The Design of Mortgage-Backed Securities and Servicer Contracts*

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Abstract

We show that renegotiation of mortgage contracts following default is strictly Pareto improving, but only if the lender is able to obtain updated information about the borrower. We apply this result to determine optimal contracts for servicers of securitized mortgages and to determine the optimal design of mortgage-backed securities (MBS). We show that, if foreclosure is inefficient and servicers are wealth constrained, then MBS's should be formed with pools of nondiversified mortgages. We also present results on the incidence of foreclosure under different security designs. Our optimal contract designs have implications for the resolution of problems with existing MBS's that have experienced significant default, and for the design of new MBS's.

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1 Introduction

In February 2011 the Wall Street Journal published an article about a disagreement among rating firms over the rating of a particular mortgage-backed security (MBS). The stated concern was that the MBS was not sufficiently diversified. An employee of Moody's is quoted as saying: "Given this geographical concentration, there is ... idiosyncratic risk." We contend that this is the wrong approach to securitization of mortgages that exhibit significant default risk. A basic result of portfolio theory is that investors can achieve diversification and elimination of idiosyncratic risk on their own by investing in a portfolio of securities. Our analysis in this paper goes further to argue that there may be a distinct disadvantage to diversification in mortgage backed securities, a "diversification discount".

There are two contracting problems in the design of MBS's: between the original borrowers and investors, and between servicers and investors. The first of these problems is present regardless of whether a loan is securitized. Securitization creates the second problem. In this paper we analyze the joint problems of the design of mortgage backed securities, the servicer contracts and the mortgage contracts.

Our main results follow from a few key assumptions about the nature of mortgages and mortgage-backed securities. We assume that: i) foreclosure is inefficient; ii) at the time that borrowers make their default decisions they have information about their collateral value that is available to the lender only at a cost;[1] iii) investors in MBS's cannot observe all of a servicer's actions and servicers are wealth constrained. The first two assumptions suggest a contracting problem between borrowers and lenders/investors regardless of whether a loan is securitized. The third assumption, however, suggests a contracting problem between servicers and investors. It constrains the ability to align servicer incentives with the interests of investors. We demonstrate that in order to efficiently address the multiple contracting problems that are inherent in securitization, the assets in any given pool should be as similar (returns as highly correlated) as possible. The only risks that should be diversified within the pool are individual risks.

Our work is particularly relevant for the securitization of mortgages that exhibit sig-

[1] We develop a simple model in which collateral value reflects a borrower's general ability to pay.

nificant default risk. Many of the early MBS's were formed by government sponsored entities (GSE's) such as Fannie Mae. A key feature of these agency securitizations is that the GSE's provide guarantees against default risk. But, starting in the late 1990's there has been significant growth in nonagency securitizations that do not provide these guarantees for investors. By 2007 non-agency securitizations accounted for nearly 20% of outstanding mortgage credit.[2] Krainer (2009) points out that the non-agency mortgage securities differ further in that they are more likely to include subprime mortgages. We have thus seen a large growth in mortgage-backed securities that exhibit significant credit risk.

We start our analysis by modeling the one borrower/one lender problem. We recognize that mortgage contracts are not renegotiation proof and we build on work by Aghion, Dewatripont and Rey (1994) and Hart and Moore (1998) to determine optimal renegotiation offers with one borrower and one lender. The renegotiation design explicitly takes into account the trade-off for the lender between the benefit of avoiding costly foreclosure and any wealth transfer to borrowers when renegotiating with defaulting borrowers. Given the information asymmetry between borrowers and lenders, this wealth transfer may come from making a concession to a borrower that is larger than the minimum necessary to avoid foreclosure and from borrowers strategically defaulting in order to obtain a concession from the lender. In order to simplify the analysis we assume a two-period debt model in which the underlying collateral (house) is the only asset that a lender can seize in the case of nonpayment. It is clear that a borrower will default if the value of this collateral has fallen below the required debt payment.[3] It is not as clear what the borrower will do if the collateral value is higher than the required payment. Strategic default in this model is defined as default that occurs not because the collateral value has dropped too low (or because a borrower is unable to pay), but because a borrower is exploiting an information

[2] Quoted from Krainer (2009), page 2.

[3] In practice, many borrowers with "underwater" mortgages, mortgages in which the principal balance exceeds the current house value, have not defaulted. Foote, Gerardi and Willen (2008) and Krainer and LeRoy (2010) provide evidence to this effect. These authors point out that borrowers who have the cash to continue servicing their mortgages hold an option. If the value of the house rises, then they gain; if it drops they can put the house to the lender later. In our simple model we assume away these dynamic aspects. We also ignore any costs to default that would lead a borrower to default only if the principal is some fixed amount above the house value. Adding these complexities will not change our main qualitative results.

asymmetry in an attempt to obtain a concession from the lender. In our analysis we explicitly consider the connection between renegotiation policies and strategic default.[4]

We obtain results that link the renegotiation process, the likelihood of foreclosure and the value of the mortgage bond to the quality of information gathering on the part of the lender. We find for example that if in equilibrium the lender will do no information gathering following a default, then the lender will optimally refuse to renegotiate with borrowers who default. In this case only nonstrategic defaults occur, and all of these defaults result in foreclosure. If it is common knowledge that the lender will do some information gathering following default, then the lender will renegotiate with defaulting borrowers. The extent of renegotiation is increasing, and the incidence of foreclosure and strategic default is decreasing, in the quality of information gathering. We then model the value of the original mortgage bond and we show that the availability of credit is increasing and the cost of credit is decreasing in the quality of information gathering on the part of the lender.

We next consider the contract problem between a servicer and the investors in a mortgage-backed security. We assume that there are N different types of mortgages and N of each mortgage type. A diversified MBS contains N mortgages, each of which is a different type. A nondiversified MBS contains N mortgages of one type. We show that diversification exacerbates the agency problem between the servicer and investors in the MBS. In a diversified MBS all information that is relevant for loan renegotiation is mortgage-specific (as opposed to pool-wide). The servicer thus has control of the information gathering and the investors cannot directly verify whether information has been gathered. The only way to align the servicer's incentives with those of the investors in a diversified MBS, so that renegotiation can proceed efficiently, is to compensate the servicer by paying him a share of the MBS proceeds. The incentive-compatible share may be quite large, but if the servicer is not wealth constrained, then the servicer can be required to pay a lump sum amount in exchange for the incentive compatible share of the MBS.

[4]The term "strategic default" is often used in practice to refer to default by a borrower who has sufficient cash flow to make the required payments, but chooses not to pay because the collateral value has fallen below the debt principal. The term is also sometimes used when discussing concerns about offering loan reworkings that might encourage defaults. Our use of the term is most consistent with this latter usage.

If the servicer is sufficiently wealth constrained, then the incentive compatible contract is prohibitively expensive. In this case the servicer is not given an incentive-compatible contract and information is not gathered following defaults. Following from our earlier results this means that if servicers are sufficiently wealth constrained, then in diversified MBS's the investors will optimally refuse to renegotiate with any defaulting borrowers, and all defaults will result in foreclosure.

In contrast, if the MBS is nondiversified, then the investor can obtain (or verify) pool-wide information that is relevant for all of the mortgages in the MBS. Making renegotiation decisions based on pool-wide information is not as efficient as making such decisions based on mortgage-specific information, but it is more efficient than not having any renegotiation-relevant information at all. If servicers are sufficiently wealth constrained, then the value of the original mortgage bonds will be higher if they are securitized into nondiversified, instead of diversified, MBS pools. This means that the availability of credit will also be higher and the cost of credit lower.

We next expand the model to allow the collateral values to be endogenous. We take into account the potential contagion effect in home mortgages: foreclosures can adversely affect the values of similar houses, possibly leading to more foreclosures, which then leads to further lowering of house values and further foreclosures. In the presence of such contagion effects, all investors are better off if they can coordinate and agree to limit the number of foreclosures. But, if mortgages have been securitized into diversified MBS's, then investors are faced with a classic prisoners' dilemma problem in which coordination is not an achievable equilibrium, and the number of foreclosures is strictly greater than what investors collectively prefer. Organizing mortgages into nondiversified MBS's increases the chance that Pareto improving coordination can be achieved.

The remainder of the paper is organized as follows. In the next section we discuss the related literature. In Section 3 we consider a single representative borrower and a single lender, and we solve for the optimal debt contract and the optimal renegotiation policy. In Section 4 we assume that $N \times N$ mortgage loans are securitized into N mortgage-backed securities. In Section 4.1 we consider diversified MBS pools and we determine the optimal servicer contract. In Section 4.2 we consider non-diversified MBS pools. In Section 5 we present results on contagion in foreclosures. Section 6 includes a summary of our work and discussions of our predictions regarding foreclosure and the relation between our work and tranching.

2 Related Literature

Ours is the only work we know of that jointly considers the design of mortgage-backed securities, sevicer contracts and the renegotiation of mortgage contracts. A key aspect in which our work differs from earlier studies of security design is that we analyze an agency problem, renegotiation of mortgages and strategic default, that occurs *after* securitization takes place, instead of before.

Our result that there is a "diversification discount" in mortgage-backed securities is in contrast to the results of DeMarzo (2005) and Riddiough (1997). DeMarzo (2005) considers the adverse selection problem that occurs when an intermediary pools mortgages and sells them to investors.[5] He also considers mortgages for which the main risk is prepayment risk, not default risk. In this context DeMarzo finds that forming a diversified pool of mortgages, and possibly also tranching, is optimal. In our paper we consider mortgages for which the main risk is default risk and we focus on the moral hazard problems that occur due to the confluence of mortgage contracting and servicer contracting problems. Riddiough (1997) also finds that diversification is optimal for reasons similar to DeMarzo's, and he considers loan renegotiation, but in a manner that is very different from ours. Riddiough's main focus is to determine which tranch claimaints should decide on renegotiation. He assumes away any possibility of strategic default and assumes that one claimant is perfectly informed about collateral value at the time of default. In our paper the possibility of strategic default and the need to gather information prior to renegotiation are central to our results.

Our nondiversification result is also in contrast to the main result of Diamond's (1984) paper on delegated monitoring. Diamond finds that diversification within the financial intermediation organization (i.e., across borrowers) is optimal because it lowers the cost of monitoring the individual borrowers. Diamond thus argues that the loan portfolio for a financial intermediary should be as diversified as possible. Diamond, however, does not

[5]The paper considers two types of adverse selection risk: when the intermediary is better informed than investors, and when investors are asymmetrically informed.

consider a wealth constraint for the intermediary. Hartman-Glaser, Piskorski and Tchistyi (2010) argue that a nondiversified pool can make it easier for investors to assess the quality of originators' screening efforts that take place prior to forming a MBS. Loutskina and Strahan (2008) examine diversification versus concentration in a slightly different context. They find empirical evidence that mortgage originators who concentrate tend to retain more mortgages and have higher profits as compared to diversified lenders.

Analogous to our work on MBS's, Bolton and Oehmke (2011) examine the effects of credit default swaps (CDS's) on renegotiation between corporate debtors and creditors. They find that CDS's increase creditors' bargaining power in renegotiation, resulting in ex-ante benefits for borrowers in the form of greater debt capacity. However, over-investment in CDS's can create ex-post costs in the form of inefficient renegotiation.

Parlour and Plantin (2008) examine the relation between loan sales by originators and related moral hazard problems. Their focus, however, is on the incentives of a loan originator to monitor prior to selling the loan, rather than on servicers' incentives to obtain information needed for renegotiation after the loan has been sold.[6] Posner and Zingales (2009) present a simple loan modification plan that is automated and based on average house prices within a zip code rather than on individual house prices. The method in which loans are renegotiated in our nondiversified MBS design is consistent with their plan.

The servicer contracting problem has been studied in the last few years, but not in conjunction with the design of the MBS or the mortgage contract. Cordell, Dynan, Lehnert, Liang and Mauskopf (2008) provide a lot of useful information regarding the servicing of securitized mortgages. Our choice of modeling assumptions has been somewhat guided by this paper. They state for example that, "Given loss rates to investors from foreclosed subprime mortgages of 50 percent or more, both investors and borrowers could be better off with more effective loss mitigation" (p.3). They also state, "Servicers of loans in private-label MBS do not have strong financial incentives to invest in additional staff or technology for loss mitigation" (p.13). Eggert (2004) describes bad servicing practices. Gan and Mayer (2007) describe ways in which servicers' incentives can deviate from investors'. Pennington-Cross and Ho (2006) look for variation across servicers for quality of

[6]They also assume that debt contracts are renegotiation proof.

servicing. They find that there is variation with regard to the probability of loan default or prepayment. Gelpern and Levitin (2009) describe attributes of mortgage contracts and pooling and servicing agreements that lead to inflexibility and thus make it difficult to avoid foreclosures.

There is potentially conflicting evidence on securitization, foreclosure rates and loan modifications. Piskorski, Seru and Vig (2010) compare mortgage loans that are securitized with loans that are held in portfolio by servicers (not securitized). They find that being held in portfolio reduces the likelihood of foreclosure. They also find that this result is stronger in periods of house price declines, a result that is consistent with our model of contagion. Adelino, Gerardi and Willen (2009), using the same data set, examine loan modifications. They find that securitized mortgage loans are not modified less than nonsecuritized mortages. Agarwal, Amromin, Ben-David, Chomsisengphet and Evanoff (2011) on the other hand do find that bank-held loans are more likely to be renegotiated than securitized loans. There is evidence consistent with our modeling approach of treating default decisions as endogenous and dependent on loan renegotiation policies. Mayer, Morrison, Piskorski and Gupta (2011) present evidence that homeowners respond to mortgage modification programs by increasing their default rates.

3 Optimal renegotiation of mortgage contracts with one borrower and one lender

In this section we develop the basic results on renegotiation of mortgage contracts. Renegotiation involves a cost-benefit tradeoff for the lender, where the benefit is avoidance of costly foreclosure and the cost is a wealth transfer to the borrower. This cost is made larger if renegotiation is done in a way that encourages strategic default by the borrower. The main result of this section is that loan renegotiation is strictly Pareto improving, but only if the lender gathers information about the borrower, subsequent to the time of the original contracting. If no information gathering will be done, perhaps because it is prohibitively expensive, then the lender will optimally refuse to renegotiate. We also show that the amount that a borrower can borrow is increasing, and the terms of the loan are improving, in the quality of post-contracting information gathering by the lender.

The model that we develop in this section is one of incomplete collateralized debt contracts with renegotiation. Incompleteness in mortgage contracts is due to the inability of the contracting players to write enforceable contracts on relevant information such as the borrower's available cash flow and the house's value. In some cases this inability is due to asymmetries of information between the borrower and lender, but even with no information asymmetries, the information cannot be verified by a third party in order to make such contracts enforceable.[7] Because contracts are incomplete, both parties may wish to renegotiate in the future. Thus, the debt contracts are not renegotiation proof. Having contracts that are not renegotiation proof is often considered a problem in contracting. This is because threats of negative consequences for parties who break the contract lose their credibility. Aghion, Dewatripont and Rey (1994), however, have pointed out that if the original contract specifies how renegotiation will work, then renegotiation may be beneficial. The Aghion, et al solution assigns all of the bargaining power in renegotiation to one party and specifies a default outcome in the case that renegotiation fails. We follow this strategy in our model of mortgage debt contracts.

Collateral (the house) plays a pivotal role in mortgage contracts and it is important to understand the role of this collateral and the reason for the collateral. In models of asymmetric information (Gale and Hellwig (1985) and Diamond (1984)) inefficient collateral serves as a sorting mechanism between low- and high-risk borrowers. Low-risk borrowers post collateral and high-risk borrowers find it too costly to imitate them. However, Bester (1994) pointed out that, if renegotiation is possible, then the high-risk borrowers will imitate the low-risk borrowers, because they know that ex post both parties will want to renegotiate to avoid an inefficient transfer of collateral. That is, if debt contracts are not renegotiation proof, then the sorting role of collateral breaks down. What we observe in mortgage markets is consistent with this breakdown. In mortgage markets all borrowers, high- and low-risk, post collateral. The best explanation for the use of collateral is simply the inherent incompleteness of mortgage contracts. Even if the lender can observe the borrower's current and future available cash flow, this cannot be verified in a way such that the borrower can take possession. (Human beings cannot be acquired in the same way that corporations can.) A contract can be written that

[7]Our notion of incompleteness is the same as that used by Hart and Moore (1988).

specifies the transfer of collateral (the house) in the case that the borrower violates any of the contract terms.[8] Because the transfer of collateral to the lender is inefficient, renegotiation may occur if the borrower defaults.[9] Transfer of collateral, i.e., foreclosure, serves as the default outcome in case renegotiation fails.

There are two time periods in our model, 0 and 1. At time 0 the initial contract is signed. This contract specifies the following cash flows between the borrower and lender: the time 0 amount loaned to the borrower and the time 1 promised cash flow from the borrower to the lender. The contract specifies consequences if payment is not made. The contract may also specify how renegotiation may be done. If foreclosure occurs at time 1, then the lender realizes a payoff of δv, where v is the realized collateral value and $\delta \in (0,1)$. The deadweight loss in foreclosure is thus $(1-\delta)v$. We employ the following notation:

$$\begin{aligned}
r_0 &= \text{time 1 promised payment from borrower to lender, based on time 0 contract} \\
\tilde{v} &= \text{time 1 value of collateral (house)} \\
v_0 &= \text{time 0 value of collateral (house)} = E_0[\tilde{v}] \\
\delta &= \text{foreclosure discount factor, } \delta \in (0,1)
\end{aligned}$$

The contract cannot be written on the realization of $\tilde{v}$. Even if this realization can be observed by both the borrower and lender, it cannot be verified by a third party. The model looks somewhat like Townsend (1979), but with the following differences: i) we assume inefficient collateral, i.e., the possibility of foreclosure where some value is lost; ii) even if the lender can at a cost determine the borrwer's cash flow, contracts cannot be written on this amount, and the lender cannot take possession of an individual borrower in the way that a lender can take possession of a corporation. We thus ignore cash flow altogether because in a two period model it is irrelevant. This model is also similar to that of Hart and Moore (1998).[10]

[8] In practice the use of collateral is complicated by the fact that noncontractible actions by the borrower, such as maintenance, affect the value of the collateral. Such concerns increase the potential benefit of renegotiating with borrowers even prior to default. In our simple two-period model the benefit to renegotiation is due solely to avoiding costly foreclosure. But, our main results do allow for renegotiation prior to default and so they would be strengthened if we extend the model to multi-periods.

[9] Hart and Moore (1998) take a similar approach. They have collateral due to contract incompleteness and they have renegotiation due to the inefficiency of transferring the collateral.

[10] Hart and Moore have three periods, but in their model the collateral has no remaining value in the

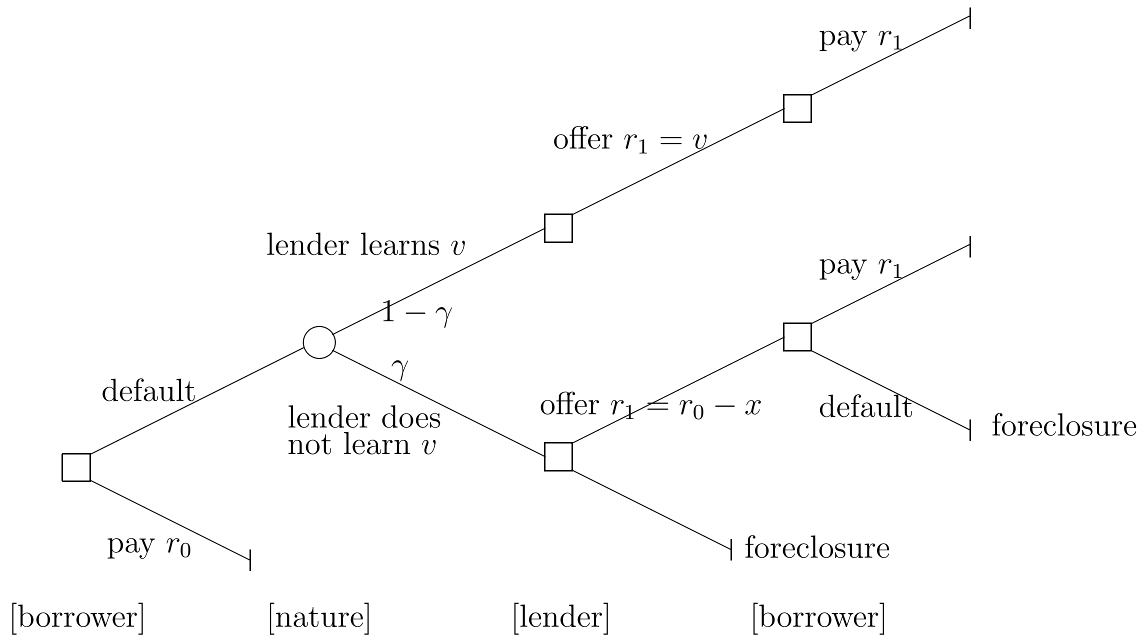

Figure 1: The time 1 game tree

At time 1 the borrower observes the realization v. The game that is played at time 1 is illustrated in Figure 1. The borrower moves first. If she makes the promised payment, r_0, then she keeps the collateral, realizes a payoff of $v - r_0$ and the game ends. If the borrower defaults, then the lender observes the realization v with probability $1 - \gamma$.[11] For the purposes of solving this game we treat γ as an exogenously given parameter and we solve the game for different values of γ. We assume that the value of γ is common knowledge.

After realizing the outcome of the information gathering the lender decides whether to renegotiate with the borrower. In renegotiation the lender makes a take-it-or-leave-it renegotiation offer, r_1, to the borrower. If the borrower accepts the offer, then she pays r_1 to the lender. If the offer is refused, then foreclosure occurs.[12] The lender has all of

final period so payment is made only in one period as in our model.

[11] If the borrower has paid r_0, then it is irrelevant whether the lender observes v.

[12] The typical mortgage contract effectively assigns the bargaining power to the lender, but in practice the borrower has some bargaining power due to the fact that legal systems in many US states prevent foreclosure from taking place immediately. (Boston Globe article February 4, 2011.) The model can be expanded along the lines of Hart and Moore (1998) so that with probability b_L the lender has all of the bargaining power in renegotiation, and with probability $1 - b_L$ the borrower has all of the bargaining power. (Because we are assuming risk neutrality all that matters is expected value.) In the interest of

the power in the renegotiation bargaining game, so if the lender has observed v, then the lender makes a renegotiated offer of $r_1 = v$ and this offer is accepted by the borrower. If the lender has not observed v, then the lender will either foreclose immediately or will make an offer of $r_1 = r_0 - x$, where x may be positive or negative. Following such an offer, the borrower may either accept the offer and pay r_1, or refuse, resulting in foreclosure.

The borrower's last decision in Figure 1 is automatic: if $v \geq r_1$, then the borrower will pay r_1 and realize the payoff $v - r_1$. Otherwise, she will reject the offer, foreclosure will occur and her payoff will be zero. The borrower's first decision, however, is not automatic. It is clear that any borrower who has realized $v < r_0$ will default, because the worst outcome of default is a payoff of zero. It is not at all clear whether a borrower with a realization $v > r_0$ will default. Such a borrower may default in the hope that the renegotiated offer will be less than r_0. We refer to such a default as a "strategic default".[13]

We begin our analysis by considering two polar cases with regard to information gathering by the lender: the case in which the lender always learns a defaulting borrower's realized collateral value v ($\gamma = 0$), and the case in which the lender has a zero probability of obtaining any information beyond the prior distribution on $\tilde{v}$ ($\gamma = 1$)

Our first proposition demonstrates that, if the lender is certain to know the collateral value of any borrower who defaults ($\gamma = 0$), then in equilibrium there will be no strategic default and no foreclosure. This result follows from the logic that a perfectly informed lender will always make a renegotiation offer that is equal to the collateral value: if the offer is higher, then the borrower will refuse the offer; if the offer is lower, then the lender is not maximizing her payoff. In equilibirum there are no foreclosures, because all renegotiation offers are accepted. And, because the lender captures the entire collateral value in renegotiation, the borrower cannot gain from strategic default.

Proposition 1. If in equilibrium the lender obtains perfect information about the collateral value, v, of any borrower who defaults, then foreclosure occurs with probability zero and all defaults are nonstrategic.

succinctness, we assume that b_L =1.

[13]In practice the term strategic default is applied to any case in which a borrower has sufficient cash flow to pay the original contracted amount, but chooses not to pay. In our simple model we use the term for the case in which a borrower behaves strategically, taking advantage of an information asymmetry to increase her expected surplus.

Proof: *If the lender knows the value v, then the lender will make a renegotiation offer of $r_1 = v$ to a borrower who has defaulted. A borrower who has defaulted will accept this offer. Any borrower with $v > r_0$ can only lose by defaulting.* ∎

We next show that there exists a "default cut-off value", v^D, that is a function of the lender's renegotiation policy. Any borrower with a realized collateral value below v^D will default; any borrower with a realized collateral value above v^D will not default. We show further that, if the lender will not obtain any new information about the collateral value of defaulting borrowers, then any possibility of a renegotiation offer with a lower payment ($r_1 < r_0$) leads to a positive probability of strategic default (i.e., v^D strictly greater than r_0).

We know that any borrower with a realized collateral value less than the promised payment ($v < r_0$) will default so as to avoid a negative payoff. Consider a borrower with a realization $v > r_0$. If this borrower makes the promised payment, then her payoff is $v - r_0 > 0$. If the borrower defaults, a strategic default, then either the borrower receives and accepts a renegotiation offer, or foreclosure occurs. If default always results in a renegotiation offer with a lower payment, then all borrowers will default in order to get the lower payment. However, if there is both a positive probability of a renegotiation offer with a lower payment and a positive probability of foreclosure, then the borrower chooses to default only if the expected gain from a lower payment in renegotiation is greater than the expected loss from foreclosure. The expected gain from a lower renegotiated payment is equal across borowers, but the expected loss from foreclosure is increasing in the collateral value, v, of the borrower. Thus, those borrowers with higher collateral values benefit less from strategic default.

Lemma 1. i) There exists a "default cut-off value", v^D, such that any borrower with realization $v < v^D$ will default and any borrower with realization $v > v^D$ will not default. ii) If $\gamma = 1$ (the lender has zero probability of learning v following a default) and if there is any possibility of successful renegotiation with a lowered payment, then $v^D > r_0$. That is, some strategic default will occur.

Proof: *See the Appendix.*

The proof of Lemma 1 shows that the lender can decrease the likelihood of strategic default by randomizing in renegotiation. Randomizing in renegotiation introduces the possibility of foreclosure, which reduces the expected payoff to borrowers who default. Borrowers with high collateral values incur high payoff reductions and so are detered from defaulting. Borrowers with collateral values only slightly above the promised payoff have little to lose and so engage in default as long as loan renegotiation is at all possible.

We now determine the equilibrium outcome for the case in which the lender has zero probability of learning the collateral value of a defaulting borrower ($\gamma = 1$). When deciding whether to make a renegotiation offer and what offer to make, the lender faces a trade-off. If the lender offers r_1 higher than v, then the borrower refuses the offer, foreclosure occurs and the lender receives δv. Alternatively, if the lender offers r_1 lower than v, then foreclosure is avoided, but the lender does not extract all of the surplus from the borrower. In addition, by making any offer $r_1 < r_0$ the lender encourages strategic default. The following proposition indicates that, unless the promised payment, r_0, is high relative to the possible collateral values, then the cost to the lender of strategic default overwhelms the benefit from renegotiation. Note that r_0 determines the proportion of borrowers who default nonstrategically (all those with $v < r_0$) and the proportion who may choose to default strategically, depending on the renegotiation policy (all those with $v \geq r_0$). Only if there is a sufficiently small proportion of borrowers for whom strategic default is possible (r_0 sufficiently high) is it in the lender's interest to offer renegotiation in the absence of information gathering. For the remainder of this section we assume that the collateral value is uniformly distributed: $\tilde{v} \sim U[v_0 - \Delta, v_0 + \Delta]$. The uniform distribution assumption is useful in that it provides quite intuitive results, given the nature of our problem which involves truncations of probability distributions at the default thresholds.[14]

Proposition 2. Suppose that, in equilibrium the lender has a zero probability of obtaining information about the collateral value of any borrower who defaults. There exists a "renegotiation cut-off value",

$$r^{noInf} = \frac{v_0 + \Delta}{2 - \delta}, \tag{1}$$

[14]Most of the results of this section have also been obtained for a triangular distribution. We have chosen not to include these derivations in the paper as they are much more complicated than those employing the uniform distribution assumption, and the main qualitative results are the same.

such that the lender's bond value is maximized if the required payment is set equal to r^{noInf}. If r_0 is less than or equal to r^{noInf}, then the lender will optimally refuse to renegotiate with any defaulting borrower; all defaults will result in foreclosure and all defaults will be nonstrategic. If r_0 is greater than r^{noInf}, then the lender will make the following renegotiated offer to all borrowers: $r_1 = r^{noInf}$.

Proof: *See the Appendix.*

Proposition 2 indicates that, if in equilibrium the lender will not obtain any information about borrowers who default, then for r_0 below a cut-off value the lender will not renegotiate at all, and for r_0 above this value the lender will offer the cut-off value payment to all borrowers.[15] Observe from equation (1) that the cut-off value payment is decreasing in the deadweight loss in foreclosure. This makes sense because greater deadweight losses increase the benefit of renegotiation.

The following Corollary follows simply from the logic that if the lender will offer the cut-off value r^{noInf} to all borrowers whenever r_0 is greater than r^{noInf}, then the initial promised payment will not be greater than r^{noInf}.

Corollary 1. If in equilibrium the lender will not obtain any information about borrowers who default, then the initial mortgage contract will call for a required payment $r_0 \leq r^{noInf}$ and the lender will refuse to renegotiate with any borrower who defaults. Only nonstrategic defaults will occur, and all of these defaults will result in foreclosure.

Proof: *Follows from Proposition 2 and the logic that r_0 will never be set greater than r^{noInf}.*

Corollary 1 states that loan renegotiation will not occur if no new information will be gathered following any default. In order to complete our analysis of the "no information" equilibrium we next determine the amount that a lender will be willing to lend, given that no information will be gathered after a loan has been made.

In order to determine the loan amount, we assume that at time zero there are many potential lenders who compete to loan money to the borrower. Thus, when the original contract is signed, the lender requires only that he break even in expected value. This

[15] Offering $r^{noInf} > r_0$ is equivalent to not renegotiating. With such offers no strategic defaults occur and all renegotiated offers are refused.

means that the lender will loan an amount equal to his time zero expected bond value. From the proof of Proposition 2 we obtain the lender's time zero bond value, given that no information will be gathered following a default:[16]

$$
\begin{aligned}
B^{noInf} &= \frac{r_0(v_0+\Delta-r_0)}{2\Delta} + \frac{\delta(r_0^2-(v_0-\Delta)^2)}{4\Delta} \\
&= r_0 - \frac{1}{4\Delta}\Big((r_0-v_0+\Delta)^2+(1-\delta)(r_0^2-(v_0-\Delta)^2)\Big) < r_0
\end{aligned}
\tag{2}
$$

If foreclosure is fully efficient ($\delta = 1$), then the maximum possible value of B^{noInf}, obtained by setting $r_0 = r^{noInf}$, is v_0. That is, if $\delta = 1$, then the borrower can borrow as much as the expected value of the house, v_0.[17] If, however, $\delta < 1$, then the maximum amount that can be borrowed is strictly less than v_0. In addition, the maximum loan amount is decreasing in $1-\delta$, the proportion of the house value that is the deadweight loss in foreclosure.[18]

At this point we have established that the lender will always renegotiate with defaulting borrowers if the lender has perfect information, and will never renegotiate if the lender is certain to have no information beyond what was available at the time that the original mortgage contract is signed. In both of these polar cases only nonstrategic defaults occur.

We now examine the intermediate case, in which following a default the lender learns v with probability $1-\gamma$, where $0 < \gamma < 1$. Referring back to Figure 1, if the lender learns the defaulter's value v, then the lender demands this full value. For a strategic default this demand results in the lender receiving an amount greater than r_0. In practice this form of punishment is meted out to strategic defaulters by charging fees to borrowers who default. If, instead, the lender does not learn the defaulter's value, then the lender can either foreclose without renegotiating, or the lender can renegotiate by offering a new payment, $r_1 = r_0 - x$. Suppose the lender renegotiates when uninformed. Any borrower who considers strategically defaulting faces a trade-off. With probability $1-\gamma$ she will lose her entire surplus, $v - r_0$; with probability γ she will increase her surplus by x. If x is nonnegative, then the indifference point for this trade-off occurs for a collateral value

[16] The following is based on the assumption that the promised payment is at least as high as the lowest possible collateral value ($r_0 \geq v_0 - \Delta$); otherwise B^{noInf} is trivially equal to r_0.

[17] In the parlance of mortgage loans, the loan-to-value ratio can be as high as one. This result was obtained by combining equations (1) and (2).

[18] See the Appendix.

equal to $r_0 + \gamma x/(1-\gamma)$. This means that any borrower with a realized collateral below the following default cut-off value will default:[19]

$$v^D = \min\left[r_0 + \frac{\gamma x}{1-\gamma}\ ,\ v_0 + \Delta\right]. \tag{3}$$

Any borrower with a realized collateral value above v^D will not default. It is clear from equation (3) that, as the quality of information gathering improves (γ decreases), the default cut-off value approaches r_0. For any borrower considering a strategic default it is the prospect of being discovered, and losing all of one's surplus, that deters the act of strategic default. And, it is the prospect of receiving a concession x that encourages default. The deterrence effect is of course strongest for borrowers with high realized collateral values.

The following proposition indicates that, if there is a positive probability that the lender becomes informed following a default ($\gamma < 1$), then the lender offers a strictly positive renegotiation concession when uninformed.[20]

Proposition 3. If the probability that the lender is uninformed, γ, is strictly less than one, then the lender will renegotiate when uninformed. When informed the lender offers $r_1 = v$. When uninformed the lender offes $r_1 = r_0 - x$, where $x > 0$. If $0 < \gamma < 1$, then the likelihood of strategic default and the likelihood of foreclosure are both decreasing in the probability, $1-\gamma$, of information acquisition by the lender.

Proof: *See the Appendix.*

Combining Proposition 3 with equation (3), it is clear that if $0 < \gamma < 1$ (the lender *may* become informed), then $v^D > r_0$; that is, strategic default occurs. As shown in the proof of Proposition 3, if the lender does not learn the collateral value following a default, then the lender will optimally offer the following renegotiation concession:[21]

$$x(\gamma) = \frac{(1-\gamma)(1-\delta)r_0}{1+(1-\gamma)(1-\delta)}. \tag{4}$$

[19] $v_0 + \Delta$ is the upper bound of the distribution on $\tilde{v}$.

[20] An essential assumption here is that γ is common knowledge. What drives the result is the borrowers' knowledge that the lender may become informed, and the lenders' knowledge that borrowers know this.

[21] For equations (4), (5) and (6) we assume that $r^{noInf} \geq r_0 \geq (1+(1-\gamma)(1-\delta))(v_0-\Delta)$. If $r_0 > r^{noInf}$, then $r_0 - x(\gamma) = r^{noInf}$. If $v_0 - \Delta \leq r_0 < (1+(1-\gamma)(1-\delta))(v_0-\Delta)$, then $r_0 - x(\gamma) = v_0 - \Delta$. These boundary solutions are presented in the Appendix.

If foreclosure is fully efficient ($\delta = 1$), then no concession is offered. This makes sense because it is the prospect of inefficient foreclosure that makes renegotiation attractive to the lender. Inserting $x(\gamma)$ into equation (3), we find that the default cut-off value is:[22]

$$v^D(\gamma) = r_0 + \frac{\gamma(1-\delta)r_0}{1+(1-\gamma)(1-\delta)} = \frac{(2-\delta)r_0}{1+(1-\gamma)(1-\delta)} \tag{5}$$

Strategic default occurs if $v^D > r_0$. It is clear from equation (5) that a necessary and sufficient condition for strategic default is that foreclosure is inefficient ($\delta < 1$) *and* there is a possibility that the lender will be uninformed ($\gamma > 0$).

Proposition 3 indicates that just the prospect that the lender will be informed sufficiently limits strategic default so that the lender is willing to renegotiate even when uninformed. The proposition further shows that better information gathering (higher value of $1-\gamma$) leads to less strategic default and less foreclosure. The final results of this section relate the original mortgage contract terms to the quality of the lender's information gathering.

From the proof of Proposition 3 we obtain the lender's bond value, given that information is obtained with probability $1-\gamma$, where $\gamma < 1$:

$$B^\gamma \;=\; r_0 - \frac{1}{4\Delta}\left(\frac{(2-\delta)r_0^2}{1+(1-\gamma)(1-\delta)} - 2r_0(v_0-\Delta) + (1-\gamma(1-\delta))(v_0-\Delta)^2\right). \tag{6}$$

The bond value is the time zero value that the lender places on the loan, and thus the amount that the lender is willing to loan at time zero. The bond value above is the second-best optimal solution, given the asymmetric information and moral hazard problems inherent in mortgage financing. A first-best world is one in which either foreclosure is fully efficient ($\delta = 1$) or the lender is certain to be informed ($\gamma = 0$). In such a world the expression in (6) simplifies to the first-best bond value:

$$B^{FB} = r_0 - \frac{(r_0-(v_0-\Delta))^2}{4\Delta}. \tag{7}$$

The following proposition presents some characteristics of the second-best bond value.

Proposition 4. For any $\gamma \in [0,1]$ the amount that the lender is willing to lend, B^γ, is less than the time zero contracted payment, r_0. If foreclosure is inefficient ($\delta < 1$) *and*

[22] The assumption that $r_0 \leq r^{noInf}$ ensures that the upper boundary in (3) is not strictly binding.

information gathering is imperfect ($\gamma > 0$), then B^γ is strictly less than the first-best bond value. B^γ is increasing in the foreclosure efficiency level (δ) and in the quality of information gathering ($1-\gamma$) by the lender.

Proof: *See the Appendix.*

In order to determine the value of information gathering we compare B^γ with B^{noInf}, the bond value when $\gamma = 1$. The value of information of quality $1-\gamma$ is thus:

$$B^\gamma - B^{noInf} = \frac{(1-\gamma)(1-\delta)}{4\Delta}\left(\frac{(2-\delta)r_0^2}{1+(1-\gamma)(1-\delta)} - (v_0-\Delta)^2\right) \tag{8}$$

This benefit is clearly decreasing in both γ and δ: better information (smaller γ) is more valuable, and information gathering is more valuable when foreclosure is more inefficient (δ smaller).[23]

4 Securitization, servicer contracts and mortgage contracts

In the previous section we developed the foundation results that enable us to examine mortgage-backed securities. In our model default always occurs when the required loan payment exceeds the value of the house.[24] Strategic default, a default when the house value is greater than the required loan payment, may occur depending on the lender's renegotiation policy. We have shown that a lender will never renegotiate if it is common knowledge that the lender will not obtain any new information about a defaulting borrower's collateral value. In this case all defaults result in foreclosure. We have also shown that there are two types of cut-off values that play key roles in the mortgage problem. For borrowers there is a "default cut-off value", v^D. Any borrower with a realized collateral value below v^D will default; any borrower with a value above v^D will not. The value of v^D (and thus the incidence of strategic default) is decreasing in the likelihood that the lender will become informed of a borrower's collateral value. The second cut-off value is

[23] The benefit of information is calculated as an a priori value, i.e., prior to default. This is the correct way to calculate the benefit because this is a rational expectations model in which borrowers respond rationally to the information policy, γ, when deciding whether to default.

[24] This aspect of the model can easily by revised by adding a cost of default that makes default automatic only when the mortgage is underwater by some strictly positive amount. Doing so, however, will add to the notation without fundamentally changing any of the results.

the lender's renegotiation cut-off value, r^{noInf}. If it is common knowledge that the lender will not gather information about about a borrower's collateral value beyond the original probability distribution, then the lender will optimally choose a required payment of r^{noInf}. In the previous section we assumed that such a lender will never agree to an original promised payment greater than r^{noInf}, thus leading to our result that no renegotiation occurs without information gathering. In this section we apply the concept of r^{noInf} to the case in which a lender does not gather new information about individual borrowers' collateral values, but does obtain new distributional information that is applicable to an entire pool of mortgages. Before doing so, however, we must extend the basic model in order to analyse mortgage-backed securities.

We now consider multiple mortgage loans and we allow loans, or rather the underlying collateral, to be of different "types". There are N different types of loans and N loans of each type. Each loan has two kinds of risk: individual risk and type risk. The type risks are independently distributed across the N types. Individual borrowers of identical type experience identical outcomes of the type risk. Within each type the individual risks, conditional on the outcome of type risk, are independently distributed.

These loans are securitized into N mortgage-backed securities (MBS), each containing N loans.[25] Once a loan has been securitized the lender, now called the investor, no longer interacts directly with the borrower. All renegotiations are carried out between a "servicer" and a borrower, with the servicer acting as the investor's agent. Each MBS has one servicer.

Because the investor is unable to communicate directly with the borrower the investor cannot directly learn the outcome v of a borrower who has defaulted. The servicer can obtain this information with probability $1 - \gamma$, but only after expending a cost, c. We assume that the investor can at a somewhat lower cost learn the outcome of a type risk. But in order for this information to be useful, the investor will need to know the type of a borrower who has defaulted.

[25]Our objective is to evaluate different methods of securitization and servicer contracts. We do not attempt to model the reasons for doing securitization. Both Parlour and Plantin (2008) and DeMarzo (2005) have presented reasons for securitizing loans. Typical reasons for an originator to sell loans have to do with risk transfer so that the originator is able to originate more loans. In our model the reason for pooling rather than selling individual loans may simply have to do with transactions costs and economies of scale. In Section 5, however, we present another reason for forming a pool.

In the following subsections we evaluate the servicer contract and the renegotiation process under two different assumptions regarding MBS design. We first consider fully diversified MBS's. In this design each MBS contains one loan of each type. We then consider nondiversified MBS's, where each MBS contains loans of only one type. The advantage of the diversified design is that the individual loan outcomes are independently distributed, resulting in a lower probability of catastrophic loss, without strategic default.[26] The disadvantage of the diversified design is that the investor is entirely dependent on the servicer to gather information needed for a renegotiation.

The following set-up is applicable to both types of MBS design. At time zero the borrower enters into a contract with the investor.[27] The borrower receives an amount of money, B, and in exchange promises to either pay r_0 at time one, or turn over the collateral (house) to the investor. The loan is then packaged into a mortgage-backed security and thereafter the borrower interacts only with the servicer, the agent of the investor. As in the previous section, at time one the borrower observes the outcome v and decides whether to pay r_0 or default on the loan. However, what happens following a default depends on the contract between the servicer and the investor.

In order to model the contract problem between the servicer and the investor we make the following assumptions:

1. The servicer can, at a cost c, gather information about a borrower's realized value, v. If c is expended, then the servicer gathers information and with probability $1-\gamma$ observes v.

2. The cost c is less than the value of information gathering. That is, $B^{\gamma} - B^{noInf} > c$. Thus, in the absence of any agency problem between the servicer and investor, information is gathered.

3. The investor cannot directly observe the actions of the servicer.

4. The investor is able to observe the total revenues resulting from all payments made by borrowers and all proceeds from foreclosures.[28]

[26] The occurance of strategic default depends on the outcome of the contracting problem. We can thus at this point only say that diversification lowers the probability of large losses from nonstrategic default.

[27] By assuming that the original contract is between the borrower and investor we are assuming away any problems due to the unbundling of origination and funding of mortgages, so that we can focus on the problems stemming from the unbundling of servicing and funding.

[28] The investor may also be able to observe the number of borrowers in the MBS pool who default and the number who experience foreclosure, but as discussed below, this information is not useful for contracting with the servicer.

5. The investor does not observe the identity of any borrower who defaults. The investor is thus unable to identify the type of a defaulting borrower in a fully diversified MBS. In a nondiversified MBS the type is observable.
6. The servicer and the investor are risk neutral. The servicer is wealth constrained.[29]

4.1 Diversified MBS

In this section we assume that the MBS's are fully diversified; each MBS contains one loan of each type. Our objective is to determine the optimal contract between the servicer and investor, for a representative MBS.

The servicer contract problem consists of two parts. The investor must decide whether to assign renegotiation authority to the servicer, and if so, specify the payment to the servicer as a function of observable outcomes. If the investor does not allocate renegotiation authority, then there is no renegotiation and all defaults lead to foreclosure. This last statement is consistent with assumption 5 above that the investor cannot confirm any information that the servicer may report to the investor. We also know from the previous section that it makes no sense to delegate renegotiation authority to the servicer without also providing sufficient incentive for the servicer to gather information about any borrower who defaults. We proceed by first determining the nature of the least cost contract between the servicer and investor that provides the servicer with sufficient incentives to collect information following a default. We next compare the investor's expected payoff under this incentive-compatible (IC) contract to the expected payoff given that renegotiation authority is not delegated so that all defaults result in foreclosure. We then present a condition such that the non-IC contract is optimal resulting in no securitized loans being renegotiated.

A fundamental result of contract theory is that, in an IC contract, the agent's (servicer's) wages should be conditioned on something that is observable by the principal (investor) and is most closely related to the actions of the agent. The investor can observe the total revenues of the MBS. It is also possible that the investor can observe the number of defaults and the number of foreclosures. We assume that the borrowers act

[29]Because we have assumed risk neutrality, if the servicer had no wealth constraint, then the optimal contract would transfer all risk to the servicer, as in Diamond (1984). The wealth constraint, if binding, precludes this.

rationally when making their default decisions, but the borrowers cannot observe the actions of the servicer; they instead condition their decisions on the nature of the contract between the servicer and investor.[30] The level of defaults is thus not dependent on the realized actions of the servicer, and so we rule out the number of defaults as a suitable observable on which to condition the servicer's wages.[31] The servicer's actions following a default do affect the likelihood of foreclosure and the amount remitted by a defaulting borrower if foreclosure doesn't occur. If the servicer expends the cost of gathering information, then the likelihood of foreclosure is lower. We contend, however, that paying the servicer a higher amount for fewer foreclosures will lead to perverse incentives. For example, the servicer can minimize the number of foreclosures simply by not expending any resources to gather information and instead offering very large renegotiation concessions. Such actions are not in the investor's interest. This leaves us with one observable on which to condition the servicer's wages in order to align the servicer's and investor's incentives: the total amount of revenue obtained from the loans in the MBS.[32]

If the servicer expends c for each defaulted loan, then the expected value of the total MBS revenues is $N \cdot B^{\gamma}$, where B^{γ} is the expected value of a single mortgage bond, given that information will be gathered in the case of default. The expected value of the total MBS revenues if the servicer does not gather information is $N \cdot B^{noInf}$.[33] Suppose that the servicer receives a fraction, z, of the total revenues from the MBS. Incentive compatibility is satisfied if:[34]

$$z \cdot N \cdot B^{\gamma} - N \cdot c \geq z \cdot N \cdot B^{noInf} \tag{9}$$

[30]It is this contract that determines whether the default cut-off value, v^D, is equal to r_0 or strictly greater than r_0.

[31]It is thus irrelevant to our problem whether the investor can or cannot observe the number of defaults in a MBS.

[32]Conditioning the wages on the amount of revenue following a default will also not align the incentives. The reason is that the servicer may take actions that will lead to a suboptimal (greater) level of strategic default.

[33]In Section 2 we determined values for B^{noInf} and B^{γ}, given uniformly distributed $\tilde{v}$. In Section 4.2 we return to this uniform example. The results in the current section are presented without reference to any specific distribution.

[34]The following may be interpreted as a sufficient condition, in that it allows for the possibility that all borrowers in the MBS default. We assume that all default decisions are made simultaneously (or without the knowledge of other defaults). If the servicer is willing to expend $N \cdot c$ to gather information, then every prospective strategic defaulter believes that with probability $1 - \gamma$ she will lose her entire collateral value following a default.

The compensation scheme described by equation (9) is sometimes referred to as a "vertical" risk-sharing scheme. If the MBS were divided into multiple tranches with different levels of risk, then the servicer would hold a fraction z of each tranch. In current practice servicers sometimes hold "first-loss" positions. That is, they hold the lowest, riskiest tranch. This may be referred to as a "horizontal" risk-sharing scheme. The problem with such horizontal schemes is that if many borrowers default, then the first-loss position is wiped out and the servicer has no incentive to minimize further losses. As such, horizontal risk-sharing schemes can be useless at exactly the times when they are most needed.[35] It may be possible to form some upper level tranches that are insensitive to the actions of the servicer, but we maintain that at least some degree of vertical risk-sharing is needed to provide proper servicing incentives, especially in times of market downturns. We thus save further discussions of tranching until later in the paper and proceed with the simple vertical incentive scheme that is described by equation (9).

Rearranging equation (9), if the servicer's contract allocates a fraction

$$z = \frac{c}{B^{\gamma} - B^{noInf}} \tag{10}$$

of the MBS revenues to the servicer, then the servicer will have sufficient incentives to gather information. We have assumed that information gathering is efficient in a first-best setting (assumption 2 above), so $z < 1$. Because the servicer is risk neutral this incentive scheme can be applied without paying excess expected wages by adding a fixed negative component to the the servicer's wage contract. In practice this means that the servicer must invest an amount W^{IR} at time zero in exchange for a share z of the MBS. The amount W^{IR} is calculated such that the servicer's incentive rationality constraint is just satisfied:[36]

$$z \cdot N \cdot B^{\gamma} - N \cdot c - W^{IR} = 0 \implies W^{IR} = \frac{N \cdot c \cdot B^{noInf}}{B^{\gamma} - B^{noInf}} \tag{11}$$

Suppose, however, that the servicer is wealth constrained so that the maximum amount the servicer can invest is $\overline{W}$. If $\overline{W}$ is less than W^{IR}, then the servicer's budget constraint is

[35] They may even be worse than useless if they are not completely wiped out, but they leave servicers with incentives to increase risk.

[36] We assume that the servicer's reservation utility is zero. This assumption is consistent with the notion that a servicer who doesn't gather information is free to pursue other opportunities, and c includes the servicer's opportunity costs.

binding and the individual rationality constraint is not. That is, in order to ensure that the contract provides incentives for the servicer to collect information, the servicer must earn an expected wage that is greater than the highest possible cost of gathering information. The investor will find it optimal to offer such an incentive compatible contract if and only if the cost to the investor does not exceed the value of information:

$$\begin{aligned} z \cdot N \cdot B^{\gamma} - \overline{W} \quad &\leq \quad N \cdot \left(B^{\gamma} - B^{noInf}\right) \implies \\ &\overline{W} \geq N \cdot \left(B^{noInf} - (1-z) \cdot B^{\gamma}\right) \end{aligned} \tag{12}$$

If condition (12) is *not* satisfied, then the servicer is *sufficiently wealth constrained*. If the servicer is sufficiently wealth constrained, then the investor optimally offers the servicer a contract that is not incentive compatible (not IC). In this case, information is not collected following default, no loans are renegotiated, borrowers default if and only if $v < r_0$ (there are no strategic defaults), and all defaults result in foreclosure.[37]

Proposition 5. If the servicer for a fully diversified MBS is sufficiently wealth constrained:

$$\overline{W} < N \cdot \left(B^{noInf} - (1-z) \cdot B^{\gamma}\right), \tag{13}$$

then no loans are renegotiated and all defaults result in foreclosure.

Proof: *Follows directly from the above derivations.*

The value of the diversified MBS is thus:

$$\max[N \cdot B^{noInf},\ N \cdot B^{\gamma} - N \cdot c - \max[W^{IR} - \overline{W}, 0]]. \tag{14}$$

The last term above, $\max[W^{IR} - \overline{W}, 0]$, is the excess that must be paid to the servicer because of the agency problem and the servicer's wealth constraint. If the servicer is sufficiently wealth constrained, then the value of each securitized mortgage in the diversified MBS is B^{noInf}, which is less than $B^{\gamma} - c$. Securitizing mortgages in this manner effectively decreases the availability of credit and increases the cost of credit.

[37] If it is possible to form an upper level tranch that is insensitive to servicer actions, then the servicer will need to hold fraction z only of the lower level tranches. This will thus ease, but not eliminate, the wealth constraint.

If servicers are sufficiently wealth constrained, then securitizing mortgages by forming diversified pools effectively makes the mortgages renegotiation proof. It is not in the interest of the investors to enter into renegotiation with borrowers. If for some reason the investors are required to offer renegotiations to defaulting borrowers,[38] then this requirement will result in an excess level of strategic default. This result should be kept in mind when considering programs to rework securitized mortgages.

4.2 Non-diversified MBS

In the previous section we showed that if mortgages are securitized into diversified pools and servicers are sufficiently wealth constrained, then it is not in the interest of investors to offer renegotiation to borrowers who default. In this section we consider nondiversified MBS's. The advantage of this organizational form is that it enables the investor to easily obtain, or verify, some information that is relevant for the entire pool of mortgages. This idea can be related to the notion of "hard" versus "soft" information.[39] In the diversified MBS the relevant information for renegotiation is soft in that it relates only to individual loans and the investor is unable to verify the information. By forming a nondiversified MBS the investor effectively hardens some of the information. Because the type information is the same for all loans in the pool, the investor can verify this information for any borrower. The type information is of course only part of the information that the investor would like to know before making a renegotiation offer. But, as we show below. in some cases, the investor optimally renegotiates after obtaining only type information. In contrast, no renegotiation takes place if the MBS is diversified and the servicer's contract does not provide sufficient incentives to gather information following defaults.

Under the nondiversified security design each MBS contains N loans that have identical type risk. We continue to assume that the investor is unable to communicate directly with borrowers, or determine individual borrower characteristics. But, because all borrowers are of the same type, the investor does know borrower type. The investor may choose to become informed about the realization of the type risk that affects all borrowers in the pool.[40] In a diversified MBS if the investor decides not to delegate renegotiation decisions

[38]Perhaps by government mandate.

[39]See Petersen (2004) for a nice description of hard versus soft information.

[40]This information will not be cost free, but we assume that the cost is low relative to the value

to the servicer, then no renegotiation takes place. In the nondiversified MBS the investor can decide not to delegate renegotiation decisions to the servicer, and instead use the type information to determine his own pool-wide renegotiation policy. Depending on the outcome of the type information, the investor instructs the servicer either to abstain from all renegotiation or to make a uniform renegotiation offer to all borrowers in the pool.[41]

In order to evaluate this new action choice and obtain tractable results we return to, and extend, the uniformly distributed example of Section 3. The time zero uncertainty about the collateral value of each borrower in the pool is described by the following distribution:

$$\tilde{v}_i = v_0 + \tilde{u}_i, \quad \tilde{u}_i \sim U[-\Delta, \Delta], \quad \Delta < v_0, \quad i \in \{1, 2, ..., N\}. \tag{15}$$

At time one each individual borrower i observes the realization of her collateral value, v_i. If the investor does not obtain the type information, then his time one uncertainty continues to be represented by (15). We assume that the outcome of the type risk is u^T, where $u^T \in [-\eta, \eta]$, $\eta < \Delta$. We also assume that conditional on the realization u^T the individual realizations are independently, uniformly distributed. More specifically, conditional on u^T:[42]

$$\tilde{u}_i = u^T + \tilde{\epsilon}_i, \quad \tilde{\epsilon}_i \sim U[-(\Delta - \eta), \Delta - \eta]. \tag{16}$$

If the investor learns the type information, then his expected value of $\tilde{v}_i$ for each borrower in the pool shifts to $v_0 + u^T$, and the size of the range of possible values decreases from 2Δ to $2(\Delta - \eta)$.

Type information can now be used to make a renegotiation offer. We know from the Section 3 results that renegotiation decisions are ideally based on full information about individual borrowers' collateral values. If, however, the servicer is sufficiently wealth constrained and so is not offered an incentive-compatible contract, then such information is not collected. Instead, in the absence of full information about individual borrowers'

of applying this information to all N loans. In the diversified MBS each piece of type information is applicable for only one loan. In addition, this information can be used only by the servicer because the investor in a diversified MBS cannot identify the type of an individual loan.

[41]If renegotiation is offered only to those borrowers who default, then it is in the best interest of every borrower to default, and renegotiation is then offered to all investors.

[42]The type risks $\tilde{u}_i^T$ are of course not uniformly distributed. For our purposes what matters is that the remaining uncertainty whenever the lender is making a renegotiation decision is uniformly distributed. This assumption allows us to determine the optimal renegotiation offer in a tractable manner.

collateral values, type information can be used to make a Pareto improving renegotiation offer to all borrowers within the MBS pool. This statement follows from Proposition 2 of Section 3. That proposition presents the concept of a renegotiation cut-off value, r^{noInf}. If the lender knows nothing beyond the original distribution on a borrower's collateral, then the lender optimally sets the promised payment to r^{noInf}. The initial contract limits the promised payment to be no greater than this amount. After learning type information, u^T, the renegotiation cut-off value changes. If the revised renegotiation cut-off value is smaller than r_0, then both the investor and the borrowers are made better off with a revised lower payment.

The following lemma is a restatement of Proposition 2 for the case in which type information is obtained.[43]

Lemma 2. If the servicer is sufficiently wealth constrained so that there is zero probability of obtaining information about the collateral value of individual borrowers, but the investor has obtained type information, then there exists a renegotiation cut-off value,

$$r^{Type} = \frac{v_0 + u^T + \Delta - \eta}{2 - \delta}, \tag{17}$$

such that the investor's expected value is maximized if all borrowers in the pool have a required payment equal to r^{Type}. If r_0 is less than or equal to r^{Type}, then the investor will instruct the servicer not to renegotiate with any defaulting borrower. If r_0 is greater than r^{Type}, then the investor will instruct the servicer to make the following renegotiated offer to all borrowers: $r_1 = r^{Type}$.

Comparing Lemma 2 with Proposition 2 we see that r^{Type} takes the same form as r^{noInf}. Each is equal to the upper end of the distribution support, divided by $2 - \delta$. The renegotiation offer described in Lemma 2 is clearly Pareto improving. For the borrowers the required payments are either unchanged or decreased. For the investor, as demonstrated in the proof of Proposition 2, the mortgage bond values are maximized if the required payment is set to r^{Type}. Lenders will thus be willing to lend more at time zero because investors will be willing to pay more for the mortgage bonds. In addition, some foreclosures are avoided by setting a lower required payment. We thus obtain the following proposition.

[43] No proof is provided, because the proof is identical to that for Proposition 2.

Proposition 6. If servicers are sufficiently wealth constrained, as specified by condition (13), then securitizing mortgages into nondiversified MBS's instead of diversified MBS's results in the following changes:
i) The maximum amount that a borrower can borrow, relative to the expected collateral (house) value, is higher.
ii) The amount that can be borrowed (bond value), relative to a given promised payment, is higher. That is, the cost of borrowing is lower.
iii) The incidence of foreclosure is lower.
Proof: *From Lemma 2 and the results of Section 3 we know that the initial bond value, as a function of the promised payment and the prior value , v_0, is higher. i) and ii) follow directly. iii) follows directly from Lemma 2.*

Proposition 6 follows directly from the results of Section 2 (single borrower, single lender) and the simple assumption that forming nondiversified MBS's enables investors to obtain type-wide information about the underlying mortgage loans. In Section 4.1 we learned that, if the servicer is sufficiently wealth constrained, then no individual information will be gathered following defaults, either in the diversified or nondiversified MBS design.[44] If MBS's are formed using the diversified design and if servicers are sufficiently wealth constrained, then no renegotiation will take place. Any realized collateral value that is less than the original contracted payment, $v < r_0$, will result in costly foreclosure. In contrast, if MBS's are organized according to the nondiversified design, then some of the foreclosures that would occur in the diversified design are avoided. The renegotiation process in the nondiversified MBS is not as efficient as what would occur if the lenders had full individual information about every defaulting borrower. But the process is elegant in that it requires only pool-wide information and it provides an improvement over the case of no renegotiation.

If MBS's are nondiversified, then some will experience large losses while others will experience relatively small losses. However, the very nature of the securitization enables investors to do their own diversification by holding a diversified portfolio of MBS's. Our

[44]The value of gathering individual information about defaulting borrowers will not be higher under the nondiversified design than under the diversified design. Thus any servicer who is sufficiently wealth constrained under the diversified design will also be sufficiently constrained under the nondiversified design.

analysis indicates that doing diversification within MBS's can result in a "diversification discount" that is caused by the combination of the following factors: i) The nature of mortgage contracts is such that allowing for renegotiation can significantly increase the value of the mortgages, but only if information is obtained about the collateral value. ii) Securitizing mortgages into diversified MBS's exacerbates the agency problem involved in gathering information that is needed for mortgage renegotiation. iii) Organizing mortgages into nondiversified pools enables investors to apply pool-wide information to form Pareto-improving pool-wide renegotiation offers.

5 Contagion

In the previous section we showed that, given the interaction between two agency conflicts, between the borrower and lender and between the servicer and investor, securitizing mortgages into nondiversified pools can strictly dominate diversified securitization. By forming a nondiversified pool the investor effectively hardens some of the information that is useful for renegotiation of mortgage loans. In this section we address another problem in mortgage markets: foreclosure contagion. Contagion occurs because foreclosure adversely affects the value of similar houses, sometimes leading to more foreclosures, which then leads to further reduction in house values and further foreclosures.[45] In this section we argue that securitizing mortgages in nondiversified, rather than diversified, pools can also mitigate this problem.

We have up to this point assumed that the realized collateral value, v_i, for each borrower is exogenously determined. We now allow these values to be influenced by the foreclosure of other properties. We assume that foreclosing on one property has a negative feedback effect on similar properties and may cause additional foreclosures that would otherwise not occur. In the presence of such contagion effects all investors are better off if they can coordinate and agree to limit the number of foreclosures. But, such an outcome may be unachievable if investors (or servicers who are making decisions) do not experience the feedback effects of their foreclosure decisions, that is, if mortgages are organized into diversified MBS's.

[45]See Frame (2010) and Lee (2008) for reviews of literature on foreclosure spillover effects.

The problem just described is a classic "prisoner's dilemma" problem. In order to present this prisoner's dilemma problem in the simplest possible manner we assume that $N = 2$, so that there are just two mortgages of each type. We also assume in this section that servicers' incentives are aligned with those of the investors so that we can focus on the coordination problem. We start by assuming that MBS's are diversified: two mortgages of one type are held in two different securities, with two different servicers.

In this section rather than add lots of new technology and assumptions about the way in which the feedback occurs we have chosen to employ an illustrative numerical example. We do, however, make use of the results we have obtained so far. All of the numbers we employ in the example are consistent with our earlier results. We assume that no information is collected about individual collateral values following a default and that $r_0 \leq r^{noInf}$. From what we learned in Proposition 2 this means that a servicer who is insensitive to any feedback effects will optimally foreclose on any borrower who defaults. For illustrative purposes suppose that the expected payoff for such a policy is 80, if the other property of the same type has not been foreclosed on. If instead the other similar property has been foreclosed on, then the expected collateral value drops significantly and the likelihood of default increases significantly. For illustrative purposes we assume that the expected payoff following the foreclosure of the related property is 40. It is clear that if foreclosures are to take place, then it is best to be the first servicer who forecloses, not the second. Suppose instead that both servicers choose to renegotiate and avoid foreclosure. We know from Proposition 2, that in the absence of any feedback effects, such renegotiation is suboptimal. Thus, the payoff must be less than 80. For illustrative purposes we assume that the payoff is 70.

Figure 2 illustrates the example just described. The top left cell presents the payoffs of 70 to each servicer if both renegotiate. If Servicer 1 deviates and forecloses instead of renegotiating, then the payoffs are represented in the bottom left cell. Servicer 1 obtains 80 for being the first to foreclose and Servicer 2 obtains only 40. The top right cell presents the outcome when Servicer 2 is the sole deviator. If both servicers foreclose without renegotiating, then each has a 50% chance of being the first to complete the foreclosure. Thus, they each obtain expected payoffs of 60. If both servicers are rational and playing a noncooperative equilibrium, then (renegotiate, renegotiate) is not an equilibrium. Each

		Servicer 2 renegotiate	Servicer 2 foreclose
Servicer 1	renegotiate	70, 70	40, 80
	foreclose	80, 40	60, 60

Figure 2: Prisoners' Dilemma Foreclosure problem

servicer has a strictly positive incentive to deviate and attempt to be the first to foreclose. The only equilibrium in the multiple servicer renegotiate/foreclose game is (foreclose, foreclose) with payoffs of 60 for each.

The prisoners' dilemma game describes the situation when mortgages are securitized into diversified pools. If each servicer makes decisions for two unrelated mortgages, then the equilibrium outcome for each mortgage type is (foreclose, foreclose). Each mortgage bond is worth 60 and each MBS has a value of 120. If instead, the mortgages are organized in nondiversified pools, then each servicer makes decisions for both mortgages of a single type. The servicers optimally choose to renegotiate and each MBS has a value of 140.[46]

This example is very simple in that we assumed $N = 2$. In practice there are many more than two of any given type of mortgage, so that even if all MBS's are nondiversified, mortgages of a given type may be distributed across multiple MBS's. However, as long as each MBS contains multiple mortgages of one type, then servicers will recognize some of the negative feedback effects of foreclosure, and they will be willing to offer more generous renegotiation terms in order to limit the occurance of foreclosures. As such, employing a nondiversified design for all MBS's should result in higher overall house values and fewer foreclosures.

[46]Depending on the timing of the payments the servicer may also be able to renegotiate one loan and then forclose on the other, thus obtaining 150.

6 Discussion

Our main argument in this paper is that foreclosure and debt renegotiation decisions are more efficiently undertaken if mortgages are securitized into nondiversified mortgage-backed securites. We thus recommend that diversification be done at the inter-security level, not at the intra-security level. As indicated by the quotes in the first paragraph of this article, our recommendation is in stark contrast to the way in which mortgages are currently securitized.

In this section we provide a general discussion of our results with a focus on practical applications. We begin by discussing our predictions regarding the incidence of foreclosure. We next discuss the implications of our results for tranching. We conclude with a brief summary of our results.

6.1 Incidence of foreclosure

All else equal we predict that the incidence of foreclosure will be lowest with nonsecuritized loans and highest if mortgages are securitized into diversified MBS's. This prediction follows both from our results on information gathering and renegotiation, and from our results on contagion. If originators wholly own the loans they originated, then they are more likely to gather information that enables loan renegotiation instead of foreclosure. In addition, originators who retain loans are likely to have multiple similar loans in their portfolios. They will be more averse to foreclosure due to concerns about contagion.

But, all else may not be equal. It is quite likely that originators choose to hold on to those mortgages for which foreclosure is relatively more efficient (δ closer to one). Also, originators who retain loans may have more local expertise than do MBS servicers, and they do not need to pay large fees to special servicers. All of these factors lead to less costly, and thus higher incidence of, foreclosure. It is thus an empirical question as to which effect dominates and whether securitized mortgages will experience more foreclosures than nonsecuritized mortgages. Piskorski, Seru and Vig (2010) find empirical evidence that nonsecuritized loans are less lilely to be foreclosed. Their evidence is thus consistent with the predictions of our model.[47] There is, however, no evidence that we know of

[47]They suggest an additional reason for this phenomenon. Originators who hold loans may postpone

regarding the relative incidence of foreclosure in more or less diversified securitization pools.

6.2 Tranching

One of the perceived advantages of securitizing mortgages is the ability to create a large number of investment grade securities from a pool of noninvestment grade securities. This magic is performed by forming a well-diversified pool of mortgages and then creating tranches. The lowest level tranches may be quite risky, but the top tranches are often perceived (rated) as being very low risk. DeMarzo (2005) demonstrates another advantage of forming diversified pools and tranching, related to adverse selection risk.

On the downside, tranching has been recognized as interfering with servicers' ability to alter loans. Cordell, Dynan, Lehnert, Liang and Mauskopf (2008) state that "tranche warfare" (p.22) can increase the time that a servicer needs to get modifications approved. Piskorski, Seru and Vig (2010) argue that tranching can "create a coordination problem amongst investors making it harder for servicers to alter mortgage contracts" (p.2).

In this paper we do not directly address the advantages or disadvantges of tranching. Our main concern is with the moral hazard problems that occur after securitization, not with the risk sharing or adverse selection problems that occur at the time of securitization. That said, our proposal with nondiversified MBS's may be quite consistent with tranching. Our proposal calls for forming nondiversified pools of mortgages and then basing renegotiation offers on verifiable pool-wide characteristics. Under this proposal it may be possible to rework loans while avoiding coordination problems across tranches. The way to do this is to include the rules for reworking mortgages in the original security prospectuses. Doing so may be impossible if the MBS pool is heterogeneous, but quite possible if there is enough similarity across the mortgages.

And while our proposal seems to obviate the stated advantages to tranching, it doesn't necessarily do so. Individual risks are diversified within our MBS's and type risks can be diversified by investing in a diversified portfolio of nondiversified MBS's, or by forming

foreclosures in order to delay accounting recognition of losses, because such recognition will affect their Tier II capital. They, however, point out that Tier II capital is also negatively affected by loan modifications. So it is not clear that this concern explains their results.

a CDO of different types of MBS's. Our proposal is that the first level of securitization should be done in a nondiversified manner.

6.3 Summary

We have presented arguments as to why the pooling of mortgages for the purpose of securitization should be done in a nondiversified manner. Our main argument is that foreclosure and debt renegotiation decisions are more efficiently undertaken if mortgages are not organized in diversified pools. We also present arguments that forming nondiversified pools can ameliorate the foreclosure contagion problem that occurs when foreclosures cause a decrease in the value of similar properties. For both of these reasons, we predict that the incidence of foreclosure will be lower if mortgages are not organized into diversified pools for securitization.

Appendix

Proof of Lemma 1.

First, suppose that the lender plays a pure strategy of always making the renegotiation offer $r_0 - x$. If $x \leq 0$, then only borrowers with $v < r_0$ will default, and all renegotiation offers are accepted with probability zero. That is, there is no renegotiation. If $x > 0$, then all borrowers will default. Now suppose that the lender plays a mixed strategy. With probability α, $0 < \alpha < 1$, the lender makes an offer $x > 0$, and with probability $1 - \alpha$ the lender forecloses. Consider a borrower with realization $v > r_0$. Such a borrower will default iff $v - r_0 \leq \alpha(v - r_0 + x)$. We can thus define the default cut-off value $v^D = r_0 + \alpha x/(1 - \alpha)$. Any borrower with a realization $v \leq v^D$ will default. A necessary condition for renegotiation to be possible is that $\alpha > 0$ and $x > 0$. In this case, $v^D > r_0$. That is, there is a strictly positive probability of strategic default. ∎

Proof of Proposition 2.

A lender who renegotiates makes a renegotiation offer $r_0 - x$, where $x \geq 0$. We assume throughout this proof that the lender has no information about the realization of $\tilde{v}$ beyond the prior distribution and the fact that a borrower has defaulted.

If the lender does not renegotiate, then the time zero bond value is:

$$B(x = 0) = sr_0 \cdot prob\{\tilde{v} \geq r_0\} \; + \; \delta E[\tilde{v}|\tilde{v} < r_0] \cdot prob\{\tilde{v} < r_0\} \tag{18}$$

If the lender renegotiates with probability α, then the bond value is:

$$\begin{aligned} B(x > 0) \;\; = \;\; & r_0 \cdot prob\{\tilde{v} \geq v^D\} \; + \; \alpha(r_0 - x)prob\{r_0 - x \leq \tilde{v} < v^D\} \\ & + (1 - \alpha)\delta E[\tilde{v}|r_0 - x \leq \tilde{v} < v^D] \cdot prob\{r_0 - x \leq \tilde{v} < v^D\} \\ & + \delta E[\tilde{v}|\tilde{v} < r_0 - x] \cdot prob\{\tilde{v} < r_0 - x\} \end{aligned} \tag{19}$$

where

$$v^D = \min\left[r_0 + \frac{\alpha x}{1 - \alpha}, \overline{v}\right] \tag{20}$$

and $\overline{v}$ is the upper end of the support of the probability distribution on $\tilde{v}$.

The benefit to renegotiating is:

$$Z \;\; \equiv \;\; B(x > 0) - B(x = 0)$$

$$
\begin{aligned}
&= \alpha(r_0 - x)prob\{r_0 - x \le \tilde{v} < r_0\} \;-\; \alpha\delta \cdot E[\tilde{v}|r_0 - x \le \tilde{v} < r_0] \cdot prob\{r_0 - x \le \tilde{v} < r_0\} \\
&\quad - ((1-\alpha)r_0 + \alpha x)prob\{r_0 \le \tilde{v} < v^D\} \qquad (21) \\
&\quad + (1-\alpha)\delta \cdot E[\tilde{v}|r_0 \le \tilde{v} < v^D] \cdot prob\{r_0 \le \tilde{v} < v^D\}
\end{aligned}
$$

The first line of equation (21) represents the benefit of avoiding foreclosure for some borrowers. This benefit is clearly positive only if δ is sufficiently low. The second and third lines of equation (21) represent the cost of encouraging strategic default.

We demonstrate in this proof that, for all valid parameter values, either $Z \le 0$, in which case the lender should not renegotiate, or $Z > 0$ and is optimized at $\alpha = 1$, in which case the lender should offer a lower payment to all borrowers. We show further that there exists a value r^{noInf}, such that $Z \le 0$ if $r_0 \le r^{noInf}$ and $Z > 0$ if $r_0 > r^{noInf}$.

We now assume that $\tilde{v} \sim U[v_0 - \Delta, v_0 + \Delta]$ and that $r_0 > v_0 - \Delta$; otherwise the result is trivial.

$$
B(x=0) \;=\; \frac{r_0(v_0 + \Delta - r_0)}{2\Delta} + \frac{\delta(v_0 - \Delta + r_0)(r_0 - (v_0 - \Delta))}{4\Delta} \qquad (22)
$$

Consider first the case such that $v^D = r_0 + \alpha x/(1-\alpha)$.

$$
\begin{aligned}
B(x>0) \;&=\; \frac{r_0(v_0 + \Delta - r_0 - \alpha x/(1-\alpha))}{2\Delta} + \frac{\alpha(r_0 - x)(x + \alpha x/(1-\alpha))}{2\Delta} \\
&\quad + (1-\alpha)\delta\left(r_0 + \frac{\alpha x/(1-\alpha) - x}{2}\right)\frac{(x + \alpha x/(1-\alpha))}{2\Delta} \\
&\quad + \frac{\delta(v_0 - \Delta + r_0 - x)(r_0 - x - (v_0 - \Delta))}{4\Delta}
\end{aligned}
$$

$$
\begin{aligned}
Z = B(x>0) \;-\; B(x=0) &= \frac{r_0(-\alpha x/(1-\alpha))}{2\Delta} + \frac{\alpha(r_0 - x)(x/(1-\alpha))}{2\Delta} \\
&\quad + \frac{\delta r_0 x}{2\Delta} + \frac{\delta x^2(2\alpha - 1)/(1-\alpha)}{4\Delta} - \frac{\delta x(2r_0 - x)}{4\Delta} \\
&= \frac{r_0(-\alpha x/(1-\alpha))}{2\Delta} + \frac{\alpha(r_0 - x)(x/(1-\alpha))}{2\Delta} \\
&\quad + \frac{\delta x^2(2\alpha - 1)/(1-\alpha)}{4\Delta} + \frac{\delta x^2}{4\Delta} \\
&= \frac{-\alpha x^2/(1-\alpha)}{2\Delta} + \frac{\delta\alpha x^2/(1-\alpha)}{4\Delta} = \frac{(\delta - 2)\alpha x^2/(1-\alpha)}{4\Delta} < 0
\end{aligned}
$$

This means that the lender will not optimally play a mixed strategy with $x > 0$.

We now consider the case such that $v^D = v_0 + \Delta$. In this case, α is set to one because all borrowers default regardless. We now directly determine the optimal value of r_1:

$$
B(r) = \frac{r(v_0 + \Delta - r)}{2\Delta} + \frac{\delta(r^2 - (v_0 - \Delta)^2)}{4\Delta}
$$

The above is maximized at $r^* = (v_0 + \Delta)/(2 - \delta)$. But any renegotiated offer must be mutually acceptable. Thus $r_1 = \min[r_0, (v_0 + \Delta)/(2 - \delta)]$. The lender will make a renegotiation offer ($x > 0$) to the entire pool iff $r_0 > (v_0 + \Delta)/(2 - \delta)$.

If instead $r_0 \leq (v_0 + \Delta)/(2 - \delta)$, then no renegotiation offer will be made and the time zero bond value will be as given in expression (22). We refer to the expression in (22) as B^{noInf}, the bond value when no information will be gathered. ∎

Maximum bond value with no information gathering.

The lender will never agree to a promised payment greater than r^* calculated above. Thus, we assume that with no information gathering $r_0 \leq (v_0+\Delta)/(2-\delta)$. Given this constraint, $\partial B^{noInf}/\partial r_0 > 0$. The maximum value of B^{noInf} thus occurs when $r_0 = (v_0+\Delta)/(2-\delta)$. We have already assumed that $\Delta < v_0$. We now restrict Δ further:[48]

$$\frac{v_0 + \Delta}{2 - \delta} \geq v_0 - \Delta \implies \left(\frac{1-\delta}{3-\delta}\right) v_0 < \Delta < v_0 \tag{23}$$

If condition (23) is satisfied, then the maximum possible value for B^{noInf} is:

$$\begin{aligned} B^{noInf}_{max} &= \frac{1}{4\Delta}\left(\frac{(v_0+\Delta)^2}{2-\delta} - \delta(v_0-\Delta)^2\right) && (24) \\ &= \frac{v_0+\Delta}{2-\delta} - \frac{1}{4\Delta}\left(\frac{3\Delta^2 + 2v_0\Delta - v_0^2}{2-\delta} + \delta(v_0-\Delta)^2\right) && (25) \end{aligned}$$

If the first inequality of condition (23) is not satisfied, then the maximum possible value for B^{noInf} is equal to the boundary value of $v_0 - \Delta$. If condition (23) is satisfied, then $\partial B^{noInf}_{max}/\partial\delta > 0$ and $\partial B^{noInf}_{max}/\partial\Delta < 0$. Also, as $\delta \to 1, B^{noInf}_{max} \to v_0$. As $\delta \to 0, B^{noInf}_{max} \to \max[(v_0+\Delta)^2/8\Delta, v_0 - \Delta]$.

Proof of Proposition 3.

A borrower who defaults will with probability $1-\gamma$ be offered $r_1 = v$, and with probability γ be offered $r_1 = r_0 - x$, where $x \geq 0$.[49] Thus, a borrower will default iff $v \leq v^D = \min[r_0 + \gamma x/(1-\gamma), \overline{v}]$. v^D is clearly increasing in γ for a given value of $x \geq 0$, but the lender's optimal value of x may be decreasing in γ. Consider first the case such that

[48]This restriction simply ensures that r^{noInf} is at least as large as the lowest possible value of $\tilde{v}$, so that we don't need to worry about boundary conditions.

[49]We assume here that $\alpha = 1$. As long as $\gamma < 1$ any borrower who strategically defaults faces a possible punishment that is equivalent from the borrower's perspective to no renegotiaiton.

$v^D = r_0 + \gamma x/(1-\gamma)$. The lender's expected bond value is:

$$\begin{aligned} B(x>0) &= r_0 \cdot prob\{\tilde{v} \geq v^D\} + (1-\gamma)E[\tilde{v}|\tilde{v} < v^D] \cdot prob\{\tilde{v} < v^D\} \\ &\quad + \gamma(r_0 - x)prob\{r_0 - x \leq \tilde{v} < v^D\} \\ &\quad + \gamma\delta E[\tilde{v}|\tilde{v} < r_0 - x] \cdot prob\{\tilde{v} < r_0 - x\} \end{aligned}$$

If $\tilde{v}$ is uniformly distributed:

$$\begin{aligned} B(x>0) &= \frac{r_0(v_0 + \Delta - v^D)}{2\Delta} + \frac{(1-\gamma)(v_0 - \Delta + v^D)(v^D - (v_0 - \Delta))}{4\Delta} \\ &\quad + \frac{\gamma(r_0 - x)(x + \gamma x/(1-\gamma))}{2\Delta} + \frac{\gamma\delta(v_0 - \Delta + r_0 - x)(r_0 - x - (v_0 - \Delta))}{4\Delta} \\ &= r_0 + \frac{r_0(v_0 - \Delta - r_0 - \gamma x/(1-\gamma))}{2\Delta} - \frac{(1-\gamma)(v_0 - \Delta)^2}{4\Delta} \\ &\quad + \frac{(1-\gamma)(r_0 + \gamma x/(1-\gamma))^2}{4\Delta} \\ &\quad + \frac{\gamma(r_0 - x)(x/(1-\gamma))}{2\Delta} + \frac{\gamma\delta(v_0 - \Delta + r_0 - x)(r_0 - x - (v_0 - \Delta))}{4\Delta} \\ &= r_0 + \frac{r_0(v_0 - \Delta - r_0)}{2\Delta} - \frac{(1-\gamma)(v_0 - \Delta)^2}{4\Delta} + \frac{(1-\gamma)(r_0 + \gamma x/(1-\gamma))^2}{4\Delta} \\ &\quad - \frac{\gamma x^2/(1-\gamma)}{2\Delta} + \frac{\gamma\delta((r_0 - x)^2 - (v_0 - \Delta)^2)}{4\Delta} \end{aligned} \tag{26}$$

Solving the FOC for x, we obtain

$$x^* = \min\left[\frac{(1-\gamma)(1-\delta)r_0}{1 + (1-\gamma)(1-\delta)}\ ,\ r_0 - v_0 + \Delta\right] \tag{27}$$

The above is strictly positive as long as $\gamma < 1$ and $\delta < 1$. It is clear that $\partial x^*/\partial\gamma \leq 0$. Also the probability of foreclosure equals $prob\{\tilde{v} < r_0 - x\}$, which is decreasing in x. If $r_0 \geq (1 + (1-\gamma)(1-\delta))(v_0 - \Delta)$, then the boundary condition is nonbinding and x^* is equal to the left hand term in (27). Assuming that this condition holds and inserting x^* into the equation for v^D:

$$v^D(\gamma) = r_0 + \frac{\gamma(1-\delta)r_0}{1 + (1-\gamma)(1-\delta)} = \frac{(2-\delta)r_0}{1 + (1-\gamma)(1-\delta)} \tag{28}$$

Our assumption that $r_0 \leq r^{noInf} = (v_0 + \Delta)(2 - \delta)$ ensures that $v^D(\gamma) \leq v_0 + \Delta$. But for completeness we now consider the case such that $v^D = v_0 + \Delta$. In this case

$$\begin{aligned} B(x>0) &= (1-\gamma)v_0 + \gamma(r_0 - x)prob\{r_0 - x \leq \tilde{v}\} \\ &\quad + \gamma\delta E[\tilde{v}|\tilde{v} < r_0 - x] \cdot prob\{\tilde{v} < r_0 - x\} \\ &= (1-\gamma)v_0 + \frac{\gamma(r_0 - x)(v_0 + \Delta - r_0 + x)}{2\Delta} + \frac{\gamma\delta((r_0 - x)^2 - (v_0 - \Delta)^2)}{4\Delta} \end{aligned}$$

Solving the FOC for x, we obtain

$$x^* = r_0 - \frac{v_0 + \Delta}{2 - \delta} \qquad \text{and} \qquad r_1 = r_0 - x^* = \frac{v_0 + \Delta}{2 - \delta} = r^{noInf} \tag{29}$$

Combining equations (30) and (27) we obtain the bond value, given that $r_0 \leq r^{noInf}$, $0 < \gamma < 1$, and $r_0 \geq (1 + (1 - \gamma)(1 - \delta))(v_0 - \Delta)$:

$$\begin{aligned} B^\gamma &= r_0 - \frac{r_0(r_0 - (v_0 - \Delta))}{2\Delta} - \frac{(1 - \gamma(1 - \delta))(v_0 - \Delta)^2}{4\Delta} + \frac{r_0^2(2 - 2\gamma - \delta + 2\gamma\delta)}{4\Delta(1 + (1 - \gamma)(1 - \delta))} \\ &= r_0 - \frac{1}{4\Delta}\left(\frac{(2 - \delta)r_0^2}{1 + (1 - \gamma)(1 - \delta)} - 2r_0(v_0 - \Delta) + (1 - \gamma(1 - \delta))(v_0 - \Delta)^2\right) \end{aligned} \tag{30}$$

We now present the boundary solution that obtains when $v_0 - \Delta \leq r_0 < (1 + (1 - \gamma)(1 - \delta))(v_0 - \Delta)$. In this case $r_0 - x^* = v_0 - \Delta$, $v^D = (r_0 - \gamma(v_0 - \Delta))/(1 - \gamma)$ and

$$B^\gamma = r_0 - \frac{(r_0 - (v_0 - \Delta))^2}{4\Delta(1 - \gamma)} \tag{31}$$

In the boundary case r_0 is low enough so that the lender optimally sets r_1 equal to the lower boundary of $v_0 - \Delta$, and foreclosure occurs with probability zero. ∎

Proof of Proposition 4.

Assume first that $r_0 \geq (1 + (1 - \gamma)(1 - \delta))(v_0 - \Delta)$. B^γ is given by equation (30). Rearranging the term inside the ( ) in (30):

$$\begin{aligned} &(2 - \delta)r_0^2 - 2\left(1 + (1 - \gamma)(1 - \delta)\right) r_0(v_0 - \Delta) + \left(1 - \gamma(1 - \delta)\right)\left(1 + (1 - \gamma)(1 - \delta)\right)(v_0 - \Delta)^2 \\ &= \left(r_0 - \left(1 + (1 - \gamma)(1 - \delta)\right)(v_0 - \Delta)\right)^2 + (1 - \delta)r_0 - \left(1 + (1 - \gamma)(1 - \delta)\right)(1 - \delta)(v_0 - \Delta) \end{aligned}$$

The condition $r_0 \geq (1 + (1 - \gamma)(1 - \delta))(v_0 - \Delta)$ is sufficient to ensure that the above is strictly positive. Thus, $B^\gamma < r_0$. The condition is also sufficient to ensure that B^{FB}, as given in equation (7), is greater than B^γ, and that $\partial B/\partial\delta > 0$ and $\partial B/\partial\gamma < 0$.

If $v_0 - \Delta \leq r_0 < (1 + (1 - \gamma)(1 - \delta))(v_0 - \Delta)$, then B^γ is given by (31) and the results are straightforward. ∎

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