.305	0.352	0.419	0.430

Table 2: **Summary Statistics.** The summary statistics in the data are taken from Chen, Collin-Dufresne, and Goldstein (2008) and Eberly, Rebelo, and Vincent (2009). The model economy is simulated for a panel of 10,000 firms over 200 periods.

This table is the first main set of results of the paper. It illustrates the effect of stochastic volatility, and of long-term debt, on firms' real and financial policies. First, note that all models replicate reasonably well the means of investment rate, profit rate and Tobin's Q. Second, there is a tension between matching the volatility of investment or profits, and that of Tobin's Q – a standard problem. As a result, our model overpredicts somewhat the volatility of investment and profits, but underpredicts the volatility of Q. (We abstract from variation in discount rates which may account for a significant fraction of the volatility of Tobin's Q; note also that the sample we use in the data is fairly stable.)

The comparison across columns shows the effect of making debt long-term, and of adding stochastic volatility. The first column shows the model statistics when debt is short-term and volatility is constant. In this case, the probability of default is small, and so are credit spreads. Adding stochastic volatility leads to somewhat higher default probabilities. This effect is amplified by long-term debt, which leads to further higher default probabilities (and hence credit spreads). Moreover, leverage falls substantially from a single period to multi period debt. This result is consistent with the findings of Leland and Toft (1996) in the context of a model without investment. Intuitively, long-term debt is more risky, leading to significant higher default risk and a reduction in debt usage.

Real policies, such as investment, are not heavily affected by long-term debt or the changes in volatility. The additional shock raises the variance of investment and profits, but does not affect the means significantly. Overall Tobin's Q is not significantly affect by the uncertainty shock.

Long-term debt, in itself, appears to have a limited effect on the volatility of investment. While long-term debt may offer some hedging advantages, this does not apparently lead to a significant change in investment policies for our current calibration.

3.6 Correlations of Investment, Leverage, Tobin's Q and credit spreads

In this subsection, we first investigate the ability of the model to replicate some standard Q-theoretic regressions. We are interested in understanding how stochastic volatility impacts the co-movements between investment, Tobin's Q, and credit spreads.

The results for the regressions of investment on Q and spreads were provided to us by Gilchrist and Zakrajsek. These authors assembled firm-level data on investment, Tobin's Q and bond yields by merging the standard Compustat database with data on individual bond issues. The credit spread of a firm is defined as the weighted average of the spread on each of its bond issues, where the weights are the market value of each issue. Spreads are observed at the monthly frequency, and the annual spread is the time-series average of the monthly spreads. The data covers the period 1983-2006 and is an unbalanced panel with 799 firms. Their results are summarized in Table 3.

Variable		i/k			$\log(i/k)$	
	(1)	(2)	(3)	(4)	(5)	(6)
$\log(c)$	-0.035		-0.034	-0.297		-0.277
	(0.005)		(0.005)	(0.024)		(0.025)
$\log(Q)$		0.051	0.0015		0.241	0.156
		(0.016)	(0.0018)		(0.049)	(0.046)
R^2 (within)	0.054	0.064	0.062	0.208	0.146	0.203

Table 3: **Data.** Regressions results from Gilchrist and Zakrajsek (2009).

Regression results for the model economy along with the data counterpart are given in Table 4. In the data, the striking fact is that while Q and investment are positively correlated, credit spreads "drive out" Q when they are both included in the regression. That is, the coefficient on Q falls, and may be insignificant, and the R^2 does not increase significantly.

When we run these regressions using simulated data from our model, we obtain results that have some of the same flavor, but we fall short of a clear replication. If volatility is constant and debt is short-term, Q is significantly correlated with investment, credit spreads are also significant, but in a joint regression Tobin's Q is much stronger. When volatility is time-varying and debt is long-term, the R^2 of the regression of investment on credit spreads becomes stronger, but including them in the regression together with Tobin's Q, does not affect the coefficient or R^2 significantly. Hence, the flavor of the result is closer to the data when we incorporate both long-term debt and stochastic volatility. However, the model does not, at this stage, generate results as sharp as the data.

Second, in the model, an increase in uncertainty increases the probability of default and hence leads to a reduction in leverage. This negative relative relation between uncertainty and leverage appears consistent with the empirical findings of Baum et al. (2008, 2009), who run regressions of leverage on volatility and obtains a clear negative effect.

3.7 Close to default vs. far from default

Our model allows us to study how firms in "financial distress" (i.e. that have a higher likelihood of default) change their real and financial policies. We calculate the correlations conditional on the spread being larger than 100bp, and compare the results to the unconditional correlation.

Model Specification	$\log(c)$	$\log(Q)$	R^2
Data	-0.035		0.054
	(0.005)		
		0.051	0.064
		(0.016)	
	-0.034	0.002	0.062
	(0.005)	(0.002)	
(1) Deterministic σ	-0.128		0.002
$\delta_b = 1$		0.232	0.083
	0.199	0.250	0.086
Far from default:	0.291	0.255	0.087
Close to default:	-0.007	-0.005	0.136
(2) Stochastic σ	-0.122		0.017
$\delta_b = 1$		0.222	0.067
	0.051	0.251	0.069
Far from default:	0.274	0.336	0.071
Close to default:	-0.011	0.011	0.011
(3) Deterministic σ	-0.113		0.025
$\delta_b = 0.2$		0.221	0.079
	0.140	0.345	0.092
Far from default:	0.761	0.534	0.130
Close to default:	-0.050	-0.039	0.058
(4) Stochastic σ	-0.115		0.026
$\delta_b = 0.2$		0.246	0.078
	0.060	0.299	0.081
Far from default:	0.606	0.602	0.115
Close to default:	-0.023	0.053	0.047
(5) Deterministic σ	-0.100		0.024
$\delta_b = 0.1$		0.238	0.085
	0.053	0.284	0.089
Far from default:	0.675	0.562	0.174
Close to default:	-0.139	-0.003	0.092
(6) Stochastic σ	-0.113		0.025
$\delta_b = 0.1$		0.259	0.084
	0.031	0.284	0.085
Far from default:	0.526	0.608	0.145
Close to default:	-0.108	0.030	0.086

Table 4: **Regression Results.** The regressions results in the data are taken from Gilchrist and Zakrajsek (2009). Regressions in the model are run on data simulated for a panel of 10,000 firms over 200 periods.

As can be seen in Table 4, this sample split shows that firms that are close to default have a stronger negative correlation of investment and credit spreads.

4 Conclusion

We introduce stochastic volatility and long-term debt in a standard model of corporate finance and investment. These features imply a higher probability of default and lower leverage. Consistent with intuition, the volatility shocks generate higher correlations between credit spreads and investment, however it fails (for our current calibration) to weaken the correlation between Q and investment. Overall, the correlation between spreads and investment is still too small, and the correlation between Q and investment is still too high, compared to the data.

In future work, we plan to introduce some extensions which might magnify the effect of stochastic volatility and long-term debt, and to calculate explicitly the agency costs of long-term debt, and how to limit these with covenants. Another possible extension of the paper is to consider the macroeconomic implications of the model, when the shock to volatility is correlated across firms. A lower volatility, such as the “Great Moderation”, would lead firms to take on more debt and increase investment. In the process, they might become more sensitive to productivity (or other) shocks.

Appendix A: A Simple Two-Period Model

This section provides a simple model to illustrate the effect of an increase in risk on investment, equity value, Tobin's Q and bond yields. There are two time periods. At time 1, the firm buys capital k, which is financed using equity s and debt b. b is the face value of debt, and $q(k, b)$ is its market price. The budget constraint is,

$$\chi q(k, b)b + s = k. \tag{6}$$

The parameter $\chi > 1$ reflects the tax shield effect (i.e. interest expenses are deductible from corporate income). To simplify we assume in this section that the deduction is done at issuance: for each dollar of debt issued, the firm receives a subsidy of $(\chi - 1)$\$.

At time 2, the firm produces and generates a profit $\pi = zk^\alpha$, where z is an idiosyncratic shock, which is distributed according to a cumulative distribution function H. We denote by h the corresponding probability distribution function. We also assume that the firm receives a growth option with value $G(z)$, i.e. $G(z)$ is the expected present discounted value of future profits if z is realized.[7] We assume that $G'(z) > 0$.

The firm will default if its total value, i.e. profits plus the growth option, is not large enough to repay its debt, i.e. if $z < z^*$, where the threshold z^* is defined through the condition,

$$z^* k^\alpha + G(z^*) = b, \tag{7}$$

If the firm does default, the absolute priority rule applies: equity holders receive nothing, while bondholders share the firm value, net of proportional bankruptcy costs. We denote by θ the recovery rate, i.e. $(1 - \theta)$ is bankruptcy costs.[8]

The market price of debt is the expected discounted payoff to debtholders. Assuming that investors are risk-neutral and have a discount factor β,

$$q(k, b) = \beta \left(\int_{z^*}^{\infty} dH(z) + \int_0^{z^*} \theta \frac{zk^\alpha + G(z)}{b} dH(z) \right). \tag{8}$$

[7] To make G endogenous, we may write a three-period model, e.g. at time 2 the firm can buy capital to take advantage of the realized z.

[8] We assume that the growth option can also be recovered at rate θ. Alternative assumptions are possible and do not affect the main result.

In this formula, the first term is the repayment of the face value in the non-default states, and the second term is the recovery of the profits and growth option, divided across all the bondholders in the event of default.

The firm equity value is the expected discounted payoff to equity holders, i.e. the expected profits and growth option net of debt repayment, in non-default states,

$$V = \beta \int_{z^*}^{\infty} (zk^{\alpha} - b + G(z))\, dH(z). \tag{9}$$

The firm picks k, b, s, and z^* to maximize its present discounted value, $V - s$, subject to equations (6), (7) and (8).[9] This problem can be written as,

$$\max_{k,b,z^*} \left\{ \beta \int_{z^*}^{\infty} (zk^{\alpha} - b + G(z))\, dH(z) - k + \chi q(k,b) b \right\}, \tag{10}$$

subject to,

$$\begin{aligned} q(k,b) &= \beta \left(\int_{z^*}^{\infty} dH(z) + \int_0^{z^*} \theta \frac{zk^{\alpha} + G(z)}{b} dH(z) \right), \\ z^* k^{\alpha} + G(z^*) &= b. \end{aligned}$$

We can rewrite this by substituting out b,

$$\begin{aligned} \max_{k,z^*} \quad & \left\{ \beta \int_{z^*}^{\infty} (k^{\alpha}(z - z^*) + G(z) - G(z^*)) dH(z) \right. \\ & \left. + \chi \beta \left((z^* k^{\alpha} + G(z^*)) \int_{z^*}^{\infty} dH(z) + \int_0^{z^*} \theta (zk^{\alpha} + G(z)) dH(z) \right) - k \right\}, \end{aligned}$$

or

$$\max_{k,z^*} \left\{ \beta k^{\alpha} E(z) + \beta (\chi - 1) \left(z^* k^{\alpha} + G(z^*) \int_{z^*}^{\infty} dH(z) \right) + \beta (\theta \chi - 1) \int_0^{z^*} (zk^{\alpha} + G(z)) dH(z) - k \right\}.$$

The first term in this expression is the expected discounted operating profit, i.e. $\beta k^{\alpha} E(z)$. The second term reflects the expected tax shield benefits in non-default states. The third term reflects the expected bankruptcy costs, net of tax shield benefits, in default states. The last term is the

[9] In this section, we assume that there are no equity issuance costs.

cost of investment.

It is easy to check that if there are no bankruptcy costs, $\theta = 1$, then it is optimal to finance with debt only. Inversely, if there is no tax shield, $\chi = 1$, then an all-equity financing is optimal. We assume that $\chi\theta < 1$, i.e. bankruptcy costs are larger than the tax shield effect conditional on default. This assumption is necessary to generate the standard trade-off.

The program (10) can be solved by writing the two first order conditions, with respect to k and z^*, which determine the optimal investment and financing. The first-order condition with respect to k yields,

$$1 = \beta\alpha k^{\alpha-1}\left\{E(z) + (\chi - 1)\, z^*\,(1 - H(z^*)) + (\theta\chi - 1)\int_0^{z^*} z dH(z)\right\}. \tag{11}$$

In the case with $\chi = \theta = 1$, we obtain the usual user cost rule: the expected marginal product of capital is equal to the cost of capital, $1 = \beta\alpha k^{\alpha-1}E(z)$. When $\theta < 1$ or $\chi > 1$, the user cost needs to be adjusted to reflect expected bankruptcy costs (which increase the user cost) and the tax shield (which decreases it).

The second first-order condition, with respect to z^*, yields, after rearrangement,

$$(\chi - 1)\,(1 - H(z^*))\, k^{*\alpha} = \chi\,(1 - \theta)\,(z^* k^{*\alpha} + G(z^*))\, h(z^*). \tag{12}$$

This equation determines the optimal z^*, and hence the probability of default $H(z^*)$. Intuitively, we can think of the choice of z^* as a leverage choice. The left side is the marginal benefit of leverage, which is the higher tax shield in non-default states, while the right side is the marginal cost of leverage, i.e. the increase in expected bankruptcy costs, which depends on the probability of having z close to z^*. Under some regularity conditions on the distribution h, equation (12) determines a unique default cutoff z^*, given the parameters χ and θ and the distribution h.[10] Equation (11) then determines the investment choice k.

[10] The technical condition (which we assume from now on) is that the function $z \to \frac{zh(z)}{1-H(z)}$ is increasing. This condition is standard in contracting problems. For instance, Bernanke, Gertler and Gilchrist (1999) make the same assumption. Most distributions satisfy this assumption, e.g. the log-normal distribution satisfies it. In our numerical example, we will use the log-normal distribution.

We define Q as the total market value of the firm, divided by its capital stock,

$$
\begin{aligned}
Q &= \frac{\text{market value(equity)} + \text{market value(debt)}}{k}, \\
&= \frac{V + \chi q(k,b)b}{k}.
\end{aligned}
$$

Note that we include the subsidy received, i.e. we count the total debt finance raised by the firm in the first period.[11]

To understand the model, Figure 3 illustrates the optimal choice of debt. More precisely, we present some firm outcomes, if the firm borrows an amount b, and makes the optimal investment k^*. The optimal level of debt b^* is denoted with a vertical line. Before picking b^*, however, the firm contemplates financing with a different amount $b \neq b^*$, and this figure illustrates the consequences of choosing a different debt level b.

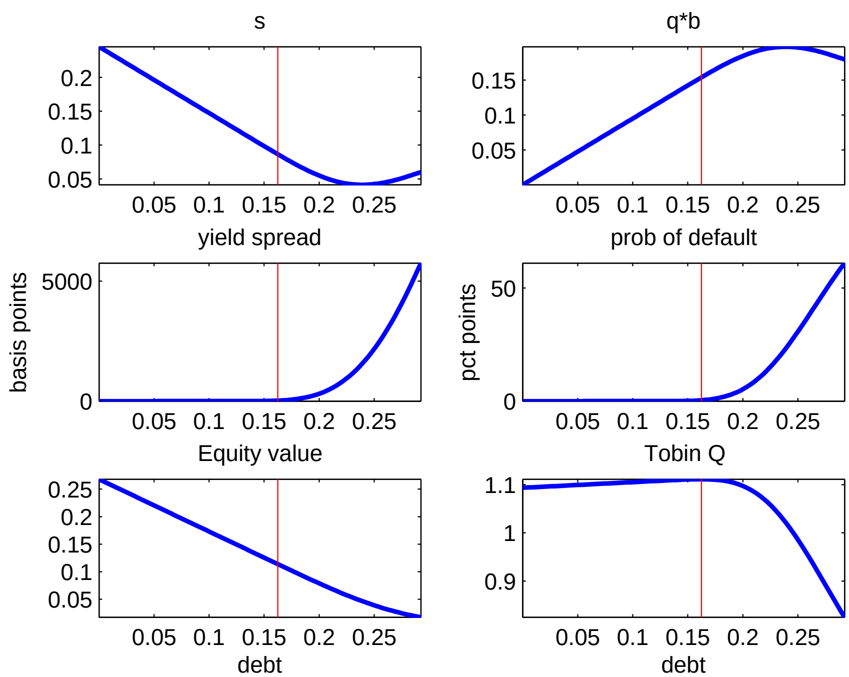

Figure 3: **Optimal Choice of Debt.** The figure shows the effects of debt b on the firm financing options s and $q * b$, the yield spread, the probability of default, the equity value V, and Tobin's Q in the baseline model. The vertical red line shows the optimal debt choice.

[11] In practice the tax shield affects future profits and thus equity value, which is why we need to include it in the firm value. Also, contrary to the empirical literature, we use the market value of debt rather than the book value. It is the theoretically cleaner definition, and the quantitative difference turns out to be small.

As illustrated in the first panel, as the firm increases its debt issuance, it simultaneously reduces the equity issuance, since the investment is fixed at k^* in this experiment. The second panel illustrates a “Laffer curve”: as the face value of debt increases, the amount actually raised first increases nearly one-for-one, then less than one-for-one, and finally falls as default becomes more likely. Default implies bankruptcy costs which will reduce the value available to creditors, hence at some point a decrease in the total amount raised. Obviously the firm never decides to increase debt beyond the maximum of this curve. Indeed, the third and fourth panel document that the probability of default, and hence the credit spread, has a “hockey stick” pattern as debt is increased, and the firm limits its debt issuance to remain safely left of the sharp increase. The last panel shows that the firm actually picks the debt level to maximize Q: ex-ante, the firm's managers decide on financial policy to maximize the total firm value (i.e. we can think of the manager as deciding on debt and equity issues so as to maximize the value he can raise by selling the firm in both debt and equity markets).

We can now use this “toy model” to perform comparative statics. As is well known, an increase in the recovery rate θ reduces the expected bankruptcy costs, and hence, according to equation (12), leads to higher leverage z^*, and, according to equation (11), to a higher capital stock. The probability of default $H(z^*)$ also rises, leading to a rise in the spread. The equity value falls as firms substitute debt for equity. Similarly, an increase in the tax shield parameter χ reduces the user cost of capital and hence leads, according to equation (11), to a higher capital stock, higher debt, as well, a higher leverage z^*, and hence a higher probability of default, higher spreads and higher equity value. Last, an increase in the mean of z leads to a higher capital, equity value, and debt, but does not affect z^* or the probability of default. Table 5 summarize the result of the comparative statics.

	k	b	z^*	Pr(default)	Spread	V
θ	+	+	+	+	+	−
χ	+	+	+	+	+	−
Mean of H	+	+	0	0	0	+
Risk of H	−	−	+	+	+	+

Table 5: **Summary of the Comparative Statics.** Effect of an increase of each parameter on the endogenous variables.

We now turn to the main experiment of this section, the effect of a mean-preserving spread of the distribution H. Because the analysis depends on the exact shape of the distribution function, no analytical result is available, but we consider some numerical examples. Specifically, we assume that z is log-normally distributed, with mean $-\frac{1}{2}\sigma^2$ and variance σ^2. As a result, an increase in σ is a mean-preserving spread of H.[12]

Figure 4 shows the responses of the endogenous variables as we increase σ. The specific parameter values used for this example are: $\beta = .95, \alpha = .9, \theta = .5, \chi = 1.03, \Gamma(z) = .4 * z$ but the results appear extremely robust to changes in these parameter values. Intuitively, the increase in volatility increases the probability of default, and hence the expected bankruptcy costs, making debt less attractive. This leads the firm to reduce its leverage, which lowers the probability of default, but does not completely offset the effect of a higher σ. The higher probability of default naturally pushes up the yield on the debt. Last, higher bankruptcy costs increase the user cost, which generates a reduction in the capital stock. Last, Tobin's Q rises with volatility. This numerical example suggests that shocks to volatility can generate the pattern of correlations needed to match the empirical evidence outlined in the introduction.

In the absence of a growth option, the model yields the same implications for all variables, except one: Tobin's Q is now constant as the volatility changes. It is easy to show that, if $\gamma = 0$, then $Q = \frac{1}{\alpha}$. Intuitively, as σ rises, both the capital stock k and the firm value fall, but their ratio Q is unchanged. This feature of the model, however, is highly specific to this example: the marginal value of capital is proportional to the average value, which is pined down by the required return on equity. While the growth option was introduced exogenously in this simple model, it arises naturally in a dynamic model as good realizations of profitability shocks allow firms to invest further. Hence, in our quantitative model (see Section 2), the growth option will be endogenously

[12] For any σ, we have $E(z) = 1$ and we can obtain simple formula for the integrals appearing in these formulas. In particular, we use the result that if $\log z$ is $N(\mu, \sigma)$, then the cumulative distribution function of z is $H(z) = \Phi\left(\frac{\log z - \mu}{\sigma}\right)$, where Φ is the standard normal CDF. Moreover the probability distribution function is,

$$h(z) = \frac{1}{\sigma z}\phi\left(\frac{\log z - \mu}{\sigma}\right) = \frac{1}{\sigma z}\frac{1}{\sqrt{2\pi}}\exp\left(-\left(\frac{\log z - \mu}{\sigma}\right)^2\right),$$

where ϕ is the standard normal PDF. Last, we have for all x,

$$\int_0^x zh(z)dz = \Phi\left(\gamma - \sigma\right)E(z)$$

with $\gamma = \frac{\log x - \mu}{\sigma}$.

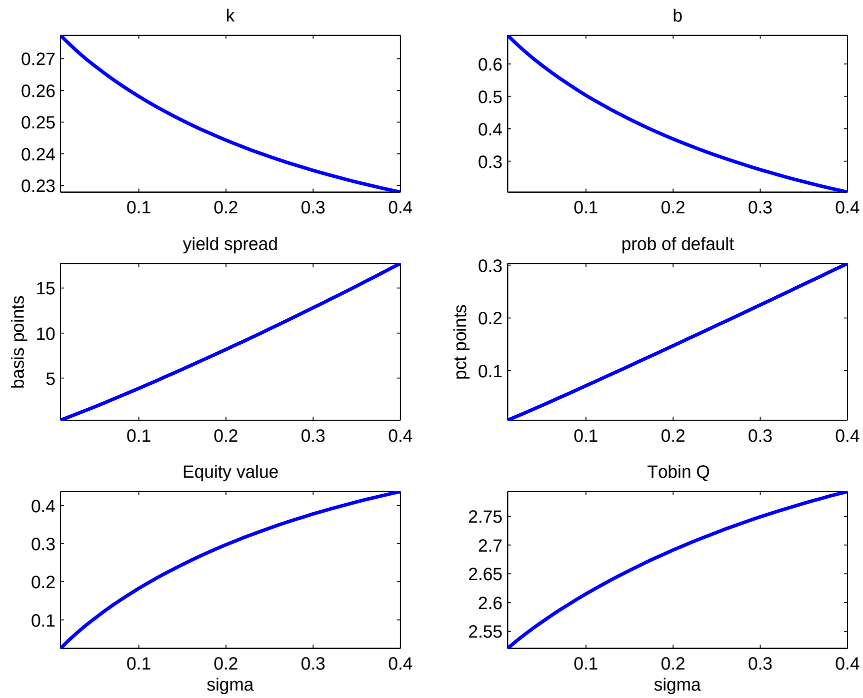

Figure 4: **Comparative Statics.** The figure shows the effects of volatility σ on the optimal firm policies k and b, the yield spread, the probability of default, the equity value V, and Tobin's Q in the model extended with a growth option.

determined by the future profits of the firm.

This two-period model has highlighted the key mechanism of the model. We now turn to a fully dynamic model with more realistic features, to provide a quantitative evaluation of the strength of the mechanism.

Appendix B: Numerical Algorithm

This section presents the numerical algorithm we use to solve our model numerically.

Timing

1. draw s
2. choose k'
3. draw m
4. default or not
5. choose b'

Recursive formulation

Look for $V(k,b,s)$, $g_k(k,b,s)$, $g_b(k,b,s,m,k')$, $\delta(k,b,s,m,k')$, $q(k',b',s)$, such that:

$$V(k,b,s) = \max_{k'} E_m \left(J(k,b,s,m,k') \right)$$

Let

$$J(k,b,s,m,k') = \max \left(0, W(k,b,s,m,k') \right)$$

$$W(k,b,s,m,k') = \max_{b'} \left\{ \Theta \left(d(k,b,s,m,k',b') \right) + \beta E_{s'|s} M(s,s') V(k',b',s') \right\}$$

Default if $\delta(k,b,s,m,k') = 1$, where

$$\delta(k,b,s,m,k') = 1_{W(k,b,s,m,k')<0}$$

$$s'|s \text{ is Markov } Q, \text{ and } m \text{ iid } [-\overline{m}, \overline{m}]$$

$$\Theta \text{ is increasing and concave:} \qquad \Theta' > 0,\ \Theta'' < 0,$$
$$\Theta(x) = x - \frac{\kappa}{2} x^2$$

$$d(k,b,s,m,k',b') = \pi(k,s) + q(k',b',s)\left(b' - (1-\delta_b)\, b \right) - \delta_b b - c(k,k') + m$$

$$
\begin{aligned}
q(k',b',s) &= E_{s',m'}\left(M(s,s')\left(\left((1-\delta_b)\,q\left(k'',b'',s'\right)+\delta_b\right)1_{\delta(k',b',s',m',k'')=0}+\xi\frac{k'}{b'}1_{\delta(k',b',s',m',k'')=1}\right)\right) \\
\text{where } k'' &= g_k(k',b',s'),\ \text{and } b''=g_b(k',b',s',m',k'')
\end{aligned}
$$

Algorithm

0. Define the grids.

1. Make a guess for $q(.)=q(k',b',s)$ and $V(.)=V(k,b,s)$.

2. For each k,b,s,k', compute the list as in LIST ALGO below.

3. For each k,b,s,k', and each interval from step 2, we compute the default thresholds. Suppose that the list implies that if $m\in[m^i,m^{i-1})$ the firm picks b'^i. Define for each $i=1...K$

$$
\Sigma_i(m)=\Theta\left(\pi(k,s)+q(k',b'^i,s)\left(b'^i-(1-\delta_b)\,b\right)-\delta_b b-c(k,k')+m\right)+\beta E_{s'|s}M(s,s')V(k',b'^i,s'),
$$

which is increasing in m. The firm defaults if $\Sigma_i(m)<0$.We compute the values $\Sigma_i(m_i)$, for all i, and check if there is a value such that $\Sigma_i(m^i)\Sigma_i(m^{i-1})<0$. If so, then there is a value of $\widehat{m}\in(m^i,m^{i-1})$ such that default is optimal for $m\in[m^i,\widehat{m})$ and b'^i is $m\in[\widehat{m},m^{i-1})$. Find this value by bisection, and call it $\widehat{m}_i$. If there is no such value, then set $\widehat{m}_i=-10000$.

Note: if $\Sigma_i(m_i)<0$ for all i, then there is default no matter what the value of m is, and if $\Sigma_i(m_i)>0$ for all i there is no default. We hence construct a new list $\left\{m^i,d^i,b'^i\right\}$. For each $i=1...K-1$: if $m<\widehat{m}^i$, $d^i=1$, and if $m>\widehat{m}^i$, $d^i=0$ and $b=b'^i$. This gives a new list. {Or keep the old list and add a second list.}

4. For each k,b,s,k' : we use a grid with M points in m to calculate the expected values. Denote the grid $\{\widetilde{m}_1,\widetilde{m}_2,..,\widetilde{m}_M\}$. We need to compute

$$
\begin{aligned}
A(k,b,s,k') &= \int_{-\overline{m}}^{\overline{m}}\max\left(0,W(k,b,s,m,k')\right)dG(m) \\
&= \sum_{i=1}^{M-1}\int_{\widetilde{m}_i}^{\widetilde{m}_{i+1}}\max\left(0,W(k,b,s,m,k')\right)dG(m)
\end{aligned}
$$

Suppose that there is a shift between $\widetilde{m}_1$ and $\widetilde{m}_2$, so that either the b'^i or the default decision d^i switches, then we appproximate the integral.

(a) For instance, if b'^i switches, we write

$$\begin{aligned}
&\int_{\widetilde{m}_i}^{\widetilde{m}_{i+1}} \max\left(0, W(k,b,s,m,k')\right) dG(m) \\
= \;& \frac{\widehat{m}^i - \widetilde{m}_i}{\widetilde{m}_{i+1} - \widetilde{m}_i}\Omega\left(k,b,s,\frac{\widetilde{m}_i + \widetilde{m}_{i+1}}{2}, k', b'^i\right) \\
&+\frac{\widetilde{m}_{i+1} - \widehat{m}^i}{\widetilde{m}_{i+1} - \widetilde{m}_i}\Omega\left(k,b,s,\frac{\widetilde{m}_i + \widetilde{m}_{i+1}}{2}, k', b'^{i+1}\right)
\end{aligned}$$

$$\Omega\left(k,b,s,m,k',b'\right) = \Theta\left(d(k,b,s,m,k',b')\right) + \beta E_{s'|s} M(s,s') V(k',b',s').$$

Note: what if two switches in the same interval? Not clear. Perhaps just increase M so as it does not happen. [Add a test].

Note 2: try alternative midpoint rules – check if it matters.

(b) For instance, if there is default switch at $\widehat{m}^i$,

$$\begin{aligned}
&\int_{\widetilde{m}_i}^{\widetilde{m}_{i+1}} \max\left(0, W(k,b,s,m,k')\right) dG(m) \\
= \;& \frac{\widehat{m}^i - \widetilde{m}_i}{\widetilde{m}_{i+1} - \widetilde{m}_i}0 + \frac{\widetilde{m}_{i+1} - \widehat{m}^i}{\widetilde{m}_{i+1} - \widetilde{m}_i}\Omega\left(k,b,s,\frac{\widetilde{m}_i + \widetilde{m}_{i+1}}{2}, k', b'^{i+1}\right)
\end{aligned}$$

Now we find the new optimal $k' = g_k(k,b,s)$ and value as

$$V^{new}(k,b,s) = \max_{k'} A(k,b,s,k'),$$

and the optimal b' is obtained as $b' = g_b(k,b,m,s,k') = g_b(k,b,m,s,g_k(k,b,s))$, **defined using the thresholds**.

$$q^{new}(k',b',s) = E_{s',m'}\left(\begin{array}{c} M(s,s')\left((1-\delta_b)\,q\left(k'',b'',s'\right)+\delta_b\right)1_{\delta(k',b',s',m',k'')=1} \\ +\xi\frac{k'-\phi(-k',k')}{b'}1_{\delta(k',b',s',m',k'')=0}\end{array}\right),$$

$$\text{where } k'' = g_k(k',b',s'),\ \text{and } b'' = g_b(k',b',s',m',k'').$$

$$
\begin{aligned}
q^{new}(k',b',s) &= \sum_{s'} Q(s,s')M(s,s') \int_{-\overline{m}}^{\overline{m}} \begin{pmatrix} \left((1-\delta_b)\, q\left(k'',b'',s'\right)+\delta_b\right) 1_{\delta(k',b',s',m',k'')=1} \\ +\xi \frac{k'-\phi(-k',k')}{b'} 1_{\delta(k',b',s',m',k'')=0} \end{pmatrix} dG(m') \\
&= \sum_{s'} Q(s,s')M(s,s') \sum_{i} \int_{\widetilde{m}_i}^{\widetilde{m}_{i+1}} \begin{pmatrix} \left((1-\delta_b)\, q\left(g_k(k',b',s'), g_b(k',b',s',m',k''), s'\right)+\delta_b\right) \\ \times 1_{\delta(k',b',s',m',k'')=1} \\ +\xi \frac{k'}{b'} 1_{\delta(k',b',s',m',k'')=0} \end{pmatrix} dG(m')
\end{aligned}
$$

To calculate this integral, we also distinguish the different cases: first, if there is a switch of b and no default:

$$
\begin{aligned}
& \int_{\widetilde{m}_i}^{\widetilde{m}_{i+1}} \begin{pmatrix} \left((1-\delta_b)\, q\left(g_k(k',b',s'), g_b(k',b',s',m',k''), s'\right)+\delta_b\right) \\ \times 1_{\delta(k',b',s',m',k'')=1} \\ +\xi \frac{k'}{b'} 1_{\delta(k',b',s',m',k'')=0} \end{pmatrix} dG(m') \\
= & \frac{\widehat{m}^i-\widetilde{m}_i}{\widetilde{m}_{i+1}-\widetilde{m}_i} B\left(k', b'^{i}, s', \frac{\widetilde{m}_i+\widetilde{m}_{i+1}}{2}, k''\right) + \frac{\widetilde{m}_{i+1}-\widehat{m}^i}{\widetilde{m}_{i+1}-\widetilde{m}_i} B\left(k', b'^{i+1}, s', \frac{\widetilde{m}_i+\widetilde{m}_{i+1}}{2}, k''\right)
\end{aligned}
$$

Note: the $\widetilde{m}_i$ depend on k', s', b'

$$
B(k',b',s',m',k'') = (1-\delta_b)\, q\left(g_k(k',b',s'), g_b(k',b',s',m',k''), s'\right) + \delta_b
$$

$$
\begin{aligned}
k'' &= g_k(k',b',s'), \\
b'' &= g_b(k',b',s',m',k'') \text{ [stored as a list]}
\end{aligned}
$$

Second, if there is a switch between non-default and default:

$$
\begin{aligned}
& \int_{\widetilde{m}_i}^{\widetilde{m}_{i+1}} \begin{pmatrix} \left((1-\delta_b)\, q\left(g_k(k',b',s'), g_b(k',b',s',m',k''), s'\right)+\delta_b\right) \\ \times 1_{\delta(k',b',s',m',k'')=1} \\ +\xi \frac{k'}{b'} 1_{\delta(k',b',s',m',k'')=0} \end{pmatrix} dG(m') \\
= & \frac{\widehat{m}^i-\widetilde{m}_i}{\widetilde{m}_{i+1}-\widetilde{m}_i} B_d\left(k', b'\right) + \frac{\widetilde{m}_{i+1}-\widehat{m}^i}{\widetilde{m}_{i+1}-\widetilde{m}_i} B\left(k', b'^{i+1}, s', \frac{\widetilde{m}_i+\widetilde{m}_{i+1}}{2}, k''\right)
\end{aligned}
$$

$$B_d(k',b') = \xi \frac{k'}{b'}$$

5. We stop the iteration if $q^{new}(k',b',s)$ and $q(k',b',s)$ **AND** $V^{new}(k,b,s)$ and $V(k,b,s)$ are close enough. If not, we update

$$V^{new\ guess}(k,b,s) = (1-\zeta_1)V(k,b,s) + \zeta_1 V^{new}(k,b,s),$$

$$q^{new\ guess}(k',b',s) = (1-\zeta_2)q(k',b',s) + \zeta_2 q^{new}(k',b',s)$$

where ζ_1 and ζ_2 are relaxation parameters (can set $\zeta_1 = 0$ typically).

LIST ALGORITHM

For each k, b, s, k', we want to find $\left\{-\overline{m} < m^{K-1} < ... < m^1 < \overline{m}\right\}$ and $\left\{b'^K < ... < b'^1\right\}$ such that b'^K is chosen for $m \in [-\overline{m}, m^{K-1})$, b'^{K-1} is chosen for $m \in [m^{K-1}, m^{K-2})$,...,and b'^1 is chosen for $m \in (m^1, \overline{m}]$.

We start with $K = 1$, and $b = b'^1 = 0$, i.e. for all $m \in [-\overline{m}, \overline{m}]$, the firm picks zero debt.

Suppose we have a list $\left\{m^{h-1}...\overline{m}\right\}$ and $\left\{b'^h...b'^1\right\}$ such that if the firm can choose only in the set $b'^1...b'^h$, it chooses b'^h if $m \in [-\overline{m}, m^{h-1})$, b'^{h-1} if $m \in [m^{h-1}, m^{h-2})$, ... and b'^1 if $m \in (m^1, \overline{m}]$.

Step 0: We now compare the value of choosing b'^h with the value of choosing the next higher level of debt, call it b'^+.

Case a. If

$$q(k',b'^+,s)\left(b'^+ - (1-\delta_b)\,b\right) < q(k',b'^h,s)\left(b'^h - (1-\delta_b)\,b\right),$$

then b'^+ is not better than b'^h, and we can go to the next higher value of debt $b'^{++} = b'^+ + 1$. {next step on the grid.}

Case b. Now suppose that

$$q(k',b'^+,s)\left(b'^+ - (1-\delta_b)\,b\right) > q(k',b'^h,s)\left(b'^h - (1-\delta_b)\,b\right).$$

Let

$$\begin{aligned}\Delta^h(m) &= \Theta\left(\pi(k,s)+q(k',b'^{+},s)\left(b'^{+}-(1-\delta_b)\,b\right)-\delta_b b-c(k,k')+m\right)\\ &\quad -\Theta\left(\pi(k,s)+q(k',b'^{h},s)\left(b'^{h}-(1-\delta_b)\,b\right)-\delta_b b-c(k,k')+m\right)\end{aligned}$$

Given that Θ is concave, Δ^h is decreasing in m. There are three subcases:

(a) $\Delta^h(-\overline{m})+\beta E_{s'|s}M(s,s')V(k',b'^{+},s')-\beta E_{s'|s}M(s,s')V(k',b'^{h},s')\leq 0$.Then b'^{h} is at least as good as b'^{+} for all m and we can drop b'^{+} and consider the next one.

(b) $\Delta^h(-\overline{m})+\beta E_{s'|s}M(s,s')V(k',b'^{+},s')-\beta E_{s'|s}M(s,s')V(k',b'^{h},s')>0$,

and $\Delta^h(\overline{m})+\beta E_{s'|s}M(s,s')V(k',b'^{+},s')-\beta E_{s'|s}M(s,s')V(k',b'^{h},s')\leq 0$. By monotonicity and continuity, there exists a unique $\widetilde{m}^h$ in $(-\overline{m},\overline{m}]$, such that

$$\Delta^h\left(\widetilde{m}^h\right)+\beta E_{s'|s}M(s,s')V(k',b'^{+},s')-\beta E_{s'|s}M(s,s')V(k',b'^{h},s')=0.$$

Find it by bisection.

(i) If $\widetilde{m}^h<m^h$, we add $(\widetilde{m},b'^{+})$ to the list of pairs and go to te next b'^{++}(i.e. go to next step 0).

(ii) If $\widetilde{m}^h>m^h$, we delete b'^{h} from the list and check with b'^{h-1}. We define $\widetilde{m}^{h-1}$ such that

$$\Delta^{h-1}\left(\widetilde{m}^{h-1}\right)+\beta E_{s'|s}M(s,s')V(k',b'^{+},s')-\beta E_{s'|s}M(s,s')V(k',b'^{h-1},s')=0.$$

Discard b'^{h-1} if $\widetilde{m}^{h-1}>m^{h-1}$, and iterate on $j=h-1,...,1$, to find $\widetilde{m}^j$ and delete b'^{j} if $\widetilde{m}^j>m^j$. We go to step 0 and go to b'^{++}.

(c) $\Delta^h(-\overline{m})+\beta E_{s'|s}M(s,s')V(k',b'^{+},s')-\beta E_{s'|s}M(s,s')V(k',b'^{h},s')>0$, and

$\Delta^h(\overline{m})+\beta E_{s'|s}M(s,s')V(k',b'^{+},s')-\beta E_{s'|s}M(s,s')V(k',b'^{h},s')>0$, then b'^{+} is always better than b'^{h} and we can drop b'^{h}. As in (b), we iterate down on the list. We go to step 0 and go to b'^{++}.

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