

Information Processing and Non-Bayesian Learning in Financial Markets

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Abstract

Many empirical studies in behavioral finance document that investors put greater weight upon dividends they experienced while being actively trading in the market. This can be interpreted as a form of availability bias as described in Kahneman, Slovic, and Tversky (1982). My paper develops an equilibrium model with overlapping generations, in which agents learn about the dividend generating process by observing past dividends with an availability bias. In the absence of private information, the availability bias and the inherent agents' heterogeneity lead to perceived noise trading. My model has applications to both long term asset pricing and intraday trading. For example in the long run it is capable of explaining a high trading volume, a connection between trading volume and volatility, excess volatility, fat tails in the return distribution, return patterns, as well as agent specific trading patterns. Extensions of the model explain further financial anomalies.

Keywords: Availability Bias, Heterogeneous Beliefs, Overlapping Generations Model, Financial Anomalies, Behavioral Finance, Heuristic Learning.

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1 Introduction

Individuals differ with respect to their past experiences. In financial markets these variations can be attributed to differences in e.g market entrance, home trading markets or trading frequency. Moreover, in financial markets the stream of new information is almost infinite and the agent can only process parts of it. Therefore, past experiences influence the information that is mentally most available to the investor. These experienced observations are empirically shown to overly influence the individual's perception of the world. As individuals differ with respect to their history this induces heterogeneity among the agents and the necessity to take heterogeneous mentally available information sets among investors into account.

Empirically the phenomenon of experienced information overly influencing expectations is very well documented. The research by Graham and Narasimhan (2004), Malmendier and Nagel (2011), and Greenwood and Nagel (2009) indicate that past experiences overly influence both individuals' perception and preferences.¹ Having observed a large negative shock, thus, leads to a higher risk aversion and more cautious future predictions. Moreover, especially young investors revise their perception frequently, showing a higher sensitivity of young agents to new experiences. Vissing-Jorgensen (2003) explains this by the idea, that young agents actually have a shorter data sample. This, however, is only true when talking about the observed data sample, and directly leads to the importance of studying individual information histories when modeling financial markets.

In this paper I introduce the idea of agents overvaluing observed relative to unobserved information in an overlapping generations equilibrium model. Within my dividend only model the market entrance of individuals determines, which past dividends were directly observed by the individuals and, therefore, also especially anchored within the individuals' minds. In my first model I distinguish between two groups of individuals. Young individuals have just entered the financial market and, therefore, have less personal experience. Thus, they are very affected by newly experienced information. In contrast the adult generation has already experienced several years of market performance. Therefore, it is not that much affected by new experienced information. However, since

¹A more extensive summary of the empirical evidence related to learning out of observations can be found in Appendix A.

they have also observed prior market performance by experience, they are also biased by these observations. I also extend the model to more generations, which leads to a higher variety in past experiences.

This difference in the reaction to current and past news leads to heterogeneity across agents. Since the dividend stream is the only information accessible, and can be observed by everyone the agents have to believe in noise-trading. This leads to a willingness to trade. This effect is especially pronounced when the observed public signal, in my case the dividend stream, strongly deviates from its long term average. Since all agents in the market have observed the last dividend by experience there is an exaggerated impact on prices - leading to overreaction. When agents leave the market after some time or make further experiences this impact fades out - leading to reversal. In the context of my model this overreaction and correction pattern is also consistent with the finding of Greenwood and Shleifer (2013). They state that investors' expectations show a strong negative correlation with model-based expected returns. In my model in times when average investors' expectations are high, prices are (too) high and, therefore, will mean-revert in the long run.

Moreover, extreme public signals lead to both significant changes in the individuals' expectations as well as to higher disagreement. This results in a positive correlation between volatility and trading volume. Furthermore, my model supports the empirical finding that returns on high-volume days tend to reverse themselves as for example described in Gallant, Rossi, and Tauchen (1992) and Campbell, Grossman, and Wang (1993). The intuition behind this finding is that on high trading volume dates the price changes are more likely to be caused by an initial overreaction than by a reversal. As a consequence in the following time the overreaction is going to fade out, which leads to reversed returns in the aftermath.

Additionally, my model provides explanations for agent specific findings. Younger agents overreact most to new observed information, because their observed data sample is very small. Therefore, they change their portfolio more drastically than older investors and tend to invest more in stock markets when recent signals have been positive.

Depending on the interpretation of an agent's life cycle my model does not only provide insights for long term financial asset pricing phenomena but also provides predictions for intraday trading. Looking at cross-listed stocks, people enter the stock market at different points in time. Especially around the opening hours of a stock exchange the model predicts a high trading volume and volatility. In this context my paper provides the testable hypothesis that the effect of high trading volume around the opening hours is more pronounced for cross-listed stocks compared to single-listed stocks. Moreover, in the case of cross-listed stocks, my model also provides predictions about which agents are generally on the buy and which agents are generally on the sell side of the transaction. On days with recent positive information, those agents that have already been in the market are more affected by these information than those just entering the market. Therefore, the entering investors generally sell their stock to the incumbents. On days with recent bad news the trading behavior is predicted to have the opposite direction.

The incorporation of availability bias into the modeling of financial markets is very recent. The most closely related paper is authored by Barberis, Greenwood, Jin, and Shleifer (2013). In their model investors form beliefs about future performance of a stock by extrapolating past returns. Similar to their model, in my model past stock market performance overly influences future expectations. However, in their model Barberis, Greenwood, Jin, and Shleifer (2013) mainly focus on infinitely lived investor with return extrapolation, whereas in my overlapping generations model agents react to new observations in different ways, depending on their history of past experiences. Therefore, apart from general predictions about prices and volatilities, I also get predictions about individual trading behavior. As market entrance determines the way individuals form their beliefs, there is disagreement about the interpretation of a new signal also among the agents, which are subject to heuristic learning.

Further models with extrapolation of past state variables (either returns or fundamentals) are Hirshleifer and Yu (2013), Alti and Tetlock (2013), Barberis and Shleifer (2003) and Fuster, Hebert, and Laibson (2011). However, in none of these models the agents differ with respect to their past observations. Therefore, my overlapping generations model is able to explain agent specific trading behavior and makes predictions in environments in which market entrance and exit of agents plays

an important role.

Apart from these papers there are mainly three major strands of literature that relate to my model.

One of the most common explanations for financial puzzles in behavioral finance is overconfidence, mostly in combination with the self-attribution bias. Among others, the articles by Daniel, Hirshleifer, and Subrahmanyam (1998), Daniel, Hirshleifer, and Subrahmanyam (2001), Gervais and Odean (2001), Odean (1998), and Scheinkman and Xiong (2003) represent this stream of literature. In these models investors believe, that with the requisite capability it is possible to beat the market. Since individuals do not know about their capability, they have to infer about it by observing their own success – thereby making mistakes. In the context of private and public information, these models can explain excess volatility, as well as price overreaction and reversal.

The second strand of literature deals with heterogeneous agents. One major strength of these papers is them explaining a high trading volume. Two reasons are often mentioned for why investors trade: to rebalance their portfolios for risk sharing and to speculate on their private information. My model considers another explanation for heterogeneity, the disagreement about the relevance of publicly known information.²

The third strand of literature deals with the effect of parameter uncertainty. Most articles in this area assume that individuals form beliefs and make investment decisions based on past returns – updating these whenever observing new information. In different modeling environments these models are able to explain several puzzles in financial markets. In a dividend model Lewellen and Shanken (2002) show that learning induces return predictability, which cannot be exploited by investors. Pástor and Veronesi (2003) and Pástor and Veronesi (2006) argue that uncertainty about the dividend growth rate increases the stock price. Moreover, David (1997) develops a model with unobservable regime shifts in the average productivity of linear technologies, which are subject to learning by a representative agent. Weitzman (2007) considers an endowment economy with unknown consumption volatility and shows that the posterior distribution of consumption growth

²Other models, which also deal with differences in the interpretation of common signals can be found in Harrison and Kreps (1978), Harris and Raviv (1993), Kandel and Pearson (1995), Scheinkman and Xiong (2003) and David (2008).

is fat tailed. For an extensive survey on articles, dealing with Bayesian learning models in financial markets, interested readers are referred to Pástor and Veronesi (2009).

My model tries to combine the strengths of these approaches by providing explanations for several anomalies found within the market. In contrast to overconfidence and other branches of the behavioral finance literature, in my model individuals need not believe in someone having the informational advantage of beating the market, but try to learn about the model's parameters – thereby committing the mistake of availability bias. This way learning still plays an important role, even after having observed more than 70 years of market data. Moreover, my model does not rely on private information, which is generally not observed by empiricists. Information only becomes heterogeneous through individuals' interpretation and their perception.

This paper is organized as follows: Section 2 develops the fundamental two generation model. Section 3 generalizes it to a multi-generation setup. Section 4 shows the model's implications with the help of simulations. Section 5 extends the model and explains further financial anomalies. Finally, Section 6 concludes.

2 The Model

This model introduces availability bias into a multi generations generalization of the model described in Lewellen and Shanken (2002). First, I convey the basic intuition of the model within a two generations setup. Afterwards, in Section 3 I will generalize my approach to a multiperiod model.

2.1 Assets

I consider a discrete time financial market with one risky and one riskless asset. The riskless asset is assumed to pay r_f in each period $t = 1, \dots, \infty$ and to have perfectly elastic supply. It can be converted into or created from one unit of consumption good in any period. Thus, its prices must equal one and r_f equals the riskfree rate of return. Moreover, one risky asset is available in the

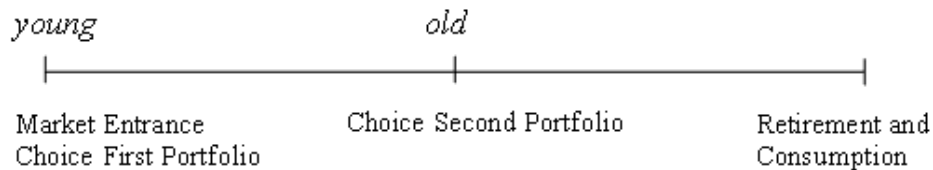


Figure 1: Individuals' Lifecycle. This figure illustrates an individual's life. In the first period the agent enters the market. The agent trades in the first and second period. Finally, he/she leaves the market in the third period.

capital market. This can be interpreted as the market portfolio and is available in one normalized unit. The dividends paid by this asset are normally distributed with mean δ and variance Σ

$$d_t \sim N(\delta, \Sigma). \quad (1)$$

2.2 Individuals

The individuals populating this economy live for three periods. My model does not take intermediary consumption into account. Thus, individuals are maximizing their utility out of terminal wealth w , by choosing the optimal portfolio holdings x

$$\max_x E[U(w(x))] = E[-\exp(-2\gamma w(x))]. \quad (2)$$

An individual's life is illustrated in Figure 1.

Each period one generation of individuals enters the financial market. I assume a constant population. Thus, there are equally many first period, second period and retired individuals within the market. However, since the retired individuals do not buy assets for the next period any more, only the first two generations actively trade on the market. Thus, in the remainder of this paper I ignore the retired generation. I call the new generation, entering the market *Young* and the generation, which has already been trading the prior period *Adult*. Since I assume a constant population, both the *Young* and the *Adult* represent half of the population

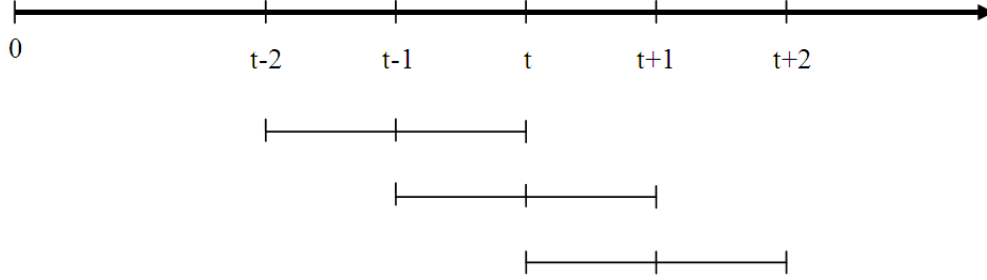


Figure 2: Overlapping Generation Model. This figure shows the overlapping generations at one point in time t . The generation having entered in $t - 2$ is retiring at time t . The generation having entered in $t - 1$ is adult at time t and trades. The generation entering in t is young and also actively participating in the market.

$$Young = Adult = \frac{1}{2}. \quad (3)$$

The overlapping generations are illustrated in Figure 2.

The young and adult individuals differ in two important aspects. First, the time horizon till retirement is shorter for the adult individuals (only one period left) than for the young individuals (two periods left). Second, in the prior period adult individuals have already been in the financial market whereas young individuals have not. Thus, adult individuals have spent more time on observing and experiencing the effect of the latest dividend, whereas the young generation has only seen it in the form of a chart or a number. Thus, adult individuals are assumed to put special weight on this last observation, especially more than on those observations prior to their entrance.

2.3 Individuals' Beliefs about Dividends

In my model individuals' expectations about dividends depend on both, past observed and past unobserved dividends, notwithstanding that observed dividends influence the perception of the dividend process more than the unobserved. In order to define which past dividends have been observable when trade is taking place and prices are determined, I have to define whether period t dividends realize prior or posterior to the determination of prices. In my model, first dividends are paid out and then prices realize. Thus, d_t is known when prices are determined and date t

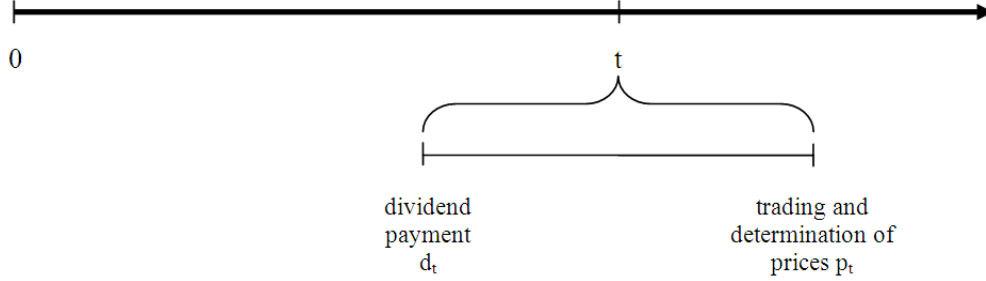


Figure 3: Timing of Dividend and Price Determination. This figure illustrates the timing of dividend and price determination. First, dividends are paid and then prices realize. Consequently, time t dividends belong to those having bought the asset in $t - 1$.

dividends belong to those, who have bought shares in $t - 1$. The intraperiod timing of dividend and price determination is illustrated in Figure 3.

Young Individuals

The young generation has just entered the financial market. Thus, the young agents have not observed the dividends themselves. Their only source of information can be a documentation of past returns in the form of charts, tables, reports etc. In short, they only had the chance of learning by reading and not by experiencing e.g. the effects on the personal portfolio. In this way, all past dividends are the same to them. Weighting all past dividends equally, they form rational expectations about the mean dividend ³

$$E_t^{s,y} [d_{t+1}] = \bar{d}_t. \quad (4)$$

For simplicity I assume as in Lewellen and Shanken (2002) that the variance Σ of the dividend process is known. Thus, I obtain

$$Var_t^{s,y} (d_{t+1}) = \Sigma. \quad (5)$$

Adult Individuals

³This assumption is equivalent to stating, that agents when entering the market have correct priors. However, this assumption is not crucial to the model, as for more generations the young agent's initial perception is of no importance for the result.

The adult generation differs from the young generation in such a way, that they have already experienced the past period and the payment of the period t dividend. Having especially experienced the last dividend, they put special weight on this observation. In my specification the adult generation's dividend expectations equals the young generation's dividend expectations, except that they are overweighting the latest dividend observation by a constant factor m

$$E_t^{s,o} [d_{t+1}] = \frac{(\bar{d}_t + md_t)}{1 + m}. \quad (6)$$

For m being equal to zero the adult generation's expectations would be rational. So for $m > 0$ the only deviation from rationality is the overweighting of those past dividends, which occurred since the individual's market entrance.

Moreover, as for the young generation, I assume the variance of the dividends to be common knowledge

$$Var_t^{s,o} (d_{t+1}) = \Sigma. \quad (7)$$

2.4 Derivation of Optimal Portfolio Choices

Agents maximize their terminal utility. By doing so they mainly differ concerning their personal expectations.

Adult Generation

The adult generation is close to retirement and, therefore, has only an one period investment horizon. The adult agents maximize their next period utility, given their current expectations

$$\max E_t^{s,o} [-\exp(-2\gamma w_{t+1}^{s,o})] \quad (8)$$

Due to the one period investment horizon terminal wealth is determined by

$$w_{t+1} = x_t^o (p_{t+1} + d_{t+1} - (1 + r_f) \cdot p_t) + (1 + r_f)w_t^o. \quad (9)$$

In this context w_t^o equals the time t wealth of the adult individuals. Moreover, it is important to notice that x_t^o equals the number of shares bought and not the amount of money invested.

Deriving Equation (8) with respect to x_t^o and solving I obtain

$$x_t^o = \frac{1}{2\gamma \text{Var}_t^{s,o}(d_{t+1} + p_{t+1})} \cdot (E_t^{s,o} [p_{t+1} + d_{t+1}] - (1 + r_f)p_t). \quad (10)$$

This equation specifies the adult generation's investment given their perception of the price and dividend process. The adult generation's perception of the dividend stream has already been specified in Equations (6) and (7). In contrast to the dividend process, however, the price process depends on both generations' perception and, therefore, will be treated later on.

Young Generation

For the young generation retirement is still two periods away. Thus, they have to maximize their wealth in two periods from now

$$\max E_t^{s,y} [-\exp(-2\gamma w_{t+2}^{s,y})].$$

Terminal wealth is determined by

$$\begin{aligned} w_{t+2}^y &= x_{t+1}^o ((p_{t+2} + d_{t+2}) - (1 + r_f)p_{t+1}) + \\ &+ (1 + r_f) ((1 + r_f)w_t^y + x_t^y ((p_{t+1} + d_{t+1}) - (1 + r_f)p_t)). \end{aligned}$$

Now I assume that the young generation does not take the influence of future prices on future portfolio choices into account.⁴ Therefore, the optimal portfolio choice at time t is given by

⁴This assumption is made for comprehensiveness of the general idea of the model. Through this assumption the qualitative results do not change. The computations, not assuming the agents being myopic, can be found in the

$$x_t^y = \frac{1}{2\gamma(1+r_f) \cdot Var_t^{s,y}(p_{t+1} + d_{t+1})} (E_t^{s,y}[p_{t+1} + d_{t+1}] - (1+r_f)p_t). \quad (11)$$

As before with the adult generation (compare Equation (10)) this equation specifies the young generation's investment given it's perceived price and dividend volatility and it's expectations concerning the future price and dividend process. The young generation's perception of the dividend stream has already been specified in Equations (4) and (7). In contrast to the dividend process, however, the price process depends on both generations' perception and therefore will be treated later on.

2.5 Market Prices and Market Clearing Condition

Having derived general expressions for the two generations' demand for the risky asset, I can now calculate prices by applying the market clearing condition (demand equals the normalized one unit supply)

$$\frac{1}{2}x_t^o + \frac{1}{2}x_t^y = 1. \quad (12)$$

For the calculation of the market clearing condition I take a weighted average formulation of the agents' demand functions. This way the representative agent formulation is invariant to the number of groups in which the economy is divided. Especially for the multi-generation extension of the two-period model, which I will deal with later on, this approach will be more intuitive.

Inserting Equations (10) and (11) into Equation (12) and solving for p_t I obtain

$$p_t = \left(\frac{1+r_f}{4\gamma Var_t^{s,o}(d_{t+1} + p_{t+1})} + \frac{1}{4\gamma Var_t^{s,y}(d_{t+1} + p_{t+1})} \right)^{-1} \cdot \left(\frac{E_t^{s,o}[d_{t+1} + p_{t+1}]}{4\gamma Var_t^{s,o}(d_{t+1} + p_{t+1})} + \frac{E_t^{s,y}[p_{t+1} + d_{t+1}]}{4\gamma(1+r_f)Var_t^{s,y}(d_{t+1} + p_{t+1})} - 1 \right) \quad (13)$$

Appendix. Appendix C treats the optimal portfolio choice in the case of two generations.

This formula expresses the price at time t as a function of the young and adult generations' subjective variances and expectations of future dividends and prices. The precise expressions for the dividends have already been specified in the previous sections. However, in contrast to the exogenous dividend process future prices are determined endogenously. Thus, agents' expectations concerning future prices do not only rely on their expectations concerning future dividends but also on the individuals' perception of the forces driving the determination of prices.

2.6 Individuals' Beliefs about Prices

Generally, it can be assumed that agents try to infer about the price properties by studying the underlying market mechanisms – finally resulting in a price process, which is structurally similar to that described in Equation (13). However, each generation does so without knowing about the other generation's perceptions.

In my model, there is only one source of information – the dividend process – which can be observed by everyone. Thus, I assume the agents to believe that their conclusions out of their observations are correct and that, therefore, everyone else, who acts rationally, must have drawn the same consequences. In technical terms this means, that individuals calculate the future price given their own expectations.

However, since the dividend expectations differ, also the future price expectations vary between the generations. This will finally lead to a deviation of the realizing prices from the subjectively expected ones. Since there is no private information within this market, the price is assumed to have no informative properties. Under the assumption, that agents trust their own expectations, the only sensible explanation for a deviation of the observed price from the subjective rational one can be the existence of noise traders, whose perceived influence on price is denoted by ϵ .⁵

In order to distinguish between the forces that really drive the market and determine prices (meaning individual expectations and the market clearing mechanism) and the individual percep-

⁵Another potential assumption would be, that agents are aware of the other individuals perception, but perceive them as wrong. This can be interpreted in the sense of Aumann (1976), that agents agree to disagree. This would lead to agents still trading, however, the noise trader variance does not occur any more.

tion of it, I now introduce a new sign B^y or B^o , which refers to the individuals perception of the process.

Consequently, the idea described above from the perspective of the young generation can be written as

$$B_t^y(p_t) = \frac{1}{1+r_f} E_t^{s,y} [p_{t+1} + d_{t+1}] - \frac{4\gamma}{2+r_f} Var_t^{s,y} (p_{t+1} + d_{t+1}) + \epsilon_t^y \quad (14)$$

The adult generation's subjective rational prices are determined accordingly

$$B_t^o(p_t) = \frac{1}{1+r_f} E_t^{s,o} [p_{t+1} + d_{t+1}] - \frac{4\gamma}{2+r_f} Var_t^{s,o} (p_{t+1} + d_{t+1}) + \epsilon_t^o \quad (15)$$

The concrete expressions for the mean and variance expectations of the dividends have already been specified in the previous section.

The additional ϵ -term in Equation (14) and Equation (15) can be attributed to the noise trading, which is perceived by the individuals due to the deviation of their expected prices from the realizing ones. I assume, and will later on see, that the perceived influence of noise traders' trading volume on price realization, meaning the deviation of the realizing price from the subjectively rational one, is normally distributed with mean zero and variance Σ_N

$$\epsilon_t \sim N(0, \Sigma_N). \quad (16)$$

As with the dividend process I assume that prior to their market entrance agents are sure about the variance in prices, which can be attributed to noise trading.

2.7 Realizing Prices

All agents assume to be rational. Therefore, the expected next period price equals the current perceived *rational* price (meaning the price which is not distorted by perceived noise traders).

Applying this insight to the price process perceived by the young generation, as given by equation (14)

$$E_t^{s,y} [p_{t+1}] = \frac{1}{r_f} \bar{d}_t - \frac{4\gamma (1+r_f)}{r_f (2+r_f)} (\Sigma + \Sigma_N). \quad (17)$$

The noise trade perceived by the adult generation differs from the young generation. But doing equivalent calculations as for the young generation I obtain the expected price of the adult generation as

$$E_t^{s,o} [p_{t+1}] = \frac{1}{r_f} \frac{(\bar{d} + md_t)}{1+m} - \frac{4\gamma (1+r_f)}{r_f (2+r_f)} (\Sigma + \Sigma_N). \quad (18)$$

Concerning the variance, I treat the sum of price and dividends. There are two reasons why the sum of price and dividends of the next periods could vary from the perspective of the young generation.⁶ The first reason is rational. It is a variation in the dividends, which is captured by the Σ . The second reason can be found in the perceived noise traders' activity, which leads to a variation in price according to Σ_N . Thus, the total perceived variance in future price plus dividend equals

$$Var_t^{s,y} (p_{t+1} + d_{t+1}) = \Sigma_N + \Sigma. \quad (19)$$

Now I can insert these results into Equation (13) and receive the following prices

$$p_t = \frac{(1+r_f)}{r_f(2+r_f)} \left(\frac{\bar{d}_t}{1+r_f} + \frac{(\bar{d} + md_t)}{1+m} - 4\gamma (\Sigma + \Sigma_N) \right). \quad (20)$$

Already from this pricing function some effects of the model becomes obvious. For $m = 0$ I simply have a model for the rational pricing mechanisms in mature markets. Only for $m > 0$ the model differs in one aspect, the period t dividend is overly influencing price (through the adult generation), thereby creating excessive volatility.

⁶The adult generation can be treated in an analog way.

Moreover, as already stated above, the price process described in Equation (20) varies over time with d_t . The first reason for this result can be seen in the variation of \bar{d}_t . This effect, though rational, is decreasing over time and therefore negligible in mature markets. The second reason for variation in prices is caused by availability bias. The corresponding term equals md_t . Since all agents commit this behavioral bias after having entered the market, this effect is persistent over time. Thus, even in mature markets its importance remains unchanged.

3 Multi Generation Model

In the two generation model the agents leave that market shortly after having learned about the dividends in the market. However, in real life agents act on the market for longer periods. Thus, the agents have longer time observe more data and revise their initial perception. Through this effect also the influence of past dividends remains in the market for a longer time. Therefore, having illustrated the basic idea with the help of a two generations model I forward to more complex models with more than only three periods and two agents.

3.1 Individuals' Beliefs

I consider a n generation model, in which individuals live for $n + 1$ periods. As the nomination of the agents now has to distinguish between more than only two generations, I enumerate them, starting with the youngest agent (agent 1: a_1) and ending with the oldest agent (agent n : a_n). Moreover, now individuals have the opportunity to learn about the dividend process for more than one period. Therefore, I need to specify the way in which way individuals form their expectations in the subsequent periods.

As in the two generation model (compare Equation (4)) the youngest generation, that enters the market, has rational expectations

$$E_t^{s,a_1} [d_{t+1}] = \bar{d}_t. \quad (21)$$

The expectations of the second youngest generation, which has been in the market for one period, corresponds to those of the adult individuals in the two period model (compare Equation (6)). Obviously this is the case, since both have the same experience background. For the older generations, which have been in the market for more than one period, I assume that these individuals put special weight on the average observed dividend

$$E_t^{s,a_i} [d_{t+1}] = \frac{\bar{d}_t + m \cdot \bar{d}_{e,t}^{a_i}}{1 + m} \quad \text{for } n \geq i \geq 2. \quad (22)$$

Hereby $\bar{d}_{e,t}$ is the mean dividend after individuals entrance at time t_e . So for the second youngest generation $\bar{d}_{e,t}^{a_2} = d_t$, for the third youngest generation $\bar{d}_{e,t}^{a_3} = \frac{1}{2}(d_t + d_{t-1})$ and for the fourth youngest generation $\bar{d}_{e,t}^{a_4} = \frac{1}{3}(d_t + d_{t-1} + d_{t-2})$ and so forth. Thus, for the k -th youngest generation ($k \geq 2$) I obtain a mean observed dividend of

$$\bar{d}_{e,t}^{a_k} = \frac{1}{k-1} \sum_{i=2}^k d_{t-i+2}. \quad (23)$$

In this specification a generation overweights all past experienced dividends equally. However, this assumption is not of crucial importance and other weighting functions can be implemented easily. Moreover, as before for the two generation model I assume the volatility of dividends and prices to be known

$$Var_t^{s,y} (p_{t+1} + d_{t+1}) = \Sigma_N + \Sigma. \quad (24)$$

3.2 Individuals' optimal portfolio choice

The next step is to determine the optimal holding of the risky asset depending upon the individual expectations. It is important to notice that the youngest generation, which has the same expectations as the *young* generation in the two generation model, now has an investment horizon of n periods. The second youngest generation, which has the same expectations as the *old* generation in the previous model, has a remaining investment horizon of $n - 1$ periods and so forth. I assume

as for the two period model, that the agents do not take the interaction between future prices and their portfolio choice into account.⁷ I obtain the optimal amount of shares held by the generations as

$$x_t^{a_j} = \frac{1}{2\gamma(1+r_f)^{(n-j)}Var_t^{a_j}(p_{t+1} + d_{t+1})} \cdot (E_t^{s,a_j}[p_{t+1} + d_{t+1}] - (1+r_f)p_t). \quad (25)$$

3.3 Market Prices and Market Clearing Condition

As in the two generation model I calculate prices by applying the market clearing condition. With n generations it now equals

$$1 = \frac{1}{n} \sum_{j=1}^n x_t^{a_j}. \quad (26)$$

Again as in Equation (12) I take the weighted average formulation, as it generally is insensitive to the number of generations in the economy.

Inserting Equation (25) into Equation (26) I receive⁸

$$p_t = G \cdot \left(\left(\sum_{i=1}^n (H_i \cdot E^{s,a_i}[p_{t+1} + d_{t+1}]) \right) - 1 \right). \quad (27)$$

As also in the two generation model, individuals believe that they have drawn the right conclusions out of the observed dividends. Consequently, agents only take differences in investment horizon, but not differences in opinion into account. Consequently, when inferring about the price process, they assume that all agents share their opinion – apart from some (imaginary) noise traders, who distort prices. In my terminology I can write this as

⁷This assumption is made for comprehensiveness of the general idea of the model. Through this assumption the qualitative results do not change. The computations, not assuming the agents being myopic, can be found in the Appendix. Appendix D treats the optimal portfolio choice in the case of N generations.

⁸The coefficient are given by $G = \left(\left(\sum_{i=1}^n \frac{1}{2n\gamma Var_t^{s,a_i}(d_{t+1}+p_{t+1})(1+r_f)^{n-i}} \right) \cdot (1+r_f) \right)^{-1}$ and $H_i = \frac{1}{2n\gamma Var_t^{s,a_i}(d_{t+1}+p_{t+1})(1+r_f)^{n-i}}$

$$B^{aj}(p_t) = \frac{E^{s,aj} [p_{t+1} + d_{t+1}]}{1 + r_f} - \frac{2n\gamma}{r_f} \left(1 - \frac{1}{(1 + r_f)^n} \right) Var_t^{s,aj} (d_{t+1} + p_{t+1}) + \epsilon_t^j. \quad (28)$$

Since now I am treating a multi generation setup of Section 2, I use the same approach for solving the problem at hand. Again, every agent assumes to be rational. Therefore, each generation expects next period's price process to equal the current perceived *rational* price (meaning the price which is not distorted by perceived noise traders)

$$E_t^{s,aj} [p_{t+1}] = \frac{1}{r_f} E_t^{s,aj} [d_{t+1}] - 2n\gamma \frac{(1 + r_f)^{(n-1)}}{(1 + r_f)^n - 1} (\Sigma + \Sigma_N). \quad (29)$$

The subjective expectations of future dividends are as above given by a weighted average of the mean dividend (from the beginning of the time series) and the mean observed dividend since market entrance. The last step in determining the equilibrium prices within this economy is inserting the individual expectations into the pricing Equation (27). This way I can specify the price at time t as a closed form expression of past dividends⁹

$$p_t = a \frac{m}{(1 + m)} \sum_{i=0}^{n-2} d_{t-i} b_i + c \cdot \frac{1}{(1 + m)} \bar{d}_t - f(\Sigma + \Sigma_N). \quad (30)$$

In this equation I can already notice two effects, which will become visible in the simulation later on. First, the longer individuals live, the longer lasts the extraordinary effect of a past dividend. Second, the more generations there are, the lower is the initial overreaction.

3.4 Trading Volume and Volatility

Since several volatility and trading volume anomalies can be explained by my model, I now derive the corresponding analytic expressions.

Volatility

⁹The coefficients are given by $a = \frac{(1+r_f)^{n-1}}{(1+r_f)^n - 1}$, $b_i = \left(\sum_{j=(i+2)}^n \frac{1}{j-1} \frac{1}{(1+r_f)^{(n-j)}} \right)$, $c = \left(\frac{m}{(1+r_f)^n - 1} + \frac{1}{r_f} \right)$ and $f = 2n\gamma \frac{(1+r_f)^{n-1}}{(1+r_f)^n - 1}$.

The variance can best be calculated from Equation (30), such that I obtain

$$Var(p_t) = \frac{t-n+2}{t^2} \left(L_1 + \frac{1}{r_f(1+m)} \right)^2 \Sigma + \sum_{i=0}^{n-2} \left(L_1 + \frac{1}{r_f(1+m)} + L_2 M_i \right)^2 \frac{\Sigma}{t^2}, \quad (31)$$

with

$$L_1 = \frac{1}{(1+r_f)^n - 1} \cdot \frac{m}{1+m}, \quad L_2 = \frac{(1+r_f)^{(n-1)}}{(1+r_f)^n - 1} \cdot \frac{m}{1+m}, \quad (32)$$

and

$$M_i = \sum_{j=i+2}^n \frac{1}{j-1} \frac{1}{(1+r_f)^{(n-j)}}. \quad (33)$$

This is generally larger than the price volatility in the case of full rationality. In the latter case ($m = 0$), the variance equals

$$Var^{m=0}(p_t) = \frac{1}{r_f^2 t} \Sigma. \quad (34)$$

Moreover, from Equation (31) I can see, that the noise trader variance, which individuals infer prior to their market entrance must be of order

$$\Sigma_N \approx \sum_{i=0}^{n-2} \left(\frac{(1+r_f)^{(n-1)}}{(1+r_f)^n - 1} \cdot \frac{m}{1+m} \cdot \left(\sum_{j=i+2}^n \frac{1}{j-1} \frac{1}{(1+r_f)^{(n-j)}} \right) \right)^2 \Sigma. \quad (35)$$

I ignore the randomness in \bar{d}_t , since in the strict sense it is no *noise* and converges to zero for mature markets ($t \rightarrow \infty$).

Trading volume

The trading activity of an individual can be defined as the change in his/her portfolio. Initially, individuals enter the market and choose their initial portfolio, thereby maximizing their expected utility. After one period the individual, now belonging to the second youngest generation, has the opportunity to change his/her portfolio. The same happens after the second period, then with the individual belonging to the third youngest generation. Generalizing this thought, I can conclude

that the trading volume of an individual after having been in the market for j periods (TV_j) can be expressed as

$$TV_t^j = x_{t+1}^{a_{j+1}} - x_t^{a_j}. \quad (36)$$

Now I insert the holdings in the risky asset, derived in Equation (25) into Equation (36) and obtain after some calculations

$$TV_t^j = C \cdot \left(\frac{1}{1+m} F \left(\frac{1+r_f}{t+1} - \frac{1}{t} \right) \sum_{i=1}^t d_i + \frac{1}{1+m} F \frac{1+r_f}{t+1} d_{t+1} \right. \quad (37)$$

$$\left. + \frac{m}{1+m} F \left(\frac{1+r_f}{j} - \frac{1}{j-1} \right) \sum_{i=2}^j d_{t-i+2} + \frac{m}{1+m} F \frac{1+r_f}{j} d_{t+1} \right.$$

$$\left. - E \cdot \left(\frac{1+r_f}{t+1} - \frac{1}{t} \right) \sum_{i=1}^t d_i + E \frac{1+r_f}{t+1} d_{t+1} \right.$$

$$\left. - B \cdot \sum_{i=0}^{n-2} A_i ((1+r_f)d_{t+1-i} - d_{t-i}) + H \right). \quad (38)$$

The exact formulae for the terms A to H are derived in Appendix B. The first two lines of Equation (38) can be attributed to the change in the agent's expectation due to the observation of a new dividend. The new observed dividend affects individuals' perception of the dividend in two ways. The first is rationally founded, however, decreasing in time (first line). The second can be attributed to availability bias. Nevertheless, it is not the absolute change in personal expectations driving the trading volume, but the change relative to the other agents, which is reflected in the market price. As with individual expectation again there are two effects driving the average dividend expectation in the market. On the one hand the rational expectation changes due to the new observation (third line) and on the other hand also the weighted irrational component in the market alters (fourth line).

4 Simulations

In order to investigate the effect of biased learning on the price process, I simulate the model for a various number of generations. My special focus will be the effects on the price path as well as the effect on volatility and trading volume. Considering the price path I can explain the overreaction and correction pattern described for example by De Bondt and Thaler (1985). Regarding the volatility I obtain the effect of both excess volatility and volatility clustering. Concerning the trading volume I get the result, that young investors trade more intensively than older investors and thereby obtain more volatile returns.

I choose my parameters as to best match empirical findings.¹⁰ However, in all cases the results, especially the direction of the effects, are insensitive to the specific parameter values chosen. For the specification of the dividend process in the base case I take a mean dividend of 0.04 and a variance of $\Sigma = (0.01)^2$. Moreover, since my model does not deal with inflation, I choose a real risk free rate of $r_f = 0.01$. The absolute risk aversion parameter γ is calibrated such that I obtain a mean price of 1 for each number of generations n .¹¹

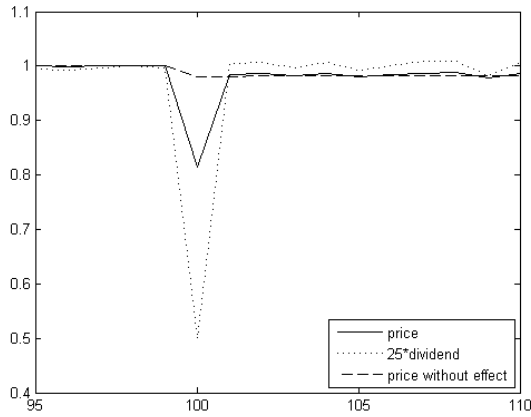
4.1 Simulation of the Price Path

When treating the two generation model in Section 2 I saw that the price is perfectly correlated with the dividend stream. However, for more than two generations this is not the case any more, since the effect of two consequent (in the case of a three generation model) or even more dividend realizations overlay each other. In order to separate the effect of one dividend realization I simulate $p = 1000$ price paths, which are independent from each other, apart from one point in time at which all are simulated to have one extreme dividend (two standard deviations below the mean). This way I calculate the mean effect of this extreme dividend on the price process. All other effects should cancel out on average. I do so for the two, three, four, five, six and eight generation model. The simulated paths can be seen in Figure 4.¹²

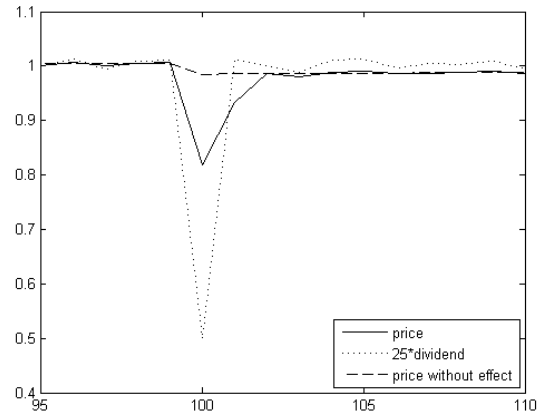
¹⁰As reference papers I chose among others Campbell, Grossman, and Wang (1993), Shiller (1981), Bansal and Yaron (2004) and Goyal and Welch (2003).

¹¹I choose γ in the basic calculations depending on the number of generations between 1.8865 and 2.8770.

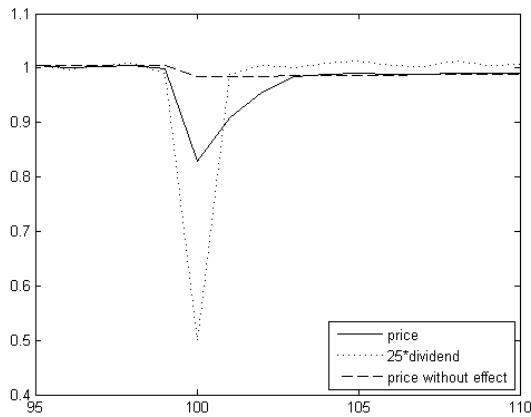
¹²Hereby it has to be noticed, that the dividends are multiplied by 25 in order to fit it into the same scale.



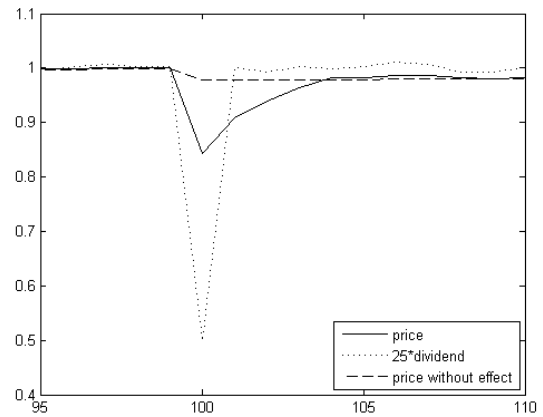
(a) Plot for Two Generations



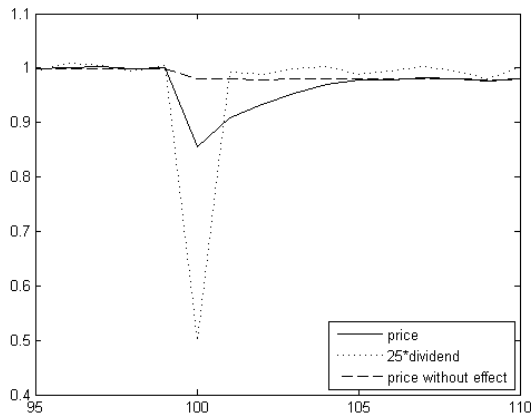
(b) Plot for Three Generations



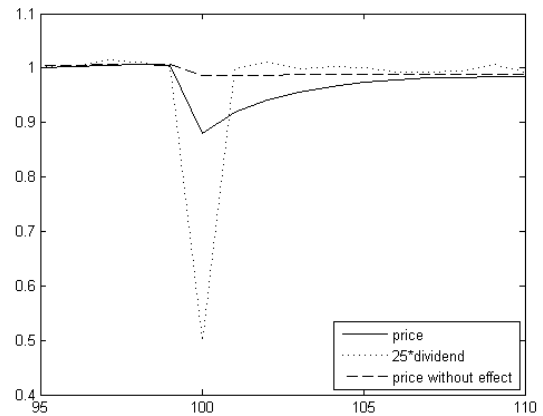
(c) Plot for Four Generations



(d) Plot for Five Generations



(e) Plot for Six Generations



(f) Plot for Eight Generations

Figure 4: Simulation of Price Process. This figure illustrates the average price reaction to an extraordinary low dividend (2 standard deviations below the mean). The average is taken out of $p = 1000$ simulations and after period 100. For the simulation I choose a mean dividend δ of 0.04 and a standard deviation of 0.01. The availability bias parameter m equals 0.2. The risk aversion parameters are chosen such that the prices are normalized to 1.

When comparing the price path with availability bias and the price path without bias, it can be seen that in the first case the price overreacts in the direction of the dividend deviation and bounces back in the subsequent periods. Moreover, the size of the overreaction and the length of the correction phase depends upon the number of simulated generations.

While the overreaction in the period following the dividend payment directly stems from an overestimation of the dividend's relevance, the fading out is caused by two different effects. First, in each period one generation leaves and a new, unbiased, generation enters the market. So in some sense, the effect of a period t dividend dies out by old generations exiting the market. The second correcting effect can be attributed to the observation of new dividends in the subsequent periods. These new dividends reduce the effect of the time t dividend on the mean dividend while being in the market ($d_{e,t+i}$). The relative importance of these two effects depends upon the number of generations modeled, or to be more precise on the length of an individual's trading life. For models with only a few generations, a relatively high percentage of the total population leaves the market in each period. So in models with only little generations the agents already have left the market before they could have learned something about the dividend stream from their own observation. In contrast for multiple generation models the dying out rate is much lower. So most of the agents observe the consequent dividends and thereby correct their beliefs and bias. Taking both effects together it becomes clear, why the correction rate is higher in the periods directly following the overreaction, than in those periods, much thereafter. Nevertheless, although the effect diminishes a lot, the effect of the dividend observation only stops to overly influence the market, when the last agent, who observed the dividend, has left the market.

4.2 Trading Volume and Volatility

When looking at the trading volume I can see that the youngest generation trades most. In other words, the youngest generation disagrees most with the representative agent's interpretation of the new observation. This finding is consistent with the empirical results of Barber and Odean (2001), that young investors trade more actively.

Trading after Period	n-Generation Model					
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 8$
1	3.16	2.79	2.76	2.83	2.89	3.08
2	-	1.07	1.35	1.54	1.67	1.91
3	-	-	0.84	0.67	0.60	0.62
4	-	-	-	1.01	0.81	0.53
5	-	-	-	-	1.07	0.73
6	-	-	-	-	-	0.90
7	-	-	-	-	-	1.02

Table 1: Trading Volume. This table illustrates the average absolute trading volume after period j . The trading volume TV_j is defined as in Equation (36). For the simulation I choose the basic parametrization, meaning a mean dividend of 0.04 and a standard deviation of 0.01. The availability bias parameter m equals 0.2. The risk aversion parameters are chosen such that the prices are normalized to 1.

Having a closer look at the trading volume I can observe, that for the two, three and four generation models the trading volume decreases with age. For the more than four generation models in contrast, the very old agents also start trading again – not as much as the youngest investors but more than the median generation. This can be explained by the fact that older generations become increasingly insensitive to new dividend observations and, therefore, also disagree with the representative investor, who places medium importance on the new observation. However, since every period a new, rational generation enters the market, this draws the representative agent back towards more rational attitudes. Consequently, the youngest generations trade most. So a market with additional rational investors would lead to an even higher trading volume of the young investors and decrease the trading volume of the older ones.

Also a relatively high return standard deviation compared to the low variation in dividends can be explained by my model (compare Table 2). As the latest dividends have been experienced by almost everyone in the market they overly influence current prices. This leads to an increase in price volatility and, consequently, also in returns.

With my current model and an assumed dividend volatility of $(0.01)^2$ and for availability bias parameters ranging between 0 and 0.3 I am able to obtain standard deviations of returns of around 0.1 to 0.15. Assuming that in real world finance dividends are smoothed compared to earnings I

σ_{return} for overreaction parameter	n-Generation Model					
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 8$
$m = 0$	0.012	0.012	0.012	0.012	0.012	0.012
$m = 0.10$	0.077	0.069	0.062	0.055	0.052	0.043
$m = 0.20$	0.134	0.120	0.105	0.094	0.087	0.070
$m = 0.30$	0.187	0.165	0.144	0.127	0.118	0.094

Table 2: Return Standard Deviation. This table shows the standard deviation in returns for varying availability bias parameters. The dividend stream is simulated with a mean of 0.04 and a standard deviation of 0.01. The risk aversion parameters are chosen such that the prices are normalized to 1.

could obtain a high return volatility with even lower availability bias levels.¹³

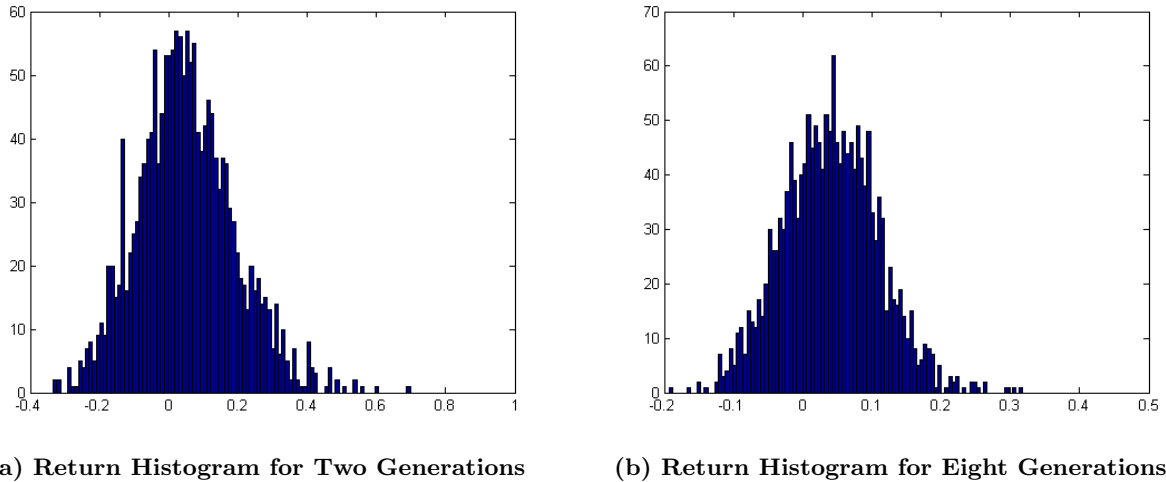


Figure 5: Return Histogram. This figure shows a return histogram for the two and the eight generation model. The dividend stream is simulated with a mean of 0.04 and a standard deviation of 0.01. The risk aversion parameters are chosen such that the prices normalize to 1.

The effect that returns vary more than under full rationality can also be seen from Figure 5. The variation in returns is much higher than the variation in dividends. Apart from a higher second central moment my distribution also has fatter tails with a kurtosis of above 3. However, the effect is not large within this basic model. Empirical evidence indicates that individuals better memorize extreme events than normal ones. In this context large negative shocks are especially kept in mind. Taking this into account I get negatively skewed returns with a higher kurtosis (cf. Section 5).

When looking at absolute returns I observe that for only few generations I obtain a significant

¹³For literature on dividend smoothing look at Lintner (1956).

Autocorrelation in σ_{return}	n-Generation Model					
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 8$
lag = 1	0.214	0.078	0.058	0.049	0.041	0.036
lag = 2	0.000	0.036	0.014	0.012	0.010	0.010
lag = 3	0.000	0.000	0.010	0.007	0.006	0.005

Table 3: Volatility Clustering. This table shows the autocorrelation in absolute returns for up to three lags. For the simulation I chose the basic parametrization, meaning a mean dividend of 0.04 and a standard deviation of 0.01. The availability bias parameter m equals 0.2. The risk aversion parameters are chosen such that the prices are normalized to 1.

autocorrelation (compare Table 3). This effect can be attributed to the mean reverting tendency in the price process. However, since the reaction to new observations decreases in the number of generations, also the autocorrelation decreases. In order to keep the effect as clean as possible in my model agents weight all their observed dividends equally. However, it can be assumed that agents weight their latest observations more than their initial ones. This way I would still be able to maintain a high autocorrelation with a high number of generations.

σ_{return}	n-Generation Model					
	$n = 2$	$n = 3$	$n = 4$	$n = 5$	$n = 6$	$n = 8$
$\text{Cor}(TV, -r_a)$	0.01	0.09	0.40	0.62	0.73	0.84

Table 4: Correlation between Trading Volume and Absolute Returns. This table shows the correlation between trading volume and absolute returns for a varying number of generations. The underlying dividend stream is simulated with a mean of 0.04 and a standard deviation of 0.01. The availability bias parameter is equal to $m = 0.2$. The risk aversion parameters are chosen such that the prices are normalized to 1.

When dealing with the correlation between volatility (in my case measured by absolute return) and trading volume, I observe close to zero correlation for the two and three generation models and high correlation for the more generation models. Generally the correlation is positive, meaning that when returns are high I can also expect a high trading volume. The absolute return - volatility correlation has two influencing factors. First, when extreme dividends are realized, then also the agents disagree most about their valuation and influence. This leads to a high trading volume. Apart from this main effect, there is another effect, which for few generation models works in the adverse direction as the first one. The second effect can be attributed to agents becoming older and wiser. Thus, they are less influenced by new information than the agents entering the market

after them.¹⁴ However, as in a real world agents should observe more than two dividends within their lifetime, which favors the more generation models, the first effect can be expected to be more influential.

5 Extensions of the model and further explanations of financial anomalies

With the basic dividend model described in this paper I want to convey the effect of availability bias in the most general framework, namely a dividend only economy. Among others I am able to explain several financial anomalies like excessive volatility, price overreaction and correction, as well as excessive trading volume. However, the explanatory potentials of this idea go far beyond those derived in this basic model.

Non-constant population (Intraday trading)

Up to now I mostly concentrated on long term effects of availability bias. However, I could also interpret the market entrance on an intraday basis. Then, however, especially for multi-listed shares the “population” is not constant. Around the market opening of a stock exchange the trading volume is found to be especially high, combined with high volatility.

Availability through News

First, the basic model mainly deals with learning by experience. However, there are also other factors influencing how available the market performance (in my model in the form of dividends) of the respective dates is within the individuals’ minds. In the basic model all past experienced dividends are equally weighted. However, the latest market performance should be more available than the the market performance at the time of market entry (especially if the time of entry has been several periods ago).

¹⁴In a two generation model, when one extremely positive dividend is followed by an extreme negative dividend the absolute return is extremely high, but there will be only a low trading volume. This can be explained by the fact that the agent that has just entered the market when the high dividend occurred was extremely influenced and bought a lot of shares. Then in the next period the agent became older and is less influenced by the new dividend, leading to the fact that he/she is disinvesting less than the new agent, who has just entered the market. Consequently, he/she keeps a relative high investment in stocks. Thus, the trading volume is low.

Therefore, I introduce apart from the former factor $\bar{d}_{e,t}$, which is the mean dividend since market entrance, another factor $\bar{d}_{a,t}$, which is a weighted average of the latest 3 time periods. This can be interpreted as the availability created through news and current hot topics. Thus, the individual expectations for the youngest generation can be specified as

$$E_t^{s,a_1} = \frac{(1+m) \cdot \bar{d}_t + f \cdot \bar{d}_{a,t}}{1+f+m}. \quad (39)$$

The expectations of the older generations is consistently specified as

$$E_t^{s,a_j} = \frac{\bar{d}_t + m \cdot \bar{d}_{e,t} + f \cdot \bar{d}_{a,t}}{1+f+m}. \quad (40)$$

This way autocorrelation is relatively stable between models with different numbers of generations.

Differences in Reactions to News

Empirical studies show that extreme information better sticks within the individuals' minds than average events. Moreover, extreme negative shocks are memorized best (compare McGraw, Larsen, Kahneman, and Schkade (2010)). This can be incorporated in the basic model in such a way that extreme and especially extreme negative events become a special weight. In the context of my model I could implement this insight by putting a greater weight upon observed dividends, when they are extreme (with an even greater weight on extremely low dividends)

$$\bar{d}_{e,t}^{a_k} = \left(\sum_{i=2}^k w_{t-i+2} \right)^{-1} \left(\sum_{i=2}^k w_{t-i+2} \cdot d_{t-i+2} \right). \quad (41)$$

The weights are given by the following formula ($0 < a < b$)

$$w(d_t) = \begin{cases} (1+a) & d_t > \bar{d}_t + \sigma \\ 1 & \bar{d}_t - \sigma \leq d_t \leq \bar{d}_t + \sigma \\ (1+b) & d_t < \bar{d}_t - \sigma. \end{cases} \quad (42)$$

This way the return distribution becomes negatively skewed with even fatter tails than in the basic model.

Differences in the Degree of Availability Bias

In markets some investors are more familiar with background statistics and better supported by financial softwares than others. Especially private investors often have to rely on their intuition, which is very unimmunized against biases like the availability bias. This can be incorporated into the basic model by adding some more rational investors to my economy. Thus, the market clearing condition (26) is modified to

$$1 = \frac{1}{n+r} \left(\sum_{j=1}^n x_t^{a_j} \right) + \frac{r}{n+r} x_t^{a_1}. \quad (43)$$

This way, the phenomenon that many private investors start investing after some favorable news and returns can be explained.

Learning about Portfolio Performance

Moreover, as described in Glaser and Weber (2009), market performance as well as past portfolio performance affect individuals' trading volume. In the context of non-Bayesian learning, this is consistent with the idea, that while being in the market, individuals have an eye on both, market and portfolio performance. Assets belonging to the individual's portfolio obtain more attention, since it is their return, which determines terminal wealth. However, agents are never the less in the market and, therefore, e.g. throughout their portfolio choice also deal with market performance. Thus, agents are biased towards both, portfolio and market returns.

6 Conclusion

Both research in empirical finance as well as in psychology stress the importance of availability bias for the decision making process in finance. Since different agents also have a varying informational background the heuristic decision making process leads to heterogeneity among agents. In my dividend only model the market entrance of individuals determines, which past dividends were directly experienced by the individuals and therefore also especially anchored within the individuals mind. Young generations have just entered the market and, thus, had only little time for learning and experiencing dividends within the market. Consequently, new dividend observations have a special impact on their perception of the dividend process. In contrast, older generations had the opportunity of observing several dividends throughout their lives. Consequently, their perception of the dividend stream varies less with new information being available.

Since the dividend stream is the only information accessible and can be observed by everyone every agent has to assume to have drawn the correct consequences out of his/her observations. Thus, the differences in reaction have to be attributed to noise trading, resulting in a willingness to trade.

Moreover, as almost all agents (every generation but the youngest) have observed the most recent dividend, it overly influences both price and dividend expectations – resulting in excessive price and return volatility. However, since this effect is fading out over time I observe a mean reverting tendency in prices (which I can call overreaction and correction).

In contrast to other behavioral biases like overconfidence and self-attribution bias the effects of heuristic learning and availability bias are rather unexplored in financial research. However, as the basic model within this paper demonstrates further research within this area, and a deeper understanding of what information is most available to agents, can lead to a better comprehension of financial markets.

A Heuristic Learning and Availability Bias

The effect of heuristic learning and availability bias is strongly supported by findings in both, psychologic and empirical finance research.

The first and most famous article describing the effect of availability bias was published by Kahneman, Slovic, and Tversky (1982). They point out that individuals focus more on information that stands out at the expense of those that stays in the background. Apart from this famous work there are many other empirical findings in psychology which support the general idea of biased learning. Many authors, for example, have shown that low probability events are underweighted till they occur and overweighted once they occur (c.f. Weber, Boeckenholt, Hilton, and Wallace (1993); Hertwig, Barron, Weber, and Erev (2004)).

Transferring the idea it means that individuals put more weight onto information they experienced and gathered through active interaction in contrast to information they only read about in books or saw on charts. Since those information that individuals' experienced are mostly the last ones of a huge time series, in the context of asset returns it can also be interpreted this as the also well known base rate underweighting bias (also described in Kahneman, Slovic, and Tversky (1982)). It points out that in experiments base rate data are often neglected or underweighted in individual predictions.

In empirical financial research the finding of availability bias is expressed by many authors. For example Graham and Harvey (2001) find that past market performance is positively correlated with the average CFO's one-year and ten-year perceived risk premium. Also Dominitz and Manski (2005) mostly support these findings, by noting that the central tendency of their expectations data shows a believe in some form of persistence.¹⁵ Moreover, they find that the expectations of a positive nominal equity return vary substantially across persons and systematically with sex, age, and schooling.

It leads us to the effect, that different investors have a varying informational background. When

¹⁵Dominitz and Manski (2005) gather their data from the Michigan Survey of Consumers (June 2002 - August 2004)

examining investors' beliefs based on the household level data underlying the index of investor optimism, Vissing-Jorgensen (2003) documents that an investor's belief about future stock returns depends on the investor's own experience measured by age, years of investment experience and own past (self-reported) portfolio returns. Apart from other effects young investors are more sensitive to past market returns than older investors. This leads the author to the conclusion that new data points should be weighted more by young investors, who effectively have a shorter data sample. This idea however can only be explained by the availability bias.

The impression that past experiences influence today attitudes and actions is shared by many empiricists. Malmendier and Nagel (2011) show that individuals who have experienced high stock market returns report lower aversion to financial risks and are more likely to participate in the stock market. Moreover, young individuals' allocation to stocks is more sensitive to recent stock market returns than that of older investors. They point out that the impact of experiences fades only slowly with time, meaning that experiences several decades ago still have impact on current risk-taking of individuals. Greenwood and Nagel (2009) show that young mutual fund managers, who had relatively more exposure to technology stocks in the late 1990s, increased their holdings particularly after quarters with high tech stock returns. Graham and Narasimhan (2004) find that corporate managers who have lived throughout the Great Depression in the 1930s choose a more conservative capital structure with less leverage.

Another branch of empirical evidence, among others represented by Barberis, Shleifer, and Vishny (1998) and Rabin (2002), concerns the so called law of small numbers. Investors subject to this bias will expect even short samples to reflect the properties of the larger population, leading to high expected returns after a period of high realized returns.

B Calculations for Volatility and Trading Volume

The trading volume after period j can be described as

$$TV_t^j = x_{t+1}^{a_{j+1}} - x_t^{a_j}. \quad (44)$$

Now I insert the holdings in the risky asset, derived in Equation (25) into Equation (44) and get

$$\begin{aligned}
TV_t^j &= \frac{1}{2\gamma(1+r_f)^{(n-j-1)}Var_{t+1}^{a_{j+1}}(p_{t+2} + d_{t+2})} \\
&\quad (E_{t+1}^{a_{j+1}}[p_{t+2} + d_{t+2}] - (1+r_f)p_{t+1}) \\
&\quad - \frac{1}{2\gamma(1+r_f)^{(n-j)}Var_t^{a_j}(p_{t+1} + d_{t+1})} \\
&\quad \cdot (E_t^{a_j}[p_{t+1} + d_{t+1}] - (1+r_f)p_t).
\end{aligned} \tag{45}$$

First, I replace $E_t^{s,a_i}[p_{t+1}]$ by the Equation (29) and $Var_t^{s,a_i}(p_{t+1} + d_{t+1})$ by specification (24) to obtain

$$\begin{aligned}
TV_t^j &= \frac{1}{2\gamma(\Sigma + \Sigma_N)(1+r_f)^{n-j}} \\
&\quad (- (1+r_f)^2 p_{t+1} + (1+r_f)p_t + (1+r_f)(1 + \frac{1}{r_f})E_{t+1}^{a_{j+1}}[d_{t+2}] \\
&\quad - (1 + \frac{1}{r_f})E_t^{a_j}[d_{t+1}] - 2n\gamma r_f \frac{(1+r_f)^{n-1}}{(1+r_f)^n - 1}(\Sigma + \Sigma_N)).
\end{aligned} \tag{46}$$

In other words, the trading volume depends on both, the changes in prices and the changes in expected dividends. For the prices I get the following expression by inserting Equation (30)

$$\begin{aligned}
(1+r_f)p_t - (1+r_f)^2 p_{t+1} &= \\
\frac{m(1+r_f)^n}{(1+m)((1+r_f)^n - 1)} \cdot \sum_{i=0}^{n-2} \left(\sum_{j=i+2}^n \frac{1}{j-1} \frac{1}{(1+r_f)^{n-j}} \right) (d_{t-i} - (1+r_f)d_{t+1-i}) \\
+ \left(\frac{m}{(1+r_f)^n - 1} + \frac{1}{r_f} \right) \frac{1}{(1+m)} \left(\frac{1}{t} - (1+r_f)\frac{1}{t+1} \right) \sum_{i=1}^t d_i \\
- \left(\frac{m}{(1+r_f)^n - 1} + \frac{1}{r_f} \right) \frac{1}{(1+m)} (1+r_f)\frac{1}{t+1} d_{t+1}
\end{aligned} \tag{47}$$

$$+2n\gamma \frac{(1+r_f)^n}{(1+r_f)^n - 1} r_f (\Sigma + \Sigma_N).$$

The dividend expectations are specified in Equation (22). Consequently, I obtain

$$\begin{aligned} (1+r_f)E_{t+1}^{a_{j+1}}[d_{t+2}] - E_t^{a_j}[d_{t+1}] = & \quad (48) \\ \frac{1}{(1+m)} \left(\frac{1+r_f}{t+1} - \frac{1}{t} \right) \sum_{i=1}^t d_i + \frac{(1+r_f)}{(1+m)(t+1)} d_{t+1} \\ + \frac{m}{(1+m)} \left(\frac{(1+r_f)}{j} - \frac{1}{j-1} \right) \sum_{i=2}^k d_{t-i+2} \\ \frac{m}{1+m} \frac{1+r_f}{j} d_{t+1}. \end{aligned}$$

Inserting Equation (47) and (48) into Equation (46) I obtain

$$\begin{aligned} TV_t^j = C \cdot (B \cdot \sum_{i=0}^{n-2} A_i (d_{t-i} - (1+r_f)d_{t+1-i}) & \quad (49) \\ + E \cdot \left(\frac{1}{t} - \frac{1+r_f}{t+1} \right) \sum_{i=1}^t d_i + E \frac{1+r_f}{t+1} d_{t+1} \\ + \frac{1}{1+m} F \left(\frac{1+r_f}{t+1} - \frac{1}{t} \right) \sum_{i=1}^t d_i + \frac{1}{1+m} F \frac{1+r_f}{t+1} d_{t+1} \\ \frac{m}{1+m} F \left(\frac{1+r_f}{j} - \frac{1}{j-1} \right) \sum_{i=2}^j d_{t-i+2} + \frac{m}{1+m} F \frac{1+r_f}{j} d_{t+1} + H), \end{aligned}$$

with

$$A_i = \sum_{j=i+2}^n \frac{1}{j-1} \frac{1}{(1+r_f)^{n-j}}, \quad (50)$$

$$B = \frac{m(1+r_f)^n}{(1+m)((1+r_f)^n - 1)}, \quad (51)$$

$$C = \frac{1}{2\gamma(\Sigma + \Sigma_N)(1 + r_f)^{n-j}}, \quad (52)$$

$$E = \left(\frac{m}{(1 + r_f)^n - 1} + \frac{1}{r_f} \right) \frac{1}{(1 + m)}, \quad (53)$$

$$F = 1 + \frac{1}{r_f}, \quad (54)$$

$$H = 2n\gamma r_f^2 \frac{(1 + r_f)^{n-1}}{(1 + r_f)^n - 1} (\Sigma + \Sigma_N). \quad (55)$$

C Non-myopic Agents - 2 Generations

I solve this model in a rational expectations equilibrium with CARA utility functions and a Gaussian-Normal environment. The solution technique is similar to the one used in Albagli (2012). I assume that

$$p_{t+1} = \bar{p} + \sigma_p \epsilon_{p1}$$

$$p_{t+2} = \bar{p} + \sigma_p \epsilon_{p2}$$

$$d_{t+1} = \bar{d} + \sigma_d \epsilon_{d1}$$

$$d_{t+2} = \bar{d} + \sigma_d \epsilon_{d2}$$

The resulting wealth at time $t = 2$ is given by:

$$\begin{aligned} w_{t+2} &= \frac{1}{\gamma} \frac{1}{\sigma_p^2 + \sigma_d^2} \cdot (\bar{p} + \bar{d} - (1 + r) \cdot (\bar{p} + \sigma_p \epsilon_{p1})) \\ &\quad \cdot (\bar{p} + \sigma_p \epsilon_{p2} + \bar{d} + \sigma_d \epsilon_{d2} - (1 + r_f)(\bar{p} + \sigma_p \epsilon_{p1})) \\ &\quad + (1 + r_f) x_t^y ((\bar{p} + \sigma_p \epsilon_{p1} + \bar{d} + \sigma_d \epsilon_{d1}) - (1 + r_f) p_t) \\ &\quad + (1 + r_f)^2 w_t^y \end{aligned}$$

I obtain

$$w_{t+2} = c + b'\epsilon + \epsilon' A \epsilon$$

where

$$\epsilon = \begin{pmatrix} \epsilon_{d1} & \epsilon_{d2} & \epsilon_{p1} & \epsilon_{p2} \end{pmatrix}^T$$

with

$$\begin{aligned} c &= \frac{[\bar{d} - r_f \bar{p}]^2}{\gamma(\sigma_p^2 + \sigma_d^2)} + (1 + r_f)^2 w_t^y \\ &+ (1 + r_f)(\bar{d} + \bar{p} - (1 + r_f)p_t)x_t^y \end{aligned}$$

$$b = \begin{pmatrix} b_1^x x_t^y & b_2 & b_3^x x_t^y + b_3 & b_4 \end{pmatrix}^T$$

with

$$\begin{aligned} b_1^x &= (1 + r_f)\sigma_d \\ b_2 &= \frac{[\bar{d} - r_f \bar{p}]\sigma_d}{\gamma(\sigma_p^2 + \sigma_d^2)} \\ b_3^x &= (1 + r_f)\sigma_p \\ b_3 &= -2 \frac{(1 + r_f)\sigma_p[\bar{d} - r_f \bar{p}]}{\gamma(\sigma_p^2 + \sigma_d^2)} \\ b_4 &= \frac{[\bar{d} - r_f \bar{p}]\sigma_p}{\gamma(\sigma_p^2 + \sigma_d^2)} \end{aligned}$$

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & a_1 & a_2 & 0 \\ 0 & 0 & a_3 & 0 \end{pmatrix}$$

with

$$a_1 = -\frac{(1+r_f)\sigma_p\sigma_d}{\gamma(\sigma_p^2 + \sigma_d^2)}$$

$$a_2 = \frac{(1+r_f)^2\sigma_p^2}{\gamma(\sigma_p^2 + \sigma_d^2)}$$

$$a_3 = -\frac{(1+r_f)\sigma_p^2}{\gamma(\sigma_p^2 + \sigma_d^2)}$$

From Vives (2010) I know

$$E[-\exp(-\gamma w)] = -(\det(\Sigma))^{-\frac{1}{2}}(\det(\Sigma^{-1} + 2\gamma A))^{-\frac{1}{2}} \quad (56)$$

$$\cdot \exp(-\gamma(c - \gamma b'(\Sigma^{-1} + 2\gamma A)^{-1}b/2)) \quad (57)$$

In my case $\Sigma = I_4$ and I obtain

$$(I_4 + 2\gamma A)^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -\frac{2\gamma a_1}{1+2\gamma a_2} & \frac{1}{1+2\gamma a_2} & 0 \\ 0 & \frac{4\gamma^2 a_1 a_3}{1+2\gamma a_2} & -\frac{2\gamma a_3}{1+2\gamma a_2} & 1 \end{pmatrix}$$

When I want to derive the FOC for Equation (57) I must obtain that the derivative of $c - \frac{\gamma}{2}b'(\Sigma^{-1} + 2\gamma A)^{-1}b$ with respect to x_t^y should be 0.

Therefore, I get

$$\frac{\partial c}{\partial x} - \frac{\gamma}{2} \left(2b_1^2 x_t^y - 2\gamma \frac{b_3^x a_1 b_2}{1 + 2\gamma a_2} + 2 \frac{b_3 b_3^x}{1 + 2\gamma a_2} + 2 \frac{(b_3^x)^2}{(1 + 2\gamma a_2)} x_t^y - 2\gamma \frac{a_3 b_4 b_3^x}{(1 + 2\gamma a_2)} \right) = 0$$

Solving for x_t^y I obtain

$$x_t^y = M[\bar{d} + \bar{p} - (1 + r_f)p_t] + N$$

where M and N are given by

$$M = \frac{(1 + 2\gamma a_2)}{\gamma(1 + r_f)(\sigma_d^2(1 + 2\gamma a_2) + \sigma_p^2)}$$

$$N = \frac{(1 + r_f)}{\gamma(\sigma_d^2(1 + 2\gamma a_2) + \sigma_p^2)} \frac{\sigma_p^2}{\sigma_p^2 + \sigma_d^2} [\bar{d} - r_f \bar{p}]$$

D Non-myopic Agents - N Generations

I conjecture the value function

$$\begin{aligned} J(W_{t+1}, M_{t+1}, j + 1, t + 1) &= \exp(-\alpha_{j+1} W_{j+1} - \frac{1}{2} M_{t+1} V_{j+1} M_{t+1}) \\ &= \exp(-\alpha_{j+1} R W_j - \alpha_{j+1} Q_{t+1} X_{j,t} - \frac{1}{2} M_{t+1} V_{j+1} M_{t+1}) \end{aligned}$$

In this case M_t is the state vector.

$$M_{t+1} = A_m M_t + B_m \epsilon_{t+1}$$

and

$$M_t = \begin{pmatrix} 1 & \sigma_p \epsilon_t^p & \sigma_d \epsilon_t^d \end{pmatrix}$$

$$A_m = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$B_m = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sigma_p & 0 \\ 0 & 0 & \sigma_d \end{pmatrix}$$

Q_t is the excess return.

$$\begin{aligned} Q_{t+1} &= D_{t+1} + P_{t+1} - RP_t \\ &= A_Q M_t + B_Q \epsilon_{t+1} \end{aligned}$$

with

$$\begin{aligned} A_Q &= \begin{pmatrix} \bar{d} - r_f \bar{p} & -(1 + r_f) & 0 \end{pmatrix} \\ B_Q &= \begin{pmatrix} 0 & \sigma_p & \sigma_d \end{pmatrix} \end{aligned}$$

I obtain

$$\begin{aligned} J(W_t, M_t, j, t) &= \exp\left(-\alpha_{j+1} R W_{j,t} - \alpha_{j+1} X A_Q M_t - \frac{1}{2} M_t^T V^{AA} M_t \right. \\ &\quad \left. + \frac{1}{2} (\alpha_{j+1} X B_Q + M_t^T V^{AB}) \Xi (V^{AB} M_t + B_Q X \alpha_{j+1}) \right) \end{aligned}$$

Deriving it with respect to X I obtain

$$X = \frac{A_Q - B_Q \Xi V^{AB}}{\alpha_{j+1} \Gamma_{j+1}} M_t.$$

From the Value function I can infer the value of the variable V

$$\begin{aligned}
 V_t = & - \frac{1}{2\Gamma} V^{AB,T} \Xi^T B_Q^T B_Q \Xi V^{AB} \\
 & - \frac{1}{2\Gamma} A_Q^T A_Q \\
 & + \frac{1}{\Gamma} A_Q^T B_Q \Xi V^{AB} \\
 & - \frac{1}{2} V^{AA} \\
 & + \frac{1}{2} V^{AB,T} \Xi V^{AB}
 \end{aligned}$$

where $V^{AB} = A_m^T V_{t+1} B_m$, $V^{AA} = A_m^T V_{t+1} A_m$, $\Gamma = B_Q^T \Xi B_Q$, and

$$\Xi = (\Sigma + B_m V_{t+1} B_m)^{-1}.$$

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