

CONFLICTING PRIORITIES: A THEORY OF COVENANTS AND COLLATERAL*

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Abstract

Debt secured by collateral is repaid ahead of unsecured debt, even if taken in violation of negative pledge covenants. We develop a model in which this priority of secured debt leads to conflicts among debt contracts, but can be optimal nonetheless. Whereas creditors' option to accelerate following covenant violations can deter dilution, preventing over-investment, their option to waive covenants allows for some dilution, preventing under-investment. The optimal debt structure manages the trade-off between over- and under-investment by blocking “bad dilution,” but not “good dilution.” It is multi-layered, including secured and unsecured debt with and without covenants. The model explains a number of facts about debt structure.

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1 Introduction

Firms finance themselves mainly with debt. They often combine several types of debt, including debt protected by covenants and debt secured by collateral.¹ Secured debt has priority over assets used as collateral—until it is paid in full, the assets cannot be sold, pledged as collateral for new debt, or used to pay other debt.² Hence, new secured debt may “leapfrog” existing unsecured debt. For protection, unsecured debt commonly includes negative pledge covenants, giving unsecured creditors the option to accelerate their debt if the borrower takes on new secured debt.³ However, these covenants are weak as secured debt retains its priority even if issued in violation of the covenant, leaving unsecured creditors with little more than the right to demand repayment from a borrower with assets already pledged elsewhere. As a result, legal scholars doubt whether negative pledge covenants are of any use at all:

The covenant does not prevent third parties from acquiring a security interest, but [is] merely...a hollow promise, for in the very act of breaching the covenant, the borrower places its assets out of reach of the negative pledgee and into the hands of the very third party against which the negative pledgee seeks protection (Bjerre (1999), p. 308).

Indeed, unsecured creditors seeking to recoup assets secured to third parties have been consistently denied in court.⁴ Hence, lawyers warn against relying solely on negative pledge covenants as protection against dilution, saying that they are no substitute for collateral.⁵ Yet, borrowers rely on such covenants even when they have assets available to pledge as collateral⁶—they do not follow a pecking order, in which they borrow unsecured only after

¹See, e.g., Erel, Julio, Kim, and Weisbach (2012) on debt’s predominance (95.6% in their sample) and, e.g., Barclay and Smith (1995), Rauh and Sufi (2010), and Colla, Ippolito, and Li (2013) on its heterogeneity.

²See, e.g., Hansmann and Kraakman (2002) and Merrill and Smith (2001) on the priority of secured debt, both outside bankruptcy, when its priority prevents assets from being sold or pledged elsewhere, and in bankruptcy, when the absolute priority rule (APR) dictates that secured debt gets paid first, unsecured debt next, and equity last. Deviations from the APR between unsecured debt and equity are not uncommon (Eberhart, Moore, and Roenfeldt (1990), Franks and Torous (1989)), and Weiss (1990)), but declining (Bharath, Panchapegesan, and Werner (2007)). However, the priority of secured over unsecured debt is typically respected, deviations occurring in none of the Ch. 7 and only 11% of the Ch. 11 bankruptcies in Bris, Welch, and Zhu (2006).

³E.g., negative pledge covenants are the fourth most common type of covenant in Billett, King and Mauer’s (2007) sample, in which they are included in 44% of the debt contracts.

⁴The oldest known ruling on the subject, *Knott v. Shepherdstown Mfg. Co.*, 5 S.E. 266, 269 (W. Va. 1888), stresses that a negative pledge covenant “creates no lien on or pledge of any property” (p. 269), but is merely a personal promise, a view upheld in later cases (see Bjerre’s (1999) footnote 40 for a list (p. 317)).

⁵E.g., an article in the *National Law Review* says that “a Negative Pledge is merely an unsecured promise and gives the Lender very little” (“Negative Pledge Pros and Cons,” April 10, 2016), expressing a view ubiquitous among lawyers (see, e.g., D’Angelo and Saccomandi (2016) and Goetz and Hoffmann (2010)).

⁶See, e.g., Badoer, Dudley, and James (2019) and Rauh and Sufi (2010).

exhausting their secured debt capacity.

If negative pledge covenants cannot enforce priority, why do borrowers rely on them so much? Why is there not a pecking order, in which borrowers first borrow secured, and use unsecured debt only once all of their pledgeable assets are used as collateral? What determines the mix of secured and unsecured debt with and without negative pledge covenants? And, taking a step back, why is secured debt given such strong priority, so that it can expressly undermine other types of debt?

To address these questions, we develop a model in which collateral serves to establish priority over assets, as in, e.g., Donaldson, Gromb, and Piacentino (2018) and DeMarzo (2019), and not to enhance pledgeability of assets, as in much of the literature (e.g., Hart and Moore (1994, 1998) and Rampini and Viswanathan (2010, 2013)).

In the model, as in practice, secured debt retains its priority even if taken in violation of negative pledge covenants, something new to the finance literature (see, however, Ayotte and Bolton (2011)). Thus, the strong priority right given to secured debt lead to conflicts among debt contracts. But we find that it can be optimal nonetheless. Whereas creditors' option to accelerate following violations can deter dilution, preventing over-investment, their incentive to waive covenants allows for some dilution, preventing under-investment. A multi-layered debt structure, including secured and unsecured debt with and without covenants, can manage the trade-off between over-investment and under-investment by blocking "bad dilution," but not "good dilution." The model explains a number of facts about debt structure, including covenant violations and waivers. It also speaks to the legal debate about the efficiency of current priority rules.

Model preview. A borrower, B, has two projects to finance sequentially via secured and/or unsecured debt with and/or without negative pledge covenants. The NPV of the first project is positive, but whether that of the second project is positive or negative is not revealed until after the first project is underway.

Financing is subject to two frictions. First, pledgeability is limited: B cannot borrow against the full value of his projects. As a result, B could be inefficiently financially constrained. Second, contracts are non-exclusive: B's debt contract with initial creditors cannot rule out new debt contracts with later creditors. In particular, B could take on new secured debt, possibly diluting his existing unsecured debt. Although B can use a negative pledge covenant to promise not to do this, he can break his promise. In this case, the different debt contracts are in conflict.

We assume that, as in practice, collateral serves to resolve this conflict, establishing priority among conflicting contracts: debt that is secured by collateral trumps debt that is not. Hence, even if it is taken on in violation of a covenant, secured debt always has

the first claim on the assets used as collateral. Yet, in the event of a violation, creditors with covenants can accelerate their debt, demanding immediate repayment, and possibly forcing B to liquidate his assets, destroying value. Nonetheless, secured debt is still paid first. It is the first claim on B's assets not only in bankruptcy, when it is paid first out of their liquidation value, but also outside bankruptcy, when it must be paid to liquidate them. Hence, what unsecured creditors can gain from acceleration is limited.

Results preview. We explore how the debt structure B chooses when he finances his first project determines the conditions under which he invests in his second project. Does he undertake it only when it has positive NPV? Or does he under- or over-invest? We derive five main results. They characterize, first, how B's debt structure—i.e. the mix of secured and unsecured debt with and without negative pledge covenants—can distort and/or correct B's investment policy and, ultimately, how the optimal debt structure depends on projects' characteristics.

Our first main result is that financing the first project via unsecured debt without covenants can lead to over-investment. Indeed, B can finance his second project via secured debt, diluting the existing unsecured debt. This effectively forces part of the project's cost onto existing creditors, so that B can find it optimal to invest even if the second project has negative-NPV. Thus, dilution of existing unsecured debt by new secured debt can be bad, because it can induce over-investment.

Our second main result is that financing the first project entirely via secured debt prevents over-investment, but can lead to under-investment. Since secured debt has priority, it cannot be diluted. This can prevent inefficient dilution, limiting over-investment. However, some dilution may be necessary to loosen financial constraints stemming from limited pledgeability—dilution can be good, because it can prevent under-investment. Thus, by blocking dilution, secured debt can cause a “collateral-overhang,” a problem that financial restructuring (i.e. renegotiation) cannot solve (Donaldson, Gromb, and Piacentino (2018)). This resonates with practitioners' intuition that secured borrowing “encumbers assets”:

Asset encumbrance not only poses risks to unsecured creditors...but also has wider...implications since encumbered assets are generally not available to obtain...liquidity (Deloitte Blogs (2014)).

Financing the first project via a mix of secured and unsecured debt, hence allowing for some limited dilution, can mitigate this inefficiency. Indeed, if less dilution is needed to finance positive-NPV projects than negative-NPV ones, B can choose a fraction of secured debt that at the same time allows enough flexibility to finance his second project if it is positive NPV but not enough if it is negative NPV. However, if more dilution is needed to

finance negative-NPV projects than positive-NPV ones, under-investment persists.

Since a simple mix of secured and unsecured debt is not always efficient, we consider the role of negative pledge covenants. Suppose the first project is financed via unsecured debt with negative pledge covenants, i.e. B promises not to borrow secured in the future. While B can still issue new secured debt, the threat of acceleration could deter him from doing so, since demanding early repayment could force liquidation, in which he loses (at least) non-pledgeable cash flows.

Our third main result is that financing the first project entirely with unsecured debt with negative pledge covenants cannot deter over-investment, because creditors' acceleration threat is not credible: they are paid after the new secured debt whether they accelerate or not and, thus, have nothing to gain from acceleration. This resonates with legal scholars' doubts on the effectiveness of negative pledge covenants (Bjerre (1999)).

Our fourth main result is that financing the first project with an appropriate mix of unsecured debt with and without negative pledge covenants can deter over-investment, without inducing under-investment. Indeed, if the debt with covenants is accelerated, it dilutes the debt without covenants. Thus, creditors have something to gain from acceleration—namely, priority—and their threat is credible. This contrasts with legal scholars' view that negative pledge covenants are not effective.

The incentive to uphold covenants is stronger when the fraction ϕ of debt with covenants is small, because creditors benefit more from acceleration when the fraction $(1 - \phi)$ of debt that can be diluted is larger. Yet, decreasing ϕ need not be desirable. Indeed, if B needs to dilute existing debt to finance a positive-NPV project, then upholding covenants can be inefficient. Thus, the fraction ϕ should be chosen, if possible, so that covenants are upheld if B is tempted to finance a negative-NPV investment, but waived if he wants to finance a positive-NPV one.

Our fifth main result is that a debt structure can be chosen to implement the efficient investment policy. The optimal debt structure is multi-layered, typically including unsecured debt with and without negative pledge covenants as well as secured debt, possibly issued in violation of those covenants. The debt structure depends on the characteristics of B's projects. If less dilution is needed to finance positive-NPV projects than negative-NPV ones, it contains only secured and unsecured debt. Otherwise, it contains debt with covenants as well, balancing the need to avoid under-investment with that to block over-investment.

This result rationalizes the priority of secured debt (in and outside bankruptcy): B can choose debt instruments appropriately to prevent “bad dilution,” but allow for “good dilution,” and, ultimately, deter both over- and under-investment. Since the quality of the second project is random, this optimal policy is state contingent. But it can be implemented

using only non-state-contingent (debt) instruments. To do so, B exploits the option to dilute unsecured debt with new secured debt. The priority of secured debt is useful: it facilitates efficient contingent dilution.

Policy. Our results speak to the costs and benefits of the absolute priority rule (APR), which prescribes that secured debt be paid first in bankruptcy. The rule is a subject of debate in the law literature; e.g., Bebchuk and Fried (1996) challenge

the desirability of a fundamental and longstanding feature of bankruptcy law: the principle that a secured creditor is entitled to receive the entire amount of its secured claim...before any unsecured claims are paid (p. 859),

arguing that the absolute priority of secured debt facilitates dilution. While our model is consistent with this conclusion, our analysis reveals that (i) relaxing the absolute priority rule could block dilution too much, reducing financial flexibility and leading to under-investment, and that (ii) the downsides of the current priority rule may be limited, because borrowers can structure their debt to block inefficient dilution but allow for efficient dilution.⁷

Stylized facts. Our model explains a number of stylized facts, including that borrowers frequently (i) use debt-structures which include a mix of simple instruments, (ii) use negative pledge covenants despite their weakness, (iii) violate covenants, (iv) receive covenant waivers following violations, (v) use covenants and collateral as parts of a multi-tiered debt structure, (vi) borrow unsecured despite having assets available to use as collateral (there is no pecking order of debt), (vii) have less financial flexibility if they borrow secured (collateral overhang), and (viii) borrow both from banks and markets, using more covenants in bank debt (see Section 8.1 for details and references).

Literature. Our paper contributes to the large finance theory literature on collateral and the smaller one on covenants.⁸ In this literature, covenants and collateral typically mitigate conflicts of interest between borrowers and creditors.⁹ We focus on how they mitigate conflicts of interest among creditors, which is arguably the main legal role of collateral and the express objective of anti-dilution covenants.¹⁰ Bolton and Oehmke (2015), Donaldson, Gromb, and Piacentino (2018), and Stulz and Johnson (1985) explore how collateral estab-

⁷See Ravid et al. (2015) for a model of debt structure in which borrowers structure their debt anticipating deviations from APR in bankruptcy.

⁸For more on collateral, see, e.g., Bester (1985), Eisfeldt and Rampini (2009), Hart and Moore (1994, 1998), and Rampini and Viswanathan (2010, 2013). For more on covenants, see, e.g., Berlin and Mester (1992), Gârleanu and Zwiebel (2009), Park (2002), Rajan and Winton (1995). There are also numerous other papers on debt structure without covenants, including, e.g., Bolton and Scharfstein (1996), Gennaioli and Rossi (2013), and Gertner and Scharfstein (1991).

⁹See, e.g., Tirole (2006) on collateral and Smith (1993) on covenants.

¹⁰Attar, Casamatta, Chassagnon, and Décamps (2015) show, however, that some covenants can help creditors to collude.

lishes priority among creditors, but do not study negative-pledge covenants and how they interact with collateral, our main focus here.

Ayotte and Bolton (2011) is the closest paper to ours in that they also allow for negative pledge covenants. Like us, they also focus on the scope of property/priority rights generating efficient investment, and rationalize aspects of current law. Unlike us, however, they do not consider efficient dilution, and they do not rationalize covenant violations (and waivers). They also abstract from acceleration and renegotiation proofness, two important features of our analysis.

Our finding that the acceleration threat can discipline a borrower is reminiscent of the idea that the option to redeem deposits on demand can discipline a bank (notably, Calomiris and Kahn (1991) and Diamond and Rajan (2001)). Debt with covenants is different from demandable debt, because it can be accelerated (i.e. redeemed) only in the event of a violation. This matters in our model, because otherwise there could be too much acceleration. Moreover, in our model a mix of different types of debt ensures acceleration is credible. If all debt has covenants, there is too little acceleration (Proposition 3).¹¹

Our paper is also related to the law literature on secured debt and priority (e.g., Bebchuk and Fried (1996), Hansmann and Kraakman (2002), Hansmann and Santilli (1997), Kronman and Jackson (1979), Schwarcz (1997), and Schwartz (1984, 1994, 1997)) and to papers on contracting subject to legal rules (e.g., Aghion and Hermalin (1990) and Gennaioli (2006)).

Finally, there is a buoyant empirical literature on secured and unsecured debt, which we relate to throughout the paper, especially in Section 8.1.

Layout. Section 2 presents the model. Section 3 presents the first- and second-best benchmarks. Section 4 studies unsecured and secured debt, and Section 5 negative pledge covenants. Section 6 includes a characterization of the equilibrium debt structure. Section 7 contains extensions. Section 8 discusses implications and related evidence. Section 9 concludes. All proofs are in the Appendix.

2 Model

We consider a model in which a borrower B finances two projects sequentially subject to financial contracting frictions. The model has one good, three dates $t \in \{0, 1, 2\}$, universal risk neutrality; limited liability, and no discounting.

¹¹Note that we abstract from “acceleration runs” by assuming that unsecured debt with negative pledge covenants is held by a single creditor or has a collective action clause. This prevents excessive acceleration and leads to the efficient outcome.

2.1 Projects

B is penniless, but has access to two investment projects, Project 0 and Project 1.

Project 0 costs I_0 at Date 0 and generates a risky payoff at Date 2 when B consumes: with probability p , the project succeeds and pays off $X_0 + Y_0$, where $X_0 \geq 0$ is pledgeable and $Y_0 \geq 0$ is not; otherwise, it fails and pays nothing. We refer to project payoffs as “cash flows.” However, as the pledgeable part could represent the value of assets used in a project, we use “pledgeable cash flows” and “assets” interchangeably (cf. footnote 15).

Project 1 can be high or low quality. Its quality $Q \in \{H, L\}$ is revealed at Date 1, with $\mathbb{P}[Q = H] =: q$.¹² The project costs I_1 at Date 1 and pays off at Date 2, when it succeeds or fails. If it succeeds, it pays off $X_1^Q + Y_1^Q$, where $X_1^Q \geq 0$ pledgeable and $Y_1^Q \geq 0$ is not. If it fails, it pays nothing.

We assume that Project 1 succeeds if and only if Project 0 does (i.e. the projects are perfectly correlated). Thus, it can be viewed as an extension of Project 0. This assumption simplifies the analysis, because it reduces the number of cases to consider, given there is only one outcome (“success”) with positive payoffs (see, however, Section 7.3).

We use the notation $X_{\text{tot.}}$ for the total pledgeable cash flow if all projects undertaken succeed:

$$X_{\text{tot.}} := \mathbb{1}_0 X_0 + \mathbb{1}_1 X_1^Q, \quad (1)$$

where $\mathbb{1}_t$ is the indicator variable,

$$\mathbb{1}_t := \begin{cases} 1 & \text{if Project } t \text{ is undertaken,} \\ 0 & \text{otherwise.} \end{cases} \quad (2)$$

Projects mature at Date 2 but can be liquidated early, before Date 2, for the expected value of their pledgeable cash flows $pX_{\text{tot.}}$. Thus, liquidation is inefficient in that it destroys all (but only) non-pledgeable cash flows.¹³

2.2 Financing

Frictions. At Date $t \in \{0, 1\}$, B can borrow from competitive creditors under two frictions.

1. *Cash flow pledgeability is limited:* X_t can be pledged to creditors, but Y_t cannot. Thus, B cannot borrow against his projects’ full value and might thus be unable to finance

¹²In Section 7.1, we allow Q to be continuous, rather than binary.

¹³The liquidation value is the price competitive market buyers who cannot capture non-pledgeable cash flows would bid for the projects, reflecting our assumption that outsiders cannot capture non-pledgeable cash flows. However, we allow for an additional liquidation discount in Section 7.4.

positive-NPV projects.

2. *Contracts are non-exclusive*: B's debt contract with initial creditors at Date 0 cannot rule out new debt contracts with later creditors at Date 1.¹⁴

Instruments. We focus on three (non-state-contingent) debt instruments: secured debt and unsecured debt with or without negative pledge covenants. We will show that it is without loss of generality in our model, in that allowing for other instruments would not improve the outcome.

1. *Secured debt* is a promise to repay a fixed face value at Date 2 with pledgeable cash flows as collateral.¹⁵

(The role of collateral depends on the priority rule, as described below.)

2. *Unsecured debt* is a promise to repay a fixed face value at Date 2 without collateral.
3. *Unsecured debt with negative pledge covenants* is unsecured debt with the option (but no obligation) to accelerate, i.e. to demand repayment of the face value at Date 1, after the borrower takes on new secured debt (i.e. violates the covenant). Covenants can be waived at any time: B can ask that the covenant be relaxed, which creditors can accept or reject.¹⁶

For simplicity, we assume that unsecured debt with negative pledge covenants is held by a single creditor. This could represent bank debt or dispersed debt with a collective action clause. This turns out to be optimal (hence without loss), and it allows us to abstract from inter-creditor coordination in our baseline analysis (cf. footnote 11).

Priority rules. Given non-exclusivity, B can enter into different contracts with different creditors that need not be consistent. In particular, B can take on more debt than he can ever repay or violate negative pledge covenants. As such, there must be rules specifying how to resolve conflicting priorities among contracts. We consider the following priority rules.

¹⁴Other papers on non-exclusive financial contracting include, e.g., Acharya and Bisin (2014), Attar, Casamatta, Chassagnon, and Décamps (2015, 2017), Bisin and Gottardi (1999, 2003), Bisin and Rampini (2005), Bizer and DeMarzo (1992), Kahn and Mookherjee (1998), Leitner (2012), and Parlour and Rajan (2001).

¹⁵As touched on above, these pledgeable cash flows can represent specific assets. However, they need not: in practice, not all secured debt is “asset based.” E.g., secured debt backed by a corporate division as collateral is based on the future cash flows of the division as a going concern, rather than the assets it currently holds. Likewise, not all unsecured debt is “cash flow based.” E.g., unsecured debt taken by a firm with unmortgaged real estate could be based on these assets in place, rather than any possible future cash flows. See Lian and Ma (2019).

¹⁶This assumption about covenant waivers plays only a small role in our analysis: since creditors are willing to relax covenants when violations do not harm (i.e. do not dilute) their debt, it allows us to restrict attention to violations that do (see the proofs of Proposition 3 and Proposition 4).

1. *Secured debt* has priority over assets used as collateral:
 - (i) Secured debt is paid ahead of unsecured debt.
 - (ii) Earlier secured debt is paid ahead of later secured debt.
 - (iii) If collateral is liquidated, secured debt is paid ahead of other claims. Specifically, if B liquidates assets used as collateral, he must pay the secured debt in full before other claims can get any part of the liquidation proceeds. (This makes security different from seniority, which affects the order of payments only in bankruptcy.)
2. *Unsecured debt (with or without covenants)* is paid in the order it matures:
 - (i) All unsecured debt maturing (or defaulted on) at the same time is paid pro rata.
 - (ii) We assume sequential service of unsecured debt: unsecured debt due at Date 1 (including accelerated debt) is paid ahead of unsecured debt due at Date 2 (but not ahead of secured debt of any maturity, in that if B liquidates collateral to pay unsecured debt, it has to pay secured debt first).

These priority rules reflect practice, as detailed in the law literature. For example, Schwartz (1989) summarizes the basic priority rules between secured and unsecured debt:

Current law regulating these priorities rests on three “priority principles”: First, if the first creditor to deal with the debt makes an unsecured loan, it shares pro rata with later unsecured creditors in the debtor’s assets on default. Second, if this initial creditor makes an unsecured loan and a later creditor takes security, the later creditor has priority over the initial creditor in the assets subject to the security interest. Third, if the initial creditor makes a secured loan, it generally has priority over later creditors in the assets in which it has security (p. 209).

Merrill and Smith (2001) emphasize that secured debt gives creditors a claim on collateral that is prioritized ahead not only of other creditors, but also ahead of potential purchasers—intuitively, you cannot sell/liquidate your house without paying off your mortgage—

a secured lender has a “priority right,” which means that under state law, the lender can enjoy this property right in the face of competing claims of purchasers, transferees, and other creditors (p. 834).

Hahn (2010) details how acceleration can dilute unsecured debt but not secured debt:

[Acceleration] facilitates collection by the speedy...creditors [i.e. those who accelerate their debt] with the potential of harming the less fortunate ones [i.e. those who do not].... Moreover, in the case of a debtor who is also indebted to secured creditors acceleration by unsecured creditors...seems somewhat futile (p. 240).

Observe that maturity and collateral are two ways to establish priority, but that collateral is stronger. Short-maturity (viz. accelerated) unsecured debt gets paid before long-maturity unsecured debt, but not before secured debt, because collateral cannot be liquidated to pay unsecured debt. Thus, these priority rules underscore the distinction between the liquidation of the assets, which happens at Date 1, and the implied liquidation of the firm, i.e. bankruptcy, which happens at Date 2.¹⁷ (See also footnote 21.)

Beyond being realistic, these priority rules turn out to be (weakly) optimal in our model (Proposition 5).

2.3 Timeline

The timeline is as follows:

Date 0: B funds Project 0 from competitive creditors or does not.

Date 1: The quality Q of Project 1 is revealed.

B funds Project 1 via secured debt¹⁸ from competitive creditors or does not.

If a covenant is violated, creditors accelerate (causing liquidation) or do not.

Date 2: If not liquidated at Date 1, projects succeed or fail (together) with probability p , and B makes repayments or defaults.

At any time, contracts can be renegotiated if doing so makes all parties (strictly) better off (cf. Section 7.5).

2.4 Assumptions

We impose three restrictions on parameters.

Assumption 1. Project 0 is efficient and Project 1 is efficient if and only if it is high quality:

$$p(X_0 + Y_0) > I_0, \tag{3}$$

$$p(X_1^H + Y_1^H) > I_1 > p(X_1^L + Y_1^L). \tag{4}$$

¹⁷Although ours is not a model of bankruptcy per se, we interpret default at Date 2 as Chapter 7 bankruptcy (liquidation). Since the model ends at Date 2, there is no scope for Chapter 11 bankruptcy (reorganization).

¹⁸Because secured debt is paid first, borrowing secured is the cheapest way to borrow at Date 1. Thus, it is optimal unless there are covenants restricting new secured debt, but not new unsecured debt. By assuming new Date-1 debt is secured, we are effectively assuming that negative pledge covenants limit new unsecured debt well. We show in Section 7.2 that this is without loss.

This implies that the efficient investment policy is state contingent.

Assumption 2. If B undertakes Project 0 and undertakes Project 1 only if it is high quality, the expected pledgeable cash flows exceed the expected investment costs:

$$pX_0 - I_0 + q(pX_1^H - I_1) \geq 0. \quad (5)$$

As we will show, this assumption implies that the efficient investment policy is implementable with exclusive contracts. This ensures that our results are driven by non-exclusivity, not just by limited pledgeability.

Assumption 3. Irrespective of Project 1's quality, the total liquidation value of Project 0 and Project 1 exceeds the face value needed to finance Project 1, i.e. for $Q \in \{H, L\}$,

$$p(X_0 + X_1^Q) > \frac{I_1}{p}. \quad (6)$$

Observe that the LHS above is the liquidation value of both projects and the RHS is the face value of secured debt F_1^s that B must take on to finance Project 1 (secured creditors' break-even condition is $pF_1^s = I_1$, given that projects succeed with probability p and pay zero otherwise). Thus, this assumption implies that dilution is not so severe that there is nothing left to pay unsecured debt after new secured debt has been paid in liquidation. Therefore, it is not a foregone conclusion that acceleration cannot benefit unsecured debt.

3 First Best and Second Best

The first-best investment policy follows immediately from Assumption 1.

Lemma 1. (First best) *The first-best investment policy is to undertake Project 0 and to undertake Project 1 if and only if it is high quality.*

Given the contracting frictions we have assumed, implementing the first-best policy could face two hurdles.

1. Non-exclusivity might allow B to *over-invest* when $Q = L$, since he could dilute his initial debt.
2. Limited pledgeability might allow B to *under-invest* when $Q = H$, since he could be inefficiently financially constrained.

Our results are driven by the trade-off between the over- and under-investment problems. However, our assumptions imply that the limited pledgeability problem is not so severe that it would prevent B from investing efficiently if he could borrow with exclusive contracts:

Lemma 2. (Second best) *The first best investment policy is implementable with exclusive contracts.*

We now ask whether B can achieve the first best with the available instruments under the associated priority rule.

4 Unsecured and Secured Debt

In this section, we study how the non-exclusivity friction affects financing and, ultimately, investment. We find conditions under which the first best is and is not implementable with only a mix of secured and unsecured debt (i.e. without covenants).

4.1 Unsecured Debt and Over-investment

Suppose that B has issued unsecured debt with face value F_0^u to finance Project 0 at Date 0. Two conditions are necessary for first best.

1. *B undertakes Project 1 if $Q = H$.* Existing debt being unsecured, B can issue debt at Date 1 secured by all assets. Hence, B is able to finance Project 1 if and only if his expected total pledgeable cash flows exceed the cost of Project 1, i.e.

$$p(X_0 + X_1^H) \geq I_1. \quad (7)$$

Given Assumption 3, this condition holds. In addition to being able to finance Project 1, B must want to do so. I.e. what B gets—the total non-pledgeable cash flows plus any residual pledgeable cash flows not used to pay creditors—must be higher if he invests than if he does not:

$$p\left(Y_0 + Y_1^H + \max\left\{0, X_0 + X_1^H - F_0^u - \frac{I_1}{p}\right\}\right) \geq p\left(Y_0 + \max\{0, X_0 - F_0^u\}\right), \quad (8)$$

where I_1/p is the face value of secured debt needed to fund Project 1. This can be simplified as

$$Y_1^H + \max\left\{0, X_0 + X_1^H - F_0^u - \frac{I_1}{p}\right\} \geq \max\{0, X_0 - F_0^u\}, \quad (9)$$

which is satisfied by Assumption 1 with $Q = H$. This simply reflects that Project 1 has positive NPV if $Q = H$ —B captures at least the NPV, and may also benefit from dilution.

2. *B does not undertake Project 1 if $Q = L$.* As above, B is able to finance Project 1 (by Assumption 3 with $Q = L$). Thus, he chooses not to invest in Project 1 only if funding it via secured debt would (weakly) decrease his payoff, or

$$Y_1^L + \max \left\{ 0, X_0 + X_1^L - F_0^u - \frac{I_1}{p} \right\} \leq \max \{ 0, X_0 - F_0^u \}. \quad (10)$$

Thus, provided Project 1's non-pledgeable cash flow Y_1^L is sufficiently low, B does not invest in it. Otherwise, he over-invests, since Date-0 creditors bear part of the investment cost, but B captures (at least) the entire non-pledgeable part of it Y_1^L —dilution is effectively a tax imposed on existing debt that subsidizes new financing/investment.

Proposition 1. (Unsecured debt) *A threshold Y_1^* exists such that the first-best investment policy can be implemented by borrowing unsecured (without covenants) at Date 0 if and only if $Y_1^L \leq Y_1^*$.*

4.2 Secured Debt and Under-investment

Now, suppose that B has issued a mix of secured debt with face value F_0^s and unsecured debt with face value F_0^u to finance Project 0 at Date 0. Since earlier secured debt is ahead of any later debt, B cannot dilute F_0^s . But he can still dilute the unsecured debt. Thus, at Date 1, B can issue debt secured by all assets not already used as collateral. If Project 1 has quality Q , his debt capacity is thus $p(X_0 + X_1^Q - F_0^s)$.

If B has enough debt capacity to finance Project 1 when $Q = H$, but not enough when $Q = L$, then he can satisfy the two conditions for efficiency:

1. *B undertakes Project 1 if $Q = H$.* This is true whenever

$$p(X_0 + X_1^H - F_0^s) \geq I_1. \quad (11)$$

2. *B does not undertake Project 1 if $Q = L$.* This is true provided that¹⁹

$$p(X_0 + X_1^L - F_0^s) < I_1. \quad (12)$$

¹⁹Note that condition (11) is sufficient for B not to invest, but not always necessary; in particular, if Y_1^L is small he will not invest anyway, as per equation (10).

Both conditions are satisfied for X_1^L sufficiently small.

Proposition 2. (Secured and unsecured debt) *The first-best investment policy can be implemented via a mix of secured and unsecured debt (without negative pledge covenants) at Date 0 if*

$$X_1^L < X_1^H. \quad (13)$$

Having financed Project 0, B's available pledgeable cash flows in excess of existing debt are $X_0 + X_1^Q - F_0^s - F_0^u$. But B might not be able to finance Project 1 out of these cash flows alone, even if it is high quality. He may need to dilute existing debt. Borrowing secured at Date 1 increases his borrowing capacity by F_0^u by diluting his unsecured debt. However, because secured debt cannot be diluted, F_0^s puts a cap on dilution and thus on borrowing capacity. If B can keep this cap loose enough to allow dilution to fund the high-quality project at Date 1, while keeping it tight enough to prevent funding the low-quality project, he can implement the first best. Otherwise, he cannot.

Whether such an F_0^s exists depends on X_1^L and X_1^H . If $X_1^L < X_1^H$, more dilution is needed to finance the low-quality project than the high-quality one. Hence, B can choose an amount of secured debt ensuring he can dilute existing debt enough to finance the high-quality project, but not to finance the low-quality one. If $X_1^L > X_1^H$, however, this is not possible.

Corollary 1. (Collateral overhang) *Suppose*

$$X_1^L \geq X_1^H. \quad (14)$$

Any secured debt level such that B cannot finance the low-quality project at Date 1, also prevents him from financing the high-quality project (even if Date-0 debt can be renegotiated).

This is the ‘‘collateral overhang problem’’ in Donaldson, Gromb, and Piacentino (2018): secured debt prevents B from diluting Date-0 creditors to fund an efficient investment—collateralization encumbers B's assets. Here, the problem arises whenever the pledgeable cash flows are lower if the project is high-quality than if it is low-quality (equation (14)). Ex interim renegotiation (i.e. at Date 1, after the project quality is revealed) cannot resolve this inefficiency because limited pledgeability makes it impossible to promise enough compensation to Date-0 creditors.

5 Negative Pledge Covenants

So far, we have shown that a debt structure containing a mix of secured and unsecured debt can sometimes but not always implement the first best. In this section, we ask whether negative pledge covenants can help. We consider first Date-0 financing entirely with unsecured debt with covenants, and then with a mix of debt with and without covenants.

5.1 Only Unsecured Debt with Covenants

Suppose Project 0 is financed entirely with covenant-protected unsecured debt with face value F_0^c . B can issue new secured debt, even in violation of its covenants, but in that case the creditor has the option to accelerate the debt. Since acceleration forces liquidation, which destroys non-pledgeable cash flows, the threat of acceleration could deter dilution, and potentially lead B to invest efficiently. The acceleration threat must be credible, however.

But what is there to gain from acceleration? The violation itself entails prioritizing the new secured debt. Thus, there is nothing to gain and the threat is not credible.

Proposition 3. *Suppose B finances Project 0 entirely via unsecured debt with negative pledge covenants. The covenant is irrelevant.*

To understand the result, suppose B violates the covenants, taking on new secured debt F_1^s , which is prioritized ahead of his existing debt F_0^c by definition (Section 2.2).²⁰ And suppose that B cannot fully repay these debts if his projects pay off (this is necessary for dilution, hence without loss). At maturity, the unsecured creditor has a claim on B's assets, which are worth $X_{\text{tot.}}$ with probability p and zero otherwise. But it is paid after F_1^s (secured debt has priority in bankruptcy), so it gets $p(X_{\text{tot.}} - F_1^s)$. Given the covenant violation, the covenant-protected creditor can, however, accelerate its debt. Acceleration forces B to liquidate the assets, which are worth $pX_{\text{tot.}}$. But, since the assets serve as collateral for the secured debt, he must first repay F_1^s (secured debt has priority over assets even outside bankruptcy), so it gets $pX_{\text{tot.}} - F_1^s$. Comparing these payoffs, we see that

$$p(X_{\text{tot.}} - F_1^s) > pX_{\text{tot.}} - F_1^s. \quad (15)$$

I.e. the creditor never accelerates. The reason is that liquidation subsidizes secured debt, since it makes it less risky: it is repaid F_1^s for sure, not just with probability p . This subsidy

²⁰Recall that we assume that B dilutes with new secured debt, and hence violates the covenants (footnote 18). Dilution with other forms of debt is easy to rule out, as discussed in Section 7.2.

is a tax on unsecured debt. To avoid it, the unsecured creditor does not accelerate. Thus, B is not deterred from taking new secured debt.

The mechanism behind this result jives with the legal arguments that the negative pledge covenant is a “hollow promise,” because, by violating it, B puts his assets out of reach of his unsecured creditors. Indeed, given the acceleration threat is not credible, the outcome is the same as if B borrowed entirely unsecured (Proposition 1).

In practice, however, not all unsecured debt has negative pledge covenants. Some does and some does not (see Section 8.1). Could using negative pledge covenants in only a fraction of the debt, paradoxically, be more effective than using them in all of it? We address this question next.

5.2 Mix of Unsecured Debt with and without Covenants

Suppose B finances Project 0 via a mix of unsecured debt with and without negative pledge covenants, with respective face values F_0^c and F_0^u . Let ϕ be the fraction of the debt with them, $\phi := \frac{F_0^c}{F_0^c + F_0^u}$. The acceleration threat could deter dilution, but it is credible only if the creditor with covenants gains from acceleration. It is still paid behind any new secured debt F_1^s , but now it is paid ahead of the other unsecured debt F_0^u —acceleration dilutes F_0^u . Hence, acceleration is incentive compatible if the benefit of this dilution is large enough.

To see this, suppose, that B violates his covenants, taking on new secured debt F_1^s . And suppose B cannot fully repay these debts if his projects pay off (this is again necessary for dilution, and without loss). At maturity, the unsecured creditor has a claim on B’s assets, which are worth X_{tot} with probability p and zero otherwise. Its claim is paid after F_1^s , but pro rata with F_0^u . Hence, it gets $\phi p(X_{\text{tot}} - F_1^s)$. Given the covenant violation, the covenant-protected creditor can, however, accelerate its debt. Acceleration forces B to liquidate the assets, which are worth pX_{tot} . But, since the assets serve as collateral, he must first repay F_1^s . Hence, the accelerating creditor gets $pX_{\text{tot}} - F_1^s$.²¹ Comparing these payoffs, we see that acceleration is credible if

$$p\phi(X_{\text{tot}} - F_1^s) < pX_{\text{tot}} - F_1^s. \quad (16)$$

²¹If we allowed for bankruptcy at Date 1, all unsecured debt would be paid off pro rata. Hence, an accelerating creditor would have incentive to avoid triggering bankruptcy. For example, it could write down some of its debt, so acceleration would allow it to extract only as much as B could pay. In practice, however, it might not know exactly how much debt to write down. Thus, there is the risk that acceleration forces the firm into bankruptcy, reducing creditors’ incentives to accelerate. Some real-world contractual provisions could mitigate this problem. In particular, a so-called “make whole premium” in debt with covenants would increase their payoff in bankruptcy, and restore their incentive to accelerate. In the model, so would decreasing the fraction of debt with covenants, by increasing the fraction $(1 - \phi)$ of debt that gets diluted if acceleration does not trigger bankruptcy.

Now, acceleration can be credible. The reason is that, although acceleration does nothing to reverse the dilution of secured debt, it now has a benefit for the covenant-protected creditor: it dilutes the unsecured debt without covenants. The accelerating creditor can get paid at Date 1, before B defaults at Date 2—if it can get its money out before B goes bankrupt, it gains effective priority over other unsecured creditors. Here is yet another side of dilution: the covenant protected-creditor’s option to dilute other unsecured debt (via acceleration) creates a credible threat to deter dilution with secured debt (with collateral).

The fraction ϕ of debt with covenants determines the strength of the acceleration threat—the smaller ϕ is, the larger the fraction $1 - \phi$ of dilutable debt, and the more there is to gain from accelerating.²² Thus, B may be able to choose ϕ to make the threat credible in the right state, deterring Date-1 investment in the low-quality project, but not in the high-quality project, i.e. satisfying the two necessary conditions for efficiency:

1. *B undertakes Project 1 if $Q = H$.* B will borrow secured in violation of covenants, only if he anticipates that the creditor with covenants will not accelerate,²³ i.e. if condition (16) does not hold for $Q = H$, or

$$p\phi(X_0 + X_1^H - F_1^s) \geq p(X_0 + X_1^H) - F_1^s. \quad (17)$$

2. *B does not undertake Project 1 if $Q = L$.* B will not issue secured debt if he anticipates that the creditor with covenants will accelerate, i.e. if condition (16) holds for $Q = L$, or,

$$p\phi(X_0 + X_1^L - F_1^s) < p(X_0 + X_1^L) - F_1^s. \quad (18)$$

There is a fraction of debt ϕ with negative pledge covenants such that these conditions are satisfied together whenever X_1^L is sufficiently large:

Proposition 4. (Covenants) *The first-best investment policy can be implemented via a*

²²This finding that decreasing ϕ makes acceleration more attractive contrasts with Gennaioli and Rossi’s (2013) that increasing the controlling creditor’s share exacerbates its liquidation bias. The difference comes from the fact that their controlling creditor is senior/secured, and hence has the most to gain from liquidation.

²³In this implementation, B takes secured debt violating the covenant and then the creditor waives it ex post. This is equivalent to an implementation in which B asks the creditor to waive the covenant ex ante and then takes secured debt without violating it. I.e. there is no distinction between asking for “forgiveness” and “permission.” This suggests that covenant violations could be even more frequent than measures of ex post violations imply, especially since, in practice, asking for “permission” could allow a borrower to circumvent any direct costs of covenant violation, beyond the risk of acceleration we model (e.g., due to lost reputation). In this case, it would also be consistent with creditors increasing interest rates, to share in the surplus created by avoiding such costs (see also footnote 16). Thanks to Adriano Rampini for pointing this out.

mix of unsecured debt with and without negative pledge covenants at Date 0 if

$$X_1^L \geq X_1^H \tag{19}$$

(even if his Date-0 debt can be renegotiated).

This says that if financing the low-quality project dilutes existing debt less than financing the high-quality project—i.e. if $X_1^L \geq X_1^H$ —then negative pledge covenants implement the first best. The result stems from there being a gain from acceleration when dilution is less severe, making the threat credible when dilution is relatively small, but not when it is large. To see why, observe that if B issues new secured debt, the existing unsecured debt is ipso facto junior. Hence, it is both debt-like and equity-like. And the more it is diluted, the closer it is to a residual claim—the more it resembles equity, a call option on B’s assets that creditors are reluctant to exercise early—and the less credible the acceleration threat is. When dilution is large, it is better not to accelerate, but to “gamble for resurrection” as in the prototypical problem of a firm in distress.

Unlike in the prototypical problem, however, this gambling incentive is what leads to the efficient action: it makes the acceleration threat credible in the right state, and hence covenants allow for some dilution—good dilution—despite their stated objective not to.

It is worth stressing that although liquidation is inefficient, B cannot renegotiate it away with his creditors, thus undermining the liquidation threat. The reason is that the extra cash flows from continuation are non-pledgeable. Hence, creditors (weakly) prefer to liquidate and seize B’s assets at Date 1. See, however, Section 7.3 on “coalitional renegotiation.”

6 Equilibrium

Our analysis implies that B can always find a debt structure to implement the first best, but how the structure looks depends on parameters. In particular, observe that the condition under which the first best policy can be implemented via a mix of secured and unsecured debt (equation (13) in Proposition 2) is the complement of that under which it can be via a mix of unsecured debt with and without covenants (equation (19) in Proposition 4). Thus, B can always choose a debt structure to implement the first best.

Proposition 5. (Characterization) *The equilibrium is (first-best) efficient and can be implemented via an appropriately chosen debt structure.*

At Date 0, B finances Project 0 by borrowing I_0 via debt with total face value

$$F_0 = \frac{I_0}{p} + \max \left\{ 0, \frac{q}{1-q} \left(\frac{I_0}{p} + \frac{I_1^H}{p} - X_0 - X_1^H \right), \frac{1-q}{q} \left(\frac{I_0}{p} - X_0 \right) \right\}, \quad (20)$$

where the proportions of this debt that are unsecured without covenants, secured, and unsecured with covenants depend on parameters as follows:

- If $Y_1^L \leq \min \left\{ X_0 - \frac{I_0}{p}, X_0 - \frac{I_0}{p} + \frac{q}{1-q} \left(X_0 + X_1^H - \frac{I_0+I_1}{p} \right) \right\}$, the debt is all unsecured without covenants.
- Otherwise, if $X_1^H > X_1^L$, an amount $F_0^s \in \left(X_0 + X_1^L - \frac{I_1}{p}, X_0 + X_1^H - \frac{I_1}{p} \right]$ is secured.
- Otherwise, the debt is unsecured, and a fraction $\phi \in \left[\frac{p(X_0+X_1^H)-I_1/p}{p(X_0+X_1^H-I_1/p)}, \frac{p(X_0+X_1^L)-I_1/p}{p(X_0+X_1^L-I_1/p)} \right]$ has negative pledge covenants.

At Date 1, B finances Project 1 by borrowing I_1 via secured debt with face value $F_1^s = I_1/p$ if $Q = H$, and does not finance it if $Q = L$.

This result rationalizes the real-world priority structure, in the sense that it allows B to use the instruments at his disposal to implement the first-best outcome. The way he uses the instruments also reflects practice, as we discuss in Section 8.1.

7 Extensions

In this section, we present extensions.

7.1 Continuum of Qualities

So far, we have stressed that we can choose the right debt structure to implement efficiency, allowing for dilution when $Q = H$, but blocking it when $Q = L$. We did this with secured debt if $X_1^H \geq X_1^L$ and with covenants if $X_1^H < X_1^L$. But are these results contingent on having just two qualities? No, the results hold for a continuum qualities as long as pledgeability is monotonic in quality, be it increasing or decreasing.

To see why, we suppose that Project 1 comes in a continuum of possible qualities, with NPV equal to $X_1^Q + Y_1^Q - I_1$. First, observe that equations (11) and (12) imply that for a given amount of secured debt F_0^s , B can fund Project 1 if and only if

$$X_1^Q \geq \frac{I_1}{p} - X_0 + F_0^s. \quad (21)$$

So it is funded only if its pledgeable cash flow is above a cutoff. Thus, if X_1^Q is increasing in Q , B can fund Project 1 with secured debt whenever Q is above the cutoff. Setting the cutoff equal to zero for the zero-NPV project implements the first-best.

Now, observe that equations (17) and (18) imply that for a given fraction ϕ of debt in place with covenants, B can fund Project 1 if and only if

$$X_1^Q < \frac{1 - p\phi}{p(1 - \phi)} F_1 - X_0 \quad (22)$$

(where $F_1 = I_1/p$ from Date-1 creditors' break-even condition). So it is funded only if its pledgeable cash flow is below a cutoff. Thus, if X_1^Q is decreasing Q , B can fund Project 1 with a mix of unsecured debt with and without covenants whenever Q is below a cutoff. Setting the cutoff equal to zero for the zero-NPV project implements the first-best.

In summary, our results obtain as long as X_1^Q is monotonic in Q . Still, our point is not that borrowers can always implement complete efficiency with the right debt structure, but rather that they should choose their debt structure weighing both the costs and benefits of dilution. The more important good dilution is relative to bad dilution, the more they should favor covenants relative to secured debt.

7.2 Pari Passu and Subordinated Debt

So far, we have focused on a debt structure involving a mix of secured debt and unsecured debt with and without covenants, which is then diluted by new secured debt. In theory, it could be diluted by new unsecured debt instead, which would not violate negative pledge covenants and hence could pose a threat to our covenants implementation. Moreover, in practice, debt structure is not so simple, and includes not only secured and unsecured debt, but subordinated debt as well. Here we explain that there is no role for dilution with new pari passu debt in our set-up, but that there can be a role for new subordinated debt in an extension.

Given unsecured debt is paid pro rata, B can dilute existing debt by taking on new unsecured debt with a high face value. Given the debt is unsecured—there is nothing pledged—it does not violate any negative pledge covenants on debt in place. We can safely abstract from this, however, because it is easy to prevent with a simple leverage covenant. There is no puzzle to explain why the threat of acceleration can be a credible way to deter a borrower from taking on new unsecured debt: unlike with new secured debt, accelerating unsecured debt does undo dilution, allowing accelerated debt to jump back ahead of new debt.

Such a leverage covenant could prevent borrowers from borrowing to finance positive NPV

projects. Hence, in practice, they typically allow borrowers to take on new subordinated debt, viz. debt paid after other unsecured debt. To see how there can be a role for subordinated debt in our model, suppose that there is a very high quality project $Q = HH$ that is self financing $pX_1^{HH} > I_1$. If B has covenants preventing him from taking on any new debt, he would not invest in this project because it would trigger acceleration. But, since its pledgeable cash flow exceeds its cost, B can finance it with subordinated debt without diluting his existing debt, something covenants should allow. Indeed, the new investment supports existing debt, since it has the first claim on its value in the event of default.

7.3 Imperfectly Correlated Cash Flows

So far, we have assumed that projects were perfectly correlated, so that Project 1 could be viewed as an enhancement of Project 0. Here, we explore what happens if they are imperfectly correlated. We find that our results are not materially different. However, the results suggest a new testable prediction: for risky firms, increasing diversification can increase reliance on collateral and covenants.

To keep things simple, we focus on the case in which F_0^s and F_1^s are such that B can repay either in full only if both projects succeed, i.e. $F_t^s \in [\max\{X_0, X_1^Q\}, X_0 + X_1^Q)$ for $Q \in \{H, L\}$ and $t \in \{0, 1\}$. This amounts to restricting attention to a risky borrower. We assume that each project succeeds with probability p , as above, but the probability that both do, denoted p' , can now be less than p . (We will not need notations for the probabilities of the other events).

We ask, first, when can B implement the first-best investment policy by financing Project 0 via a mix of secured and unsecured debt? As in Section 4.2, we look for an amount of secured debt F_0^s such that B's debt capacity is sufficient to finance Project 1 if and only if $Q = H$. Now we have that

$$\text{B's debt capacity} = p'(X_0 + X_1^Q - F_0^s). \quad (23)$$

Hence the conditions for efficient investment are just as in equations (11) and (12) in the baseline case, except with p replaced by p' . Thus, our results on secured debt are qualitatively unchanged. However, since $p' \leq p$ —the probability both projects succeed is less than the probability that one does—this suggests that F_0^s should be higher than in the baseline case of perfect correlation.

Now, as in Section 5.2, we look for a fraction of debt with covenants ϕ such that acceleration is IC following investment in Project 1 if and only if $Q = L$. We have that acceleration

is IC whenever

$$p(X_0 + X_1^Q) - F_1^s \geq \phi p'(X_0 + X_1^Q - F_1^s). \quad (24)$$

Hence, the conditions for efficient investment are just as in equations (17) and (18) in the baseline case, except with p replaced by p' on the RHS (but not on the LHS). Thus, our results on covenants are qualitatively unchanged. However, since $p' \leq p$, this suggests that ϕ should be higher than in the case of perfect correlation.

7.4 Liquidation Discount

So far, we have assumed that liquidation destroyed only the non-pledgeable value, which cannot be captured by outsiders, but none of the pledgeable value, which can be. Here, we briefly consider what happens if there is an additional cost of early liquidation, for example because it entails not only a transfer of control, but also early termination. Specifically, we assume that the liquidation value is $\lambda p X_{\text{tot}}$, for $\lambda < 1$. The analysis of debt structure containing only secured and unsecured debt without covenants is unchanged, as it does not involve liquidation. What changes is the analysis of debt structure with covenants. To see how, we look for a fraction of debt with covenants ϕ such that acceleration is IC following investment in Project 1 if and only if $Q = L$, as in Section 5.2. Now, we have that acceleration is IC whenever

$$p\lambda(X_0 + X_1^Q) - F_1^s \geq \phi p'(X_0 + X_1^Q - F_1^s). \quad (25)$$

Hence, the conditions for efficient investment are just as in equations (17) and (18) in the baseline case, except with an additional λ on the LHS. Thus, our results on covenants are qualitatively unchanged. However, since the LHS is smaller than in the baseline case, this suggests that ϕ should be lower than in above. This suggests the additional empirical prediction that when liquidation is more costly, firms should use more covenants.

7.5 Coalitional Renegotiation and Intercreditor Agreements

So far, we have shown that our results are renegotiation proof under the assumption that renegotiation must make all parties better off. Here, we explore whether our results are robust to “coalitional renegotiation,” whereby B colludes with one creditor to expropriate another. This matters, because such renegotiation undermines the acceleration threat in our analysis in Section 5.2. In particular, B could offer collateral to a creditor with covenants as a bribe not to accelerate.²⁴

²⁴This would be a second lien, paid after F_1^s , but ahead of F_0^u . Its payoff from accepting the bribe is thus (up to) $p(X_{\text{tot}} - F_1^s)$, which exceeds his payoff from acceleration of $pX_{\text{tot}} - F_1^s$.

To rule out multi-party renegotiations, we can appeal to multi-party contractual arrangements, called “inter-creditor agreements” in practice.²⁵ When B borrows at Date 0, he includes clauses in his debt by which his creditors commit to each other not to change the priority of their debt. If one creditor violates this agreement, he must compensate the injured creditor. With such an agreement, creditors benefit from renegotiation only if they are collectively better off. I.e., renegotiation is feasible only if it makes all parties better off, as per our original criterion.

8 Empirical Content and Discussion

In this section, we describe the empirical relevance of our findings and discuss their practical and theoretical implications.

8.1 Empirical Relevance

Consistent evidence. Our findings are consistent with a number of stylized facts in the literature:

1. **Covenant use.** Borrowers frequently use negative pledge covenants despite their weakness (e.g., Billet, King, and Mauer (2007) and Ivashina and Vallée (2018)). Thus, we respond to the puzzle stressed by, e.g., Bjerre (1999):

Some may wonder why, given their weakness, costs, and difficulties, lenders bother with negative pledge covenants at all.... [B]orrowers have strong incentives to breach the covenant if necessary financing is available only on a secured basis. [...] The foregoing simply raises, however, the broader question of why lenders ever agree to lend on an unsecured basis, with or without a negative pledge covenant, if collateral is available (pp. 338–339).

In our theory, the borrower uses negative pledge covenants (rather than secured debt) in part because of their weakness: because they allow for efficient dilution.

2. **Covenant violations.** Covenants are frequently violated (e.g., Chava and Roberts (2008), Dichev and Skinner (2002), Nini, Smith, and Sufi (2012), and Roberts and Sufi (2009)).

In our theory, the optimal debt structure allows the borrower to violate covenants by borrowing secured in order to finance efficient investments.

²⁵Thanks to Ken Ayotte for pointing this out.

3. **Covenant waivers.** Following violations, covenants are typically waived and debt is rarely accelerated (e.g., Beneish and Press (1993, 1995), Gopalakrishnan and Prakash (1995), Nini, Smith, and Sufi (2012), and Sweeney (1994)).

In our theory, the borrower violates covenants only in anticipation of their being waived. However, covenants are useful nonetheless because they would not always be waived if violated, which discourages violation. Thus, our theory illustrates how the fact that covenants are waived when violated does not mean they are useless. (Recall that, in our model, waivers could equivalently be granted in anticipation of a violation—there is no distinction between asking for “forgiveness” and “permission”; see footnote 23.)

4. **Debt structure.** Debt secured by collateral and debt protected by covenants can coexist as part of a multi-layered debt structure (Rauh and Sufi (2010)).

In our theory, the borrower exploits complementarities among different types of debt to implement the first-best investment policy. For example, he uses new secured debt to dilute existing unsecured debt, gaining financial flexibility. And he uses negative pledge covenants to prevent excessive dilution. These covenants have teeth only because they can be accelerated at the expense of other debt.

5. **No pecking order.** Borrowers do not use the claims with highest priority first and then lower priority claims. In particular, (i) firms borrow unsecured even when they have assets that they could use as collateral (Rampini and Viswanathan (2013)), (ii) borrowers include deductibles in senior debt to allow for dilution (Ivashina and Vallée (2018)), and (iii) borrowers do not use tight covenants in all of their unsecured debt (e.g., Billett, King, and Mauer (2007) and Rauh and Sufi (2010)).

In our theory, the borrower does not borrow entirely secured, because otherwise there is a collateral-overhang problem, and he does not include covenants in all his debt, because otherwise there is an over-investment problem.

6. **Collateral overhang.** Using collateral to secure debt reduces future debt capacity (Badoer, Dudley, and James (2019)).

In our theory, using collateral limits the borrower’s ability to dilute, reducing debt capacity.

7. **Creditor types.** Borrowers have public and private debt at the same time, and private debt has tighter covenants than public debt (Gopalakrishnan and Prakash (1995)).

In our theory, creditors holding debt protected by negative pledge covenants must be able to enforce or waive covenants optimally following violations, whereas those holding plain unsecured debt should be passive. Thus, we suggest that debt with covenants is more likely to be held by large creditors such as banks, whereas debt without can be held by more dispersed creditors/bondholders.²⁶

New predictions. Our model suggests that borrowers choose their debt structure to manage the trade-off between under-investment and over-investment. This leads to the following predictions, which have yet to be tested directly (to our knowledge), but seem to be consistent with some existing indirect evidence (cf. Proposition 6).

Prediction 1. *Firms relatively more exposed to under-investment problems use covenants.*

Under-investment problems are likely to be most severe in growth firms, which could have good investment opportunities but little pledgeable assets. Thus, the prediction is in line with the fact that covenant use increases in growth opportunities (Billett, King, and Mauer (2007)).

Prediction 2. *Firms relatively more exposed to over-investment problems use collateral.*

Over-investment problems are likely to be severe in distressed firms, which have incentive to gamble for resurrection, tunnel, strip assets, and shift risk. Thus, the prediction is in line with the fact that the use of secured debt, rather than covenants, increases in financial distress (Badoer, Dudley, and James (2019), Benmelech, Kumar, and Rajan (2019), and Rauh and Sufi (2010)).

Prediction 3. *Decreasing the pledgeability of good investment opportunities increases covenant use and decreases collateral use.*

Over time, good investment opportunities seem to have become more likely to use intangible capital, as reflected by the increasing fraction of intangibles on the asset-side of the balance sheets. Thus, the prediction is in line with the fact that secured debt has become less important on the liabilities side (Benmelech, Kumar, and Rajan (2019)).

Another new prediction is based on the extension in Section 7.4, in which we introduce a liquidation discount:

Prediction 4. *Increasing the cost of liquidation increases covenant use.*

²⁶Thus, we provide an explanation for the role of a large creditor with concentrated control rights: it needs to be able to waive covenants. This complements explanations in the literature, based on, e.g., creating incentives to monitor (Park (2002)).

8.2 Discussion of Implications

Covenants vs. collateral. The literature stresses the substitutability of covenants and collateral; for example, Schwartz (1989) says that “Secured debt and covenants are substitutes (both are issued to protect against dilution)” (p. 1418). Indeed, this is true in our model. But we show that there is also a complementarity between covenants and collateral: covenants can implement efficiency only in conjunction with collateral. Although you need covenants to promise not to use collateral—not to dilute unsecured debt inefficiently—you also need collateral to break that promise—to dilute efficiently.

Maturity vs. collateral. Folk wisdom suggests that maturity and collateral are substitutes.²⁷ Indeed, shortening maturity and pledging collateral are two ways to establish priority in our model. But they can still be complements: shortening maturity via acceleration is not only a way for unsecured creditors to get priority, it is also a way for them to prevent secured creditors from getting priority, since the acceleration threat makes it unattractive for the borrower to pledge collateral to new creditors.

Demandable debt vs. short-term debt vs. option to accelerate. The literature has stressed how demandable and short-term debt can discipline borrowers (e.g., Bolton and Scharfstein (1990) and Calomiris and Kahn (1991)). Likewise, the option to accelerate debt disciplines the borrower in our model. But, in contrast, the option to accelerate is only available conditional on a covenant violation, which prevents excessive acceleration. Thus, covenants provide a kind of contingent debt structure, in which a fraction of debt is demandable following a violation. The borrower chooses this fraction such that debt does not discipline too much, i.e. does not deter good dilution.

Assets vs. collateral. In our model, sometimes the borrower chooses to borrow via unsecured debt, even when he has assets available to use as collateral for secured debt. The reason is that unsecured embeds an option to dilute, which helps him to maintain financial flexibility. This contrasts with much of the literature on collateral, in which borrowers cannot choose between secured and unsecured debt. Notably, in models with collateral constraints (e.g., Kiyotaki and Moore (1997)), all debt is effectively secured by the same assets. The reason is that these models effectively assume that contracts are exclusive. Hence, there is no option to dilute existing debt.

Security vs. seniority. By the APR, secured debt is senior in bankruptcy (at least up to the value of the collateral). But secured debt also has priority rights outside of bankruptcy.

²⁷This folk wisdom seems to come from a combination of theories; for example, Hertzberg, Liberman, and Paravisini (2018) say “In theory, lenders can partially mitigate these inefficiencies by using contract terms...such as high collateral (Bester (1985), short maturity (Flannery (1986)), or strict covenants (Levine and Hughes (2005)).”

Assets used as collateral for secured debt cannot be sold or used as collateral for new debt. This matters because it limits what unsecured debt can gain from acceleration—whereas accelerated debt gets paid before long-term senior debt, it is not paid before secured debt. In our model, this allows the borrower to calibrate his debt structure so the acceleration threat is effective—it deters bad dilution—but not too effective—it does not deter good dilution.

Debt vs. debt. The literature stresses how covenants address conflicts between debt and equity. Notably, Smith and Warner (1979) say

In this paper, we examine how debt contracts are written to control the bondholder-stockholder conflict. We investigate the various kinds of bond covenants which are included in actual debt contracts (p. 117).

Our analysis suggests that conflicts among different debts could be as important as conflicts between debt and equity—indeed, negative pledge covenants need not exist at all in our model if creditors did not have conflicting priorities.

Debt vs. equity. In our model, unsecured debt without covenants is paid at the end of the queue—it is always paid behind both secured debt and accelerated debt with covenants. Hence, it is similar to outside equity, the bottom tranche of corporate capital structure. But it is not the same. The reason is that it is paid after debt with covenants only in the event of acceleration, and is otherwise *pari passu*, a contingency necessary to make acceleration incentive compatible. Indeed, it is debt that implements the necessary contingent payoffs, even though such contingencies are more commonly associated with equity.

Creditors vs. creditors. In our model, the threat of acceleration helps to mitigate conflicts among debts. But to make the acceleration threat credible, creditors are pitted against each other—one creditor has the incentive to accelerate only to dilute another’s debt. Thus, efficiency relies on how some conflicts among multiple creditors mitigate others. One creditor, with negative pledge covenants, must act strategically, deciding whether to accelerate its debt or waive a covenant violation. Such a large, strategic creditor could represent a bank. Other creditors, without negative pledge covenants, are passive by comparison. Whether they are diluted or not depends on what the borrower and the bank do. These creditors could represent bondholders. Indeed, in practice, bank debt is concentrated and relatively covenant heavy, whereas bonds are dispersed and relatively covenant lite.

Dilution vs. dilution. Debt dilution is largely viewed as a “serious danger” for firms (Schwartz (1997)) and, likewise, a “major problem” for countries (Eyigungor (2013)). Indeed, dilution can be bad in our model: dilution via collateral can lead to over-investment and dilution via acceleration can lead to inefficient liquidation. But it can also be good: dilution

via collateral can prevent under-investment and dilution via acceleration creates a threat that deters other, inefficient dilution.²⁸ The optimal debt structure—the amount of secured debt and the amount of unsecured debt with negative pledge covenants—allows for good dilution while preventing bad dilution.

Contingent outcomes vs. non-contingent contracts (and contingent debt structure). The literature has paid a lot of attention to contingent contracting. In corporate finance, it has also focused a lot on the debt vs. equity decision, and explored how contingent contracts can be implemented via a mix of debt and equity, as well as some other instruments, such as credit lines. Our model is about implementing a contingent contract too; for the equilibrium to be efficient, B should invest if $Q = H$ but not if $Q = L$. But we focus on the debt vs. debt decision, and show that the efficient strategy can be implemented with a variety of debt contracts that are not contingent at all. Rather, contingencies are implemented via contingent dilution, which itself is implemented by mixing debts with different covenants and priorities. The mix of debt contracts B uses resembles firms’ real-world funding structure: it is almost all debt, but debt is heterogeneous.

Absolute vs. partial priority. The absolute priority rule dictates secured debt is paid in full before anyone else is paid anything. Bebchuk and Fried (1996) argue that such absolute priority of secured debt can create inefficiencies, because it gives secured debt the power to defeat other claims. We argue that this is not always a bad thing, because dilution can be good, helping to overcome limited pledgeability. Moreover, we show how borrowers can use a mix of different types of (non-contingent) debt to allow for contingent dilution, allowing efficient dilution, but still preventing inefficient dilution.

The price of debt with vs. without covenants. How do covenants affect debt pricing? Their being prevalent in contracts suggests they might matter a lot.²⁹ But their being enforced seldom could suggest they might not. In our model, debt with covenants has the same price as debt without, even when covenants are effective (see Lemma 3 in the Appendix). Indeed, covenants are effective exactly because there is debt without covenants that can be diluted—it is this option to dilute that makes the acceleration threat credible. However, all debt, not just that with covenants, is more valuable because some of it has covenants (which discipline the borrower through the acceleration threat) and some of it does not (which makes this threat credible). This is consistent with evidence in Bradley and Roberts (2015) which finds that firms’ bonds have lower yields when their loans have more covenants.

Flexibility vs. rigidity. In many models, covenants are hard restrictions, and hence

²⁸Optimal “dilutable debt” also appears in Diamond (1993), Donaldson and Piacentino (2017), and Hart and Moore (1995).

²⁹Matvos (2013) and Green (2018) use structural models to argue that covenants are economically valuable.

impose the cost of limited flexibility. In ours, in contrast, covenants can be violated, and indeed bring the benefit of increased flexibility with respect to secured debt.

9 Conclusion

We present a model of financial contracting in which contracts are non-exclusive, and hence can conflict: contracts may contain covenants putting restrictions on other contracts, but these covenants can be violated. In this case, a priority rule is needed to resolve conflicts among contracts. Hence, contracts are meaningful only with respect to the priority rule.

In practice, secured debt has priority. This creates the risk of dilution: new secured debt overrides existing unsecured debt. Given this priority, negative pledge covenants restricting new secured debt might seem futile—they can be overridden by the very dilution they are supposedly there to prevent. But we show that this can be a good thing. The reason is that in addition to the usual bad side of dilution (it leads to over-investment), there are good sides as well. First, it can loosen borrowing constraints that could be too tight due to limited pledgeability, and hence prevent over-investment. Second, it subsidizes accelerating creditors, hence making their threat credible and preventing bad dilution. In our environment, a borrower who understands the existing priority structure can choose his debt structure to get the good sides of dilution without the bad, and hence implement the efficient investment policy. Hence, our model rationalizes the existing priority rules.

A Proofs

A.1 Proof of Lemma 1

The result follows immediately from Assumption 1. \square

A.2 Proof of Lemma 2

We show that B and a creditor can commit to an exclusive contract at Date 0 with Date-2 repayments R^H given success if $Q = H$ and R^L given success if $Q = L$ such that:

1. Irrespective of Project 1's quality, B's pledgeable cash flow suffices to meet the promised repayments given success at Date 2 (under the first-best investment policy):

$$X_0 + X_1^H \geq R^H, \quad (26)$$

$$X_0 \geq R^L. \quad (27)$$

2. Given repayments R^H and R^L , the creditor is willing to participate at Date 0, i.e. her expected repayment exceeds her expected investment costs (under the first-best investment policy):

$$p(qR^H + (1 - q)R^L) \geq I_0 + qI_1. \quad (28)$$

Assumption 2 and Assumption 3 imply these inequalities can be satisfied. One easy way to see this is to make the first two bind, so $F_0^H = X_0 + X_1^H$ and $F_0^L = X_0$. In this case, the third (inequality (28)) reduces to Assumption 2.

Note that this result does not rely on debt being state-contingent. It is also implementable with defaultable debt: letting F_0 be the face value associated with lending I_0 at Date 0 and F_1 with lending I_1 at Date 1, just set

$$R^L \equiv \min\{X_0 + X_1^H, F_0 + F_1\} \quad \text{and} \quad R^H \equiv \min\{X_0, F_0\}. \quad (29)$$

\square

A.3 Proof of Proposition 1

To prove the proposition, we consider face value F_0^u if B follows the efficient strategy and determine when B has no incentive to deviate and invest if $Q = L$. (We know B will invest

if $Q = H$ irrespective of F_0^u .) If $Q = H$, B can borrow I_1 with secured debt with face value F_1 such that $p \min\{X_0 + X_1^H, F_1\} = I_1$. Thus, by Assumption 3, $F_1 = I_1/p$.

Case 1: $X_0 + X_1^H \geq I_0/p + I_1/p$ and $X_0 \geq I_0/p$.

In this case, if the projects succeed, B is able to pay I_0/p to Date-0 creditors irrespective of Q and so

$$F_0^u = I_0/p. \quad (30)$$

Condition (10) becomes

$$Y_1^L + \max\left\{0, X_0 + X_1^L - \frac{I_0 + I_1}{p}\right\} \leq X_0 - \frac{I_0}{p}. \quad (31)$$

There are two subcases, depending on whether B defaults on Date-0 creditors if he invests when $Q = L$ and the projects succeed.

Subcase 1.1 $X_0 + X_1^L > I_0/p + I_1/p$.

In this case, B does not default. As a result, he would bear the full negative value of Project 1 when $Q = L$ and so does not undertake it in that case.

Subcase 1.2 $X_0 + X_1^L < I_0/p + I_1/p$.

In this case, if B invests in Project 1 when $Q = L$ and the projects succeed, he defaults on Date-0 creditors. Hence, condition (10) becomes

$$Y_1^L \leq X_0 - \frac{I_0}{p}. \quad (32)$$

Summing up, B will undertake Project 1 when $Q = L$ if

$$X_0 + X_1^L < \frac{I_0 + I_1}{p} \quad \text{and} \quad Y_1^L > X_0 - \frac{I_0}{p}. \quad (33)$$

By Assumption 1, $Y_1^L < I_1/p - X_1^L$, so one condition implies the other: in this case, there is over-investment if and only if $Y_1^L > X_0 - I_0/p$, and conversely, B will not undertake Project 1 when $Q = L$ if and only if

$$Y_1^L \leq X_0 - I_0/p. \quad (34)$$

Case 2: $X_0 + X_1^H < I_0/p + I_1/p$ and $X_0 \geq I_0/p$.

In this case, if B undertakes Project 1 and the projects succeed, he defaults on his Date-0 debt for $Q = H$ but not for $Q = L$. Hence, F_0^u is given by the following break-even condition

for Date-0 creditors:

$$I_0 = p \left(q \left(X_0 + X_1^H - \frac{I_1}{p} \right) + (1 - q)F_0^u \right) \quad (35)$$

so

$$F_0^u = \frac{I_0/p - q \left(X_0 + X_1^H - I_1/p \right)}{1 - q}. \quad (36)$$

Note that given $X_0 + X_1^H < I_0/p + I_1/p$, Assumption 2 implies $F_0^u \leq X_0$, so B does not default if $Q = L$. Thus, condition (10) becomes

$$Y_1^L + \max \left\{ 0, X_0 + X_1^L - F_0^u - \frac{I_1}{p} \right\} \leq X_0 - F_0^u \quad (37)$$

There are two subcases, depending on whether B defaults if B undertakes Project 1 when $Q = L$ and the projects succeed.

Subcase 2.1: $X_0 + X_1^L \geq F_0^u + I_1/p$.

In that case, B would not default and so would bear the full negative value of Project 1. Hence, he does not undertake Project 1 if $Q = L$.

Subcase 2.2: $X_0 + X_1^L < F_0^u + I_1/p$. In that case, B would default and condition (10) becomes

$$Y_1^L \leq X_0 - F_0^u, \quad (38)$$

which, by Assumption 1, implies the subcase's condition, i.e.

$$X_0 + X_1^L < F_0^u + \frac{I_1}{p}. \quad (39)$$

Hence, B does not undertake Project 1 when $Q = L$ if and only if condition (38) holds which, plugging in for F_0^u , can be rewritten as

$$(1 - q)Y_1^L \leq X_0 - \frac{I_0}{p} + q \left(X_1^H - \frac{I_1}{p} \right). \quad (40)$$

Case 3: $X_0 < I_0/p$. In this case, B defaults if $Q = L$ but not if $Q = H$ and the projects succeed. Thus, Date-0 creditors' break-even condition is

$$I_0 = p(qF_0^u + (1 - q)X_0) \quad (41)$$

so

$$F_0^u = \frac{I_0/p - (1-q)X_0}{q}. \quad (42)$$

Note that given $X_0 < I_0/p$ in this case, Assumption 2 implies that $F_0^u + I_1/p \leq X_0 + X_1^H$, so B does not default if $Q = H$ and the projects succeed. In this case B always defaults if $Q = L$. Hence, inequality (10) reduces to $Y_1^L \leq 0$, which is never satisfied.

Efficiency conditions. In summary, efficient investment requires that $X_0 - I_0/p \geq 0$ (from Case 3) and that (from Case 1)

$$Y_1^L \leq X_0 - \frac{I_0}{p} \quad \text{if} \quad X_0 + X_1^H - \frac{I_0 + I_1}{p} \geq 0 \quad (43)$$

and (from Case 2)

$$Y_1^L \leq X_0 - \frac{I_0}{p} + \frac{q}{1-q} \left(X_0 + X_1^H - \frac{I_0 + I_1}{p} \right) \quad \text{if} \quad X_0 + X_1^H - \frac{I_0 + I_1}{p} < 0. \quad (44)$$

Taken together, equations (43) and (44) can be written as

$$Y_1^L \leq \min \left\{ X_0 - \frac{I_0}{p}, X_0 - \frac{I_0}{p} + \frac{q}{1-q} \left(X_0 + X_1^H - \frac{I_0 + I_1}{p} \right) \right\}. \quad (45)$$

which defines Y_1^* in the proposition.

(Finally, note that we can omit the condition that $X_0 \geq I_0/p$, since it is implied by the condition that $Y_0 \leq X_0 - I_0/p$.) \square

A.4 Proof of Proposition 2

Immediately from equations (11) and (12), efficiency is implementable whenever there is a face value F_0^s such that

$$X_0 + X_1^L - \frac{I_1}{p} \leq F_0^s < X_0 + X_1^H - \frac{I_1}{p} \quad (46)$$

The RHS is positive by Assumption 3; hence, F_0^s exists whenever the LHS is less than the RHS, or $X_1^H > X_1^L$, which is the condition in the proposition. \square

A.5 Proof of Corollary 1

The baseline result follows from the observation that the inequalities (11) and (12) cannot be satisfied at once if $X_1^L \geq X_1^H$, which is the condition in the corollary (cf. the proof of

Proposition 2).

Renegotiation proofness. First, observe that, by hypothesis, the L -quality project cannot be financed, or

$$p(X_0 + X_1^L - F_0^s) < I_1 \quad (47)$$

and, also by hypothesis, $X_1^H < X_1^L$, so

$$p(X_0 + X_1^H - F_0^s) < I_1. \quad (48)$$

Now suppose (in anticipation of a contradiction) that B can renegotiate with his creditors to do the H -quality project at Date 1, i.e. that he can reallocate cash flow to make everyone strictly better off (and hence agree to renegotiation). This requires that Date-0 creditors get at least pF_0^s (which they get if they do not renegotiate) and Date-1 creditors get at least I_1 (which they pay to invest). Since B can promise creditors only the pledgeable cash flow, it must be that there is enough pledgeable cash flow to make all creditors better off, or

$$p(X_0 + X_1^H) > pF_0^s + I_1, \quad (49)$$

which contradicts the inequality (48). Hence, renegotiation is not feasible. \square

A.6 Proof of Proposition 3

The argument for why the single creditor never accelerates is in the text. Without the acceleration threat, unsecured debt with negative pledge covenants is equivalent to unsecured debt. Hence, the outcome is that described in Proposition 1.

A.7 Proof of Proposition 4

Before starting the proof, we write down creditors' payoffs from accelerating or not. First, observe that if B borrows at Date 1, he always borrows fully secured, to maximize the benefit of dilution. Hence, from Date-1 creditors' break-even condition, the face value of Date-1 debt is

$$F_1 = \frac{I_1}{p} \quad (50)$$

Now, we denote the total face value of Date-0 debt with and without covenants F_0^c and F_0^{nc} respectively, with $F_0^c + F_0^c \equiv F_0$, and, likewise, amount borrowed with and without covenants by I_0^c and I_0^{nc} respectively, with $I_0^c + I_0^{nc} \equiv I_0$, and, by the definition of ϕ , $I_0^c \equiv \phi I_0$. There

are three relevant cases:³⁰

1. **B does not borrow at Date 1.** In this case, B repays in full at Date 2 if $X_{\text{tot.}} \geq F_0$ and defaults otherwise, in which case creditors are paid pro rata:
 - Unsecured creditors with covenants get $p \min\{F_0^c, \phi X_{\text{tot.}}\}$.
 - Unsecured creditors without covenants get $p \min\{F_0^{\text{nc}}, (1 - \phi)X_{\text{tot.}}\}$.
2. **B borrows secured at Date 1, but debt is not accelerated.** In this case, B repays in full at Date 2 if $X_{\text{tot.}} \geq F_0 + F_1$ and defaults otherwise, in which case he repays the secured debt first and the unsecured debt pro rata:
 - Secured creditors break even, getting F_1 with probability p (recall that $F_1 = I_1/p$ from equation (50)).
 - Unsecured creditors with covenants get $p \min\{F_0^c, \phi(X_{\text{tot.}} - F_1)\}$.
 - Unsecured creditors without covenants get $p \min\{F_0^{\text{nc}}, (1 - \phi)(X_{\text{tot.}} - F_1)\}$.
3. **B borrows secured at Date 1, and debt is accelerated.** In this case, B repays in full at Date 1 if $pX_{\text{tot.}} \geq F_0 + F_1$ and defaults otherwise, in which case he repays secured debt first, the accelerating unsecured creditors (those with covenants) next, and other unsecured creditors last:
 - Secured creditors get F_1 (given $pX_{\text{tot.}} \geq F_1$ by Assumption 3).
 - Unsecured creditors with covenants get $\min\{F_0^c, pX_{\text{tot.}} - F_1\}$.
 - Unsecured creditors without covenants get either the smaller of their face value and the assets remaining after all other creditors have been repayed: $\min\left\{F_0^{\text{nc}}, pX_{\text{tot.}} - F_1 - \min\{F_0^c, pX_{\text{tot.}} - F_1\}\right\}$.

Before moving on the main argument, we prove a lemma that said that the interest rates on debt with and without covenants are the same under the efficient strategy.

Lemma 3. *If B does the first-best strategy, then the interest rates on debt with and without covenants coincide: $F^c/I_0^c = F_0^{\text{nc}}/I_0^{\text{nc}}$. Hence $I_0^c = \phi I_0$ and $I_0^{\text{nc}} = (1 - \phi)I_0$.*

Proof. There are three cases.

³⁰We omit cases in which B takes on new unsecured debt at Date 1, because it is easy to show that doing so is dominated by taking on new secured debt at Date 1.

Case 1: $p(X_0 + X_1^H) > I_0 + I_1^H$ **and** $pX_0 \geq I_0$. In this case, all debt is repaid in full in the event of success and repaid nothing otherwise. Thus, $F_0^c = I_0^c/p$ and $F_0^{\text{nc}} = I_0^{\text{nc}}/p$. Hence $F_0 = I_0/p$ which implies that $F_0^c = \phi I_0$ and $F_0^{\text{nc}} = (1 - \phi)F_0 = (1 - \phi)I_0$.

Case 2: $p(X_0 + X_1^H) < I_0 + I_1$ **and** $pX_0 \geq I_0$. In this case, B following success if $Q = H$, but not if $Q = L$. Using $I_0^c = \phi I_0$ and $I_0^{\text{nc}} = (1 - \phi)I_0$, creditors' break-even conditions are

$$\phi I_0 = p \left(q\phi \left(X_0 + X_1^H - \frac{I_1}{p} \right) + (1 - q)F_0^{\text{nc}} \right), \quad (51)$$

$$(1 - \phi)I_0^{\text{nc}} = p \left(q(1 - \phi) \left(X_0 + X_1^H - \frac{I_1}{p} \right) + (1 - q)F_0^{\text{nc}} \right), \quad (52)$$

having used $F_1 = I_1/p$. Solving for F_1^c and F_1^{nc} above gives the result.

Case 3: $pX_0 < I_0$. In this case, B defaults given success if $Q = L$ but not if $Q = H$. Again, we use $I_0^c = \phi I_0$ and $I_0^{\text{nc}} = (1 - \phi)I_0$ to write creditors' break-even conditions:

$$\phi I_0 = p \left(qF_0^c + (1 - q)\phi X_0 \right), \quad (53)$$

$$(1 - \phi)I_0 = p \left(q(1 - \phi)F_0^{\text{nc}} + (1 - q)(1 - \phi)X_0 \right). \quad (54)$$

Again, solving for F_0^c and F_0^{nc} gives the result. □

We now turn to the proof of the proposition. As we argued in the text, acceleration must be incentive compatible following a covenant violation if $Q = L$ but not if $Q = H$. Rather than the conditions (17) and (18), in which we assumed that B never repaid in full, we now have

$$\min \left\{ \phi F_0, p \left(X_0 + X_1^H \right) - \frac{I_1}{p} \right\} \leq p \min \left\{ \phi F_0, \phi \left(X_0 + X_1^H - \frac{I_1}{p} \right) \right\}, \quad (55)$$

$$\min \left\{ \phi F_0, p \left(X_0 + X_1^L \right) - \frac{I_1}{p} \right\} \geq p \min \left\{ \phi F_0, \phi \left(X_0 + X_1^L - \frac{I_1}{p} \right) \right\}. \quad (56)$$

Before launching into the main argument, we can dispense with a few cases relatively easily:

1. We can focus on cases in which there is dilution if $Q = L$, or $F_0 > X_0 + X_1^L - I_1/p$. Otherwise, B will not finance Project 1 in this case, since it has negative NPV.

2. By implication, we can focus on cases in which there is dilution if $Q = H$ as well, since $X_1^H < X_1^L$ by hypothesis.
3. We can focus on cases in which the accelerated debt is not paid in full if $Q = L$, since we are looking only for a sufficient condition (and otherwise acceleration is always IC).

Now, the ICs can be simplified to read

$$\min \left\{ \phi F_0, p(X_0 + X_1^H) - \frac{I_1}{p} \right\} \leq q\phi \left(X_0 + X_1^H - \frac{I_1}{q} \right), \quad (57)$$

$$p(X_0 + X_1^L) - \frac{I_1}{p} \geq p\phi \left(X_0 + X_1^L - \frac{I_1}{p} \right). \quad (58)$$

To get sufficient conditions, we can split (57) in two, and write

$$p(X_0 + X_1^H) - \frac{I_1}{p} \leq p\phi \left(X_0 + X_1^H - \frac{I_1}{p} \right), \quad (59)$$

$$p(X_0 + X_1^H) - \frac{I_1}{p} \leq \phi F_0, \quad (60)$$

$$p(X_0 + X_1^L) - \frac{I_1}{p} \geq p\phi \left(X_0 + X_1^L - \frac{I_1}{p} \right). \quad (61)$$

Combining the above, we have

$$\max \left\{ \frac{p(X_0 + X_1^H) - I_1/p}{F_0}, \frac{p(X_0 + X_1^H) - I_1/p}{p(X_0 + X_1^H - I_1/p)} \right\} \leq \phi \leq \frac{p(X_0 + X_1^L) - I_1/p}{p(X_0 + X_1^L - I_1/p)}. \quad (62)$$

Now, we can do away with the max above. Recall that we are focused on a case in which there is dilution (hence default) given success if $Q = H$. It follows that $F_0 > X_0 + X_1^H - I_1/p > p(X_0 + X_1^H - I_1/p)$ and hence

$$\max \left\{ \frac{p(X_0 + X_1^H) - I_1/p}{F_0}, \frac{p(X_0 + X_1^H) - I_1/p}{p(X_0 + X_1^H - I_1/p)} \right\} = \frac{p(X_0 + X_1^H) - I_1/p}{p(X_0 + X_1^H - I_1/p)}. \quad (63)$$

So we can implement the first best if we can find ϕ satisfying

$$\frac{p(X_0 + X_1^H) - I_1/p}{p(X_0 + X_1^H - I_1/p)} \leq \phi \leq \frac{p(X_0 + X_1^L) - I_1/p}{p(X_0 + X_1^L - I_1/p)}. \quad (64)$$

Since the LHS is always less than one and the RHS is greater than zero by Assumption 3, such a ϕ exists whenever the LHS is less than the RHS, or $X_1^H \leq X_1^L$, which is the condition in the proposition.

Renegotiation proofness. This argument hinges on acceleration being a credible threat when $Q = L$, even though liquidation is inefficient. To complete the proof, we show that this is robust to the possibility of renegotiation. For renegotiation to be feasible, all parties, i.e. (i) B, (ii) Date-1 secured creditors, (iii) Date-0 creditors, both protected by covenants and not, must be strictly better off. However, if B avoids liquidation and continues, the most he can promise his creditors is $p(X_0 + X_1^L)$. But this is only equal to the liquidation value that creditors are already dividing up among themselves. Hence, there is no way to make them collectively better off.

A.8 Proof of Proposition 5

The expression face value F_0 follows from equations (30), (36), and (42) in the proof of Proposition 1. The regions in which B uses secured debt or covenants and the ranges of σ_0 and ϕ follow from Proposition 2 and Proposition 4 (and their proofs).

References

- Acharya, V. and A. Bisin (2014). Counterparty risk externality: Centralized versus over-the-counter markets. *Journal of Economic Theory* 149(C), 153–182.
- Aghion, P. and B. Hermalin (1990). Legal restrictions on private contracts can enhance efficiency. *Journal of Law, Economics, & Organization* 6(2), 381–409.
- Attar, A., C. Casamatta, A. Chassagnon, and J.-P. Décamps (2015). Multiple lenders, strategic default, and the role of debt covenants. Working paper, Toulouse School of Economics.
- Attar, A., C. Casamatta, A. Chassagnon, and J.-P. Décamps (2017). On the role of menus in sequential contracting: A multiple lending example. Working paper, Toulouse School of Economics.
- Ayotte, K. and P. Bolton (2011). Optimal property rights in financial contracting. *Review of Financial Studies* 24(10), 3401–3433.
- Badoer, D., E. Dudley, and C. James (2019). Priority Spreading of Corporate Debt. *The Review of Financial Studies*.
- Barclay, M. J. and C. W. Smith (1995). The priority structure of corporate liabilities. *The Journal of Finance* 50(3), 899–916.
- Bebchuk, L. A. and J. M. Fried (1996). The uneasy case for the priority of secured claims in bankruptcy. *The Yale Law Journal* 105(4), 857–934.
- Beneish, M. D. and E. Press (1993). Costs of technical violation of accounting-based debt covenants. *The Accounting Review* 68(2), 233–257.
- Beneish, M. D. and E. Press (1995). The resolution of technical default. *The Accounting Review* 70(2), 337–353.
- Benmelech, E., N. Kumar, and R. Rajan (2019). The decline of secured debt. Working paper, Northwestern University.
- Berlin, M. and L. J. Mester (1992). Debt covenants and renegotiation. *Journal of Financial Intermediation* 2(2), 95–133.
- Bester, H. (1985). The level of investment in credit markets with imperfect information. *Institutional and Theoretical Economics* 141(4), 503–515.

- Bharath, S., V. Panchapegesan, and I. Werner (2007, October). The Changing Nature of Chapter 11. Working Paper Series 2008-4, Ohio State University, Charles A. Dice Center for Research in Financial Economics.
- Billett, M., T.-H. D. King, and D. Mauer (2007). Growth opportunities and the choice of leverage, debt maturity, and covenants. *The Journal of Finance* 62(2), 697–730.
- Bisin, A. and P. Gottardi (1999). Competitive equilibria with asymmetric information. *Journal of Economic Theory* 87, 1–48.
- Bisin, A. and P. Gottardi (2003). Competitive markets for non exclusive contracts with adverse selection: The role of entry fees. *Review of Economic Theory* 6(2), 313–338.
- Bisin, A. and A. A. Rampini (2005). Exclusive contracts and the institution of bankruptcy. *Economic Theory* 27(2), 277–304.
- Bizer, D. S. and P. M. DeMarzo (1992, February). Sequential Banking. *Journal of Political Economy* 100(1), 41–61.
- Bjerre, C. S. (1999). Secured transactions inside out: Negative pledge covenants, property and perfection. *Cornell Law Review*.
- Bolton, P. and M. Oehmke (2015). Should derivatives be privileged in bankruptcy? *Journal of Finance* 70(6), 2352–2394.
- Bolton, P. and D. S. Scharfstein (1990, March). A theory of predation based on agency problems in financial contracting. *American Economic Review* 80(1), 93–106.
- Bolton, P. and D. S. Scharfstein (1996, February). Optimal Debt Structure and the Number of Creditors. *Journal of Political Economy* 104(1), 1–25.
- Bradley, M. and M. R. Roberts (2015). The structure and pricing of corporate debt covenants. *Quarterly Journal of Finance* 05(02), 1550001.
- Bris, A., I. Welch, and N. Zhu (2006). The costs of bankruptcy: Chapter 7 liquidation versus chapter 11 reorganization. *The Journal of Finance* 61(3), 1253–1303.
- Calomiris, C. W. and C. M. Kahn (1991, June). The Role of Demandable Debt in Structuring Optimal Banking Arrangements. *American Economic Review* 81(3), 497–513.
- Chava, S. and M. Roberts (2008). How does financing impact investment? the role of debt covenants. *Journal of Finance* 63(5), 2085–2121.

- Colla, P., F. Ippolito, and K. Li (2013). Debt specialization. *The Journal of Finance* 68(5), 2117–2141.
- D’Angelo, B. and F. Saccomandi (2016). Negative pledge pros and cons. <https://www.jdsupra.com/legalnews/negative-pledge-pros-and-cons-81258/>.
- Deloitte Blogs (2014). Asset encumbrance: The elephant in the room?
- DeMarzo, P. (2019). Presidential address: Collateral and commitment. *The Journal of Finance* 74(4), 1587–1619.
- Diamond, D. W. (1993). Seniority and maturity of debt contracts. *Journal of Financial Economics* 33(3), 341–368.
- Diamond, D. W. and R. G. Rajan (2001). Liquidity risk, liquidity creation, and financial fragility: A theory of banking. *Journal of Political Economy* 109, 287–327.
- Dichev, I. D. and D. J. Skinner (2002). Large sample evidence on the debt covenant hypothesis. *Journal of Accounting Research* 40(4), 1091–1123.
- Donaldson, J. R., D. Gromb, and G. Piacentino (2018). The paradox of pledgeability. *Journal of Financial Economics*, forthcoming.
- Donaldson, J. R. and G. Piacentino (2017). Netting. Working paper, Washington University in St. Louis.
- Eberhart, A., W. Moore, and R. Roenfeldt (1990). Security pricing and deviations from the absolute priority rule in bankruptcy proceedings. *The Journal of Finance* 45(5), 1457–1469.
- Eisfeldt, A. L. and A. A. Rampini (2009). Leasing, ability to repossess, and debt capacity. *Review of Financial Studies* 22(4), 1621–1657.
- Erel, I., B. Julio, W. Kim, and M. S. Weisbach (2012). Macroeconomic conditions and capital raising. *The Review of Financial Studies* 25(2), 341–376.
- Eyigungor, B. (2013). Debt dilution: when it is a major problem and how to deal with it. *Business Review* (Q4), 1–8.
- Flannery, M. (1986). Asymmetric information and risky debt maturity choice. *The Journal of Finance* 41(1), 19–37.

- Franks, J. and W. Torous (1989). An empirical investigation of u.s. firms in reorganization. *The Journal of Finance* 44(3), 747–769.
- Gârleanu, N. and J. Zwiebel (2009). Design and renegotiation of debt covenants. *Review of Financial Studies* 22(2), 749–781.
- Gennaioli, N. (2006). Contracting in the shadow of the law. Working paper, Institute for International Economic Studies, Stockholm University.
- Gennaioli, N. and S. Rossi (2013). Contractual resolutions of financial distress. *The Review of Financial Studies* 26(3), 602–634.
- Gertner, R. and D. Scharfstein (1991). A theory of workouts and the effects of reorganization law. *The Journal of Finance* 46(4), 1189–1222.
- Goetz, E. and K. Hoffmann (2010). The negative pledge. <https://www.jdsupra.com/legalnews/negative-pledge-pros-and-cons-81258/>.
- Gopalakrishnan, V. and M. Prakash (1995). Borrower and lender perceptions of accounting information in corporate lending agreements. 9(1).
- Green, D. (2018). Corporate refinancing, covenants, and the agency cost of debt. Working paper, Massachusetts Institute of Technology.
- Hahn, D. (2010). The roles of acceleration. *DePaul Business and Commerce Law Journal* 8.
- Hansmann, H. and R. Kraakman (2002). Property, contract, and verification: The *Numerus Clausus* problem and the divisibility of rights. *The Journal of Legal Studies* 31(S2), S373–S420.
- Hansmann, H. and M. Santilli (1997). Authors’ and artists’ moral rights: A comparative legal and economic analysis. *The Journal of Legal Studies* 26(1), 95–143.
- Hart, O. and J. Moore (1994, November). A theory of debt based on the inalienability of human capital. *The Quarterly Journal of Economics* 109(4), 841–79.
- Hart, O. and J. Moore (1995). Debt and Seniority: An Analysis of the Role of Hard Claims in Constraining Management. *American Economic Review* 85(3), 567–85.
- Hart, O. and J. Moore (1998, February). Default and renegotiation: A dynamic model of debt. *The Quarterly Journal of Economics* 113(1), 1–41.

- Hertzberg, A., A. Liberman, and D. Paravisini (2018). Screening on loan terms: Evidence from maturity choice in consumer credit. *The Review of Financial Studies* 31(9), 3532–3567.
- Ivashina, V. and B. Vallée (2018). Weak credit covenants. Working paper, Harvard Business School.
- Kahn, C. M. and D. Mookherjee (1998). Competition and incentives with nonexclusive contracts. *The RAND Journal of Economics* 29(3), 443–465.
- Kiyotaki, N. and J. Moore (1997). Credit cycles. *Journal of Political Economy* 105(2), 211–248.
- Kronman, A. T. and T. H. Jackson (1979). Secured financing and priorities among creditors. *Yale Law Journal* 88.
- Leitner, Y. (2012). Inducing agents to report hidden trades: A theory of an intermediary. *Review of Finance* 16, 1013–1042.
- Levine, C. B. and J. S. Hughes (2005). Management compensation and earnings-based covenants as signaling devices in credit markets. *Journal of Corporate Finance* 11(5), 832 – 850.
- Lian, C. and Y. Ma (2019). Anatomy of corporate borrowing constraints. Working paper, MIT.
- Matvos, G. (2013). Estimating the benefits of contractual completeness. *The Review of Financial Studies* 26(11), 2798–2844.
- Merrill, T. W. and H. E. Smith (2001). The property/contract interface. *Columbia Law Review* 101(4), 773–852.
- Nini, G., D. C. Smith, and A. Sufi (2012). Creditor control rights, corporate governance, and firm value. *The Review of Financial Studies* 25(6), 1713–1761.
- Park, C. (2002). Monitoring and structure of debt contracts. *The Journal of Finance* 55(5), 2157–2195.
- Parlour, C. A. and U. Rajan (2001). Competition in loan contracts. *American Economic Review* 91(5), 1311–1328.
- Rajan, R. and A. Winton (1995). Covenants and collateral as incentives to monitor. *Journal of Finance* 50(4), 1113–46.

- Rampini, A. A. and S. Viswanathan (2010). Collateral, risk management, and the distribution of debt capacity. *The Journal of Finance* 65(6), 2293–2322.
- Rampini, A. A. and S. Viswanathan (2013). Collateral and capital structure. *Journal of Financial Economics* 109(2), 466 – 492.
- Rauh, J. and A. Sufi (2010). Capital structure and debt structure. *The Review of Financial Studies* 23(12), 4242–4280.
- Ravid, S. A., R. Sverdlove, A. Bris, and G. Coiculescu (2015). Conflicts in bankruptcy and the sequence of debt issues. *Journal of Financial and Quantitative Analysis* 50(6), 1353–1388.
- Roberts, M. and A. Sufi (2009). Control rights and capital structure: An empirical investigation. *The Journal of Finance* 64(4), 1657–1695.
- Schwarcz, S. L. (1997). The Easy case for the priority of secured claims in bankruptcy. *Duke Law Journal*.
- Schwartz, A. (1984). The continuing puzzle of secured debt. *Vanderbilt Law Review* 37(5).
- Schwartz, A. (1989). A theory of loan priorities. *The Journal of Legal Studies* 18(2).
- Schwartz, A. (1994). The absolute priority rule and the firm’s investment policy. *Washington University Law Review* 72(3), 1213–1229.
- Schwartz, A. (1997). Priority contracts and priority in bankruptcy. *Cornell Law Review* 82, 1396–1419.
- Smith, C. W. (1993). A perspective on accounting-based debt covenant violations. *The Accounting Review* 68(2), 289–303.
- Smith, C. W. and J. B. Warner (1979). On financial contracting: An analysis of bond covenants. *Journal of Financial Economics* 7(2), 117 – 161.
- Stulz, R. and H. Johnson (1985). An analysis of secured debt. *Journal of Financial Economics* 14(4), 501–521.
- Sweeney, A. P. (1994). Debt-covenant violations and managers’ accounting responses. *Journal of Accounting and Economics* 17(3), 281 – 308.
- Tirole, J. (2006). *The Theory of Corporate Finance*. Princeton University Press.

Weiss, L. A. (1990). Bankruptcy resolution: Direct costs and violation of priority of claims.
Journal of Financial Economics 27(2), 285 – 314.