Misvaluation of Investment Options*

Evgeny Lyandres Egor Matveyev Alexei Zhdanov

April 2017

Abstract

We study whether investment options are correctly priced. We build a real options model of optimal investment in the presence of demand uncertainty. We structurally estimate the model and classify stocks into undervalued and overvalued based on the difference between observed and model-implied firm values. A long-short strategy that buys undervalued and shorts overvalued stocks generates annualized alphas between 10% and 17%. This relation is only present in subsamples of firms with high proportions of investment options. We interpret these findings as evidence of misvaluation of investment options, leading to mispricing in equity markets that is gradually corrected over time.

Keywords: Equity misvaluation, Investment options, Optimal investment, Demand uncertainty, Expected returns, Structural estimation.

*Lyandres is at Boston University, email: lyandres@bu.edu. Matveyev is at the University of Alberta, email: egor.matveyev@ualberta.ca. Zhdanov is at the Pennsylvania State University, email: auz15@psu.edu. We thank Yakov Amihud, George Aragon, Ilona Babenko, Malcolm Baker, Hank Bessembinder, Oliver Boguth, Kobi Boudoukh, Alex Boulatov, Andrea Buffa, Maria Chaderina, Tony Cookson, Andrea Gamba, Yaniv Grinstein, Zhiguo He, Mike Hertzel, Dirk Jenter, Shimon Kogan, Seokwoo Lee, Laura Lindsey, Dmitry Livdan, Dmitry Makarov, Roni Michaely, Dino Palazzo, Alex Philipov, Farzad Saidi, Enrique Schroth, David Schreindorfer, Per Strömberg, Yuri Tserlukevich, Mihail Velikov, Vikrant Vig, Jessie Wang, and seminar participants at the Pennsylvania State University, Arizona State University, University of Alberta, Boston University, George Mason University, Universidad de los Andes, 2016 IDC Summer Finance Conference, 2016 Gerzensee ESSFM meetings, 2016 International Moscow Finance Conference, and 2017 European Winter Finance Summit for helpful comments.

Scheduled: 2017 ABFER CF, 2017 FIRS, 2017 WFA.

I really feel the valuation we've gotten is more than we have any right to deserve honestly.
Elon Musk, CEO of Tesla Motors.
December 2014.

1 Introduction

A central question in financial economics is whether stock market investors value financial assets correctly. It is often argued by behavioral economists that various assets are systematically mispriced by market participants (e.g., Baker and Wurgler (2007) and Hirshleifer (2015), among many others). Our goal in this paper is to understand whether misvaluation in equity markets is driven, in part, by investors' inability to correctly price firms' investment (growth) options.

Investment options are arguably one of the most important components of firm value.[1] At the same time, they are the most difficult component to value. Optionality embedded in any investment decision makes the usual valuation techniques, such as discounted cash flow (DCF) and valuation by multiples, less appropriate. First, it is more challenging to project cash flows of a growth firm as its future cash flows depend on its future investment policy. Second, a firm's risk changes as the firm exercises its investment options, invalidating the assumption of constant discount rate embedded in a typical DCF valuation approach (e.g., Berk, Green and Naik (1999) and Carlson, Fisher and Giammarino (2004)). Since DCF and valuation by multiples are two methods that are predominantly used by equity analysts, it is possible that the market's valuation of growth options is at times incorrect. As a result, firms' equity may be mispriced. Importantly, valuation errors in investment options do not necessarily affect equity mispricing in a particular direction. Rather, firms with abundant growth options are more likely to be either overpriced or underpriced relative to those with scarce growth options.

Our analysis of whether complexity of valuing growth options contributes to equity mispricing consists of three steps. We start by building a real options model, which avoids the challenges described above, and is therefore a more appropriate tool for valuing growth firms. Our model is closely related to Pindyck (1988) and Abel and Eberly (1996). It features a firm that has a

[1] A long literature in financial economics has documented the significance of growth options. See, for example, Pindyck (1991).

continuum of expansion options, and faces uncertain demand for its goods or services. Demand uncertainty translates into the stochastic nature of the firm's profits. The firm can purchase and install additional units of capital at any time. The optimal investment policy of the firm is to exercise investment options and acquire additional capital when the realization of demand shock is sufficiently high. We solve the model for the optimal investment policy and derive a theoretical firm value.

Second, we estimate the model on a broad cross-section of publicly traded U.S. firms. Our estimation procedure minimizes valuation errors – the difference between the observed and theoretical values of the firm – at the industry level. We run the estimation procedure each month within each industry, and obtain estimated theoretical value for each firm each month. We use the estimated theoretical firm value to compute a misvaluation measure, which is the ratio of the firm's actual market value relative to its value implied by the model. Ratios higher than one indicate overvaluation relative to the model, and ratios lower than one indicate undervaluation. Importantly, we rely only on publicly available information at a particular point in time to estimate theoretical firm values and resulting misvaluation.

To the best of our knowledge, our paper represents the first effort in the literature to measure equity mispricing that comes from misvaluation of growth options at the individual firm level. To be able to clearly attribute equity mispricing to misvaluation of investment options, our model has only the elements essential for valuing investment options. While the model abstracts from some important aspects, such as a firm's financing decision, it is specific enough to be able to value a firm's investment options better than a typical DCF model. In addition, we designed our model to be solvable in closed form. This ensures computational feasibility since we are estimating the model each month for each industry.

Third, we study the empirical properties of our misvaluation measure and its relation to future returns. We start our empirical analysis by sorting all stocks every month into ten portfolios based on the misvaluation measure. We find that the most misvalued stocks – both undervalued and overvalued – are smaller, younger, less liquid, invest more in R&D, have more volatile returns, have lower analyst coverage and higher analysts' forecast dispersion, and have lower institutional ownership. These results make economic sense – it is harder to value firms that fall into these categories, and therefore these firms are more likely to be misvalued.

We find a strong relation between misvaluation and subsequent risk-adjusted returns. Mean value-weighted monthly excess return of stocks belonging to the most undervalued decile in the previous month is 1.05%, while for stocks belonging to the most overvalued decile it is 0.15% – an annualized difference of 11.4%. This difference persists but becomes economically weaker at the 3-month and 12-month horizons, which implies that mispricing is gradually corrected.

This difference in excess returns is not due to difference in loadings on known risk factors. We estimate alphas from both established asset pricing models, such as CAPM, Fama and French (1993) 3-factor model, and Carhart (1997) 4-factor model, as well as more recent models, such as Fama and French (2015) 5-factor model and Hou, Xue and Zhang (2015) 4-factor q model, with or without the momentum factor. The differences in risk-adjusted returns between the most undervalued and the most overvalued portfolios are large – they range between 0.79% and 1.31% per month – and are highly statistically significant. These differences are robust to varying the thresholds used for assigning firms to the most undervalued and most overvalued portfolios, and to various changes in the model's estimation procedure.

To confirm these non-parametric portfolio-level results, we estimate parametric firm-level Fama and MacBeth (1973) regressions. We regress excess returns on the logarithm of misvaluation measure, controlling for the usual cross-sectional determinants of stock returns. The coefficient on log misvaluation is highly statistically significant and also economically large: increasing misvaluation measure by one standard deviation is associated with approximately 30 basis points reduction in following-month excess returns, with 90 basis points reduction in returns over the next 3 months, and with 2 percentage points reduction in returns over the next year.

To explore the role of investment options in the relation between misvaluation and future returns, we decompose each firm's value into the value of growth options (GO) and the value of assets already in place (AP). We compute GO and AP values for each firm each month. We then construct the ratio of a firm's GO value to the value of its AP. Firms with higher GO/AP ratios are those whose value is derived to a larger extent from expansion options as opposed to existing assets.

We then sort all stocks into equally-sized terciles based on industry-level GO/AP ratios. Within each tercile, we sort stocks into deciles based on misvaluation measure. The idea is that if equity mispricing is driven by misvaluation of investment options – and if our model can correctly value these options – then the strategy that buys undervalued and shorts overvalued stocks is expected to perform better in subsamples of firms with more growth options.

The results of this double-sorting analysis show that mispricing is concentrated among high GO/AP firms. The differences in mean risk-adjusted returns between the most undervalued and the most overvalued portfolios range between highly statistically significant 0.75% and 1.31% per month in the highest GO/AP tercile. In the lowest GO/AP tercile, these differences range between statistically insignificant 0% and 0.69% per month.

Since estimated GO/AP ratios are a product of a structural model, they may embed model misspecification rather than the true value of firms' investment options. We therefore repeat the double-sorting analysis, replacing GO/AP ratio with market-to-book (M/B) ratio, which is a commonly accepted measure of firms' investment opportunities. A potential challenge with using M/B ratio is that it is directly related to misvaluation. To avoid this problem, we use industry-level M/B ratios because they are orthogonal to the firm-level misvaluation by construction. The results of this analysis are similar to GO/AP sorts, and are consistent with mispricing being driven by investors' difficulties in valuing growth options.

To provide additional evidence on the mispricing of growth options, we perform a counterfactual analysis. We shut down growth options in the valuation model and assume that each firm's value is derived only from its existing assets. By construction, the counterfactual model is not able to identify any mispricing of investment options. We then estimate the model, compute firms' misvaluation relative to this model, assign firms to misvaluation deciles, and repeat the asset pricing tests. The resulting pattern of risk-adjusted returns across misvaluation deciles is non-monotonic. In addition, the differences in risk-adjusted returns between the two extreme misvaluation deciles are insignificant, confirming the growth-options-based explanation of equity mispricing.

We also examine the relation between misvaluation and future returns during times of high and low investor sentiment. Since stock prices are more likely to deviate from their fundamental values when investor sentiment is high, we expect to see a stronger relation between misvaluation and future returns in times of high sentiment. Our results support this prediction. In addition, we show that the effect of investor sentiment on the relation between misvaluation and future returns is driven mostly by growth firms, consistent with the growth-option-based explanation of equity misvaluation.

Our paper contributes to several strands of literature. The first is the literature that examines implications of real options models for equity returns. For example, Carlson, Fisher and Giammarino (2004) model the role of growth option exercise in the dynamics of firm betas. Aguerrevere (2009) studies the effect of real options on equities' risk and return in the presence of product market competition. Hackbarth and Morellec (2008) model the dynamics of betas around takeover transactions in a real options framework. Babenko, Boguth and Tserlukevich (2016) examine the relation between idiosyncratic cash flow shocks and systematic risk. Sagi and Seasholes (2007) analyze the effect of growth options on the dynamics of return autocorrelations. Liu, Whited and Zhang (2009) estimate a q-theoretic model, which ties expected returns to firms' observable characteristics, and use it to study the cross-section of average returns.

A common goal of these papers is to explain the relation between moments of expected returns and growth options in a rational framework. Our paper, on the other hand, attempts to capture firm-level mispricing of growth options. Our paper's contribution, therefore, is not an analysis of the effects of firm characteristics on risk and expected returns, but an attempt to examine whether values of investment options are adequately reflected in firms' market valuations.

Our paper also contributes to the literature that structurally estimates dynamic corporate finance models.[2] The paper that is most closely related to ours is Warusawitharana and Whited (2016). Their model features an exogenous misvaluation shock that affects a firm's financing and investment policies. Warusawitharana and Whited (2016) estimate the model for an average firm and use it to study the response of managers to misvaluation shocks and their effects on shareholder value. In contrast, our approach is to capture firm-level misvaluation: in every month and for each firm our model produces a unique firm-specific misvaluation measure.

[2]See, for example, Strebulaev and Whited (2012) for an overview.

Our paper is also related to the literature on implied cost of capital (e.g., Claus and Thomas (2001), Gebhardt, Lee and Swaminathan (2001), Easton (2004), and Hou, van Dijk and Zhang (2012)). This literature typically equates a firm's market values to the present value of its estimated future cash flows to obtain "implied cost of capital," and studies the relation between implied cost of capital and future returns. Similar to these studies, we compare firms' market values to estimated model values and examine the association between misvaluation and future returns. We contribute to this literature by explicitly accounting for investment options in our valuation model and showing that misvaluation of growth options contributes to firms' mispricing.

2 Model

Our model features a firm that is characterized by a starting level of capital stock. We assume that the firm operates infinitely. The firm faces stochastic demand for its products or services. It can purchase and add additional units of capital to its existing capital stock, K_t, at any point in time t. The price of buying and installing one unit of capital is η. Note that η captures both the purchase price of capital as well as any potential proportional installation costs. Capital depreciates at a rate λ per unit of time. The firm's instantaneous operating profit is given by

$$\pi(K_t, x_t) = (1 - \tau)x_t K_t^{\theta}, \tag{1}$$

where $0 < \theta < 1$ is the curvature of the production function, τ is the corporate tax rate, x_t is the non-negative stochastic demand process, and K_t is the capital stock. Demand process x_t follows a geometric Brownian motion:

$$dx_t = \mu x_t dt + \sigma x_t dB_t, \tag{2}$$

where μ is the drift parameter, σ is the volatility parameter, and B_t is standard Brownian motion. The profit function specified in (1) is equivalent to an environment in which a firm has a Cobb-Douglas production function and faces isoelastic demand for its products.[3] We further assume that there exists a tradable asset whose value is perfectly correlated with x_t and hence the risk-neutral

[3]For details, see Appendix A and Morellec (2001).

measure $\mathbb{Q}$ exists.

The firm's optimal investment policy is to purchase and install an infinitesimally small amount of additional capital as soon as x_t reaches the optimal investment boundary, $X(K_t)$. This optimal investment boundary is increasing in K_t – a better capitalized firm optimally waits longer, until a higher realization of x_t is reached, before installing additional capital. Function $X(K_t)$ divides the (x_t, K_t) plane into two regions. If $x_t < X(K_t)$, then the firm is in the inaction region, as the marginal increase in firm value due to potential investment is lower than the cost of purchasing and installing new capital, η. If $x_t > X(K_t)$, then an immediate lumpy investment of ΔK_t, such that $x_t = X(K_t + \Delta K_t)$, is optimal. We prove in Appendix B that $X(K_t)$ has the following functional form:

$$X(K_t) = \frac{\beta_1}{\beta_1 - 1} \frac{(r - \mu + \lambda\theta)\eta K_t^{1-\theta}}{(1-\tau)\theta}, \tag{3}$$

where β_1 is the positive root of quadratic equation

$$\frac{1}{2}\sigma^2\beta(\beta - 1) + [\mu - \lambda(\theta - 1)]\,\beta - (r + \lambda) = 0. \tag{4}$$

Given the optimal investment policy, we show in Appendix B that the value of the firm is given by

$$V(K_t, x_t) = \underbrace{\frac{(1-\tau)x_t K_t^{\theta}}{r - \mu + \lambda\theta}}_{\text{Value of AP}} + \underbrace{\left[\frac{(1-\tau)\theta}{\beta_1\,(r - \mu + \lambda\theta)}\right]^{\beta_1} \left(\frac{\beta_1 - 1}{\eta}\right)^{\beta_1 - 1} \frac{x_t^{\beta_1}}{(\beta_1(1-\theta) - 1)K_t^{\beta_1(1-\theta)-1}}}_{\text{Value of GO}}. \tag{5}$$

The value of assets in place in the first term of equation (5) equals the expectation (under $\mathbb{Q}$) of all future cash flows generated by existing capital. The value of investment options in the second term of equation (5) is the sum of the present values of cash flows generated by additional capital that the firm optimally installs over time, net of the costs of acquiring capital.

Before we estimate our model, it is useful to examine its comparative statics. Figure 1 provides the values of investment options and assets in place as functions of the curvature of the production function θ, depreciation rate λ, purchase price of capital η, and the volatility of cash flows σ.[4] Panel A shows that the value of assets in place decreases in the depreciation rate λ: faster depreciating

[4]The remaining parameter values in Figure 1 are set as follows: $\lambda = 0.1$, $\sigma = 0.2$, $\theta = 0.25$, $\eta = 1$, $\lambda = 0.1$, $r = 0.05$, $\mu = 0.01$. The qualitative comparative statics are insensitive to the choice of these parameter values.

capital generates lower cash flows in the future. Furthermore, as equation (3) suggests, the effect of depreciation on the value of assets in place is amplified when the curvature of the production function, θ, is higher: it is costlier to lose capital due to depreciation when its productivity is higher. The value of growth options, on the other hand, is increasing in λ because the loss of capital due to depreciation makes it optimal to install additional units of capital at a faster rate, and a firm with a higher depreciation rate is forced to invest in new capital more aggressively. At the same time, as economic intuition suggests, depreciation negatively impacts total firm value (the sum of the values of assets in place and growth options). Therefore, overall firm value is decreasing in λ.

The curvature of the production function, θ, has a positive effect on the values of both assets in place and growth options. More productive capital is reflected in higher values of existing production assets as well as options to expand capital stock in the future. Importantly, as Figure 1 demonstrates, the value of growth options is more sensitive to θ than the value of assets in place. In particular, as θ approaches $1 - \frac{1}{\beta_1}$, the value of growth options becomes infinitely high. For the value of investment options to be finite, the production function has to be sufficiently concave: $\theta < 1 - \frac{1}{\beta_1}$. When estimating the model, we make sure that this constraint on θ is always satisfied.

Finally, volatility of demand, σ, has a positive effect on the value of expansion options due to the fundamental positive relation between the value of an option and the volatility of the underlying process. On the other hand, an increase in the purchase price of capital, η, has a negative effect on the value of expansion options. The reason is that more expensive capital forces the firm to slow down its investment and reduces the present value of its investment options. Neither σ nor η influence the value of assets in place.

Our goal is to estimate parameters of the model described above on a large panel of firms and, therefore, for computational feasibility it is necessary to have an analytical solution for the value of the firm. For this reason, the model contains only the essential elements that allow us to value firms' investment options and assets in place, and abstracts from many realistic features. First, we do not include disinvestment options in the model (e.g., Abel and Eberly (1996) and Morellec (2001)), and assume that any purchase of capital is completely irreversible. Second, we do not model financing decisions and the option to default on debt (e.g., Eisdorfer, Goyal and Zhdanov (2016)). Instead, in our empirical implementation we approximate the market value of the firm by the sum of its market value of equity and book value of debt. Third, we do not allow for a feedback from misvaluation to firms' investment decisions (e.g., Warusawitharana and Whited (2016)). Fourth, we abstract from competition in product markets and its effect on optimal exercise of investment options (e.g., Grenadier (2002) and Novy-Marx (2007)). It is possible to integrate many of these features into our model. However, since our paper is the first to estimate and measure mispricing of growth options at the individual firm level, our goal is to have a model that is simple and transparent, yet has the capacity to relate the values of growth options and assets in place to firm fundamentals.

3 Model estimation and calibration

This section explains our estimation and identification approach. Our dataset covers U.S. publicly traded firms over the period 1980-2014. Since the primary goal of our paper is to understand whether investors systematically misprice firms' investment options, we compare firms' theoretical values, which include values of their investment options, to firms' market values. We estimate the model's parameters at a monthly frequency at the industry level.

To obtain firms' market values and corresponding theoretical values, we use data from annual Compustat files and monthly CRSP files. We use Standard & Poor's Global Industry Classification Standard (GICS) to define industries. GICS is the most common classification used in the financial industry. Furthermore, Bhojraj, Lee and Oler (2003) compare GICS, NAICS, and SIC industry classifications, and find that GICS classification is significantly better at explaining stock return comovement, as well as cross-sectional variation in valuation multiples, forecasted and realized growth rates, R&D expenditures, and various key financial ratios. This is important, since our identifying assumption is that some of the model's parameters are common for all firms belonging to a particular industry. We exclude financial firms (GICS Sector 40) and regulated utilities (GICS Sector 55). Financial firms typically use productive capital that differs from the capital used in other sectors of the economy. Regulated utilities, on the other hand, may not invest optimally because their investment process is often subject to frictions that we do not model. This leaves us with 8 GICS sectors and 56 industries. In untabulated tests we also use 3-digit SIC codes and

2-digit NAICS codes to define industries and obtain qualitatively similar results.

The two firm-level variables that determine the differences in theoretical values across firms in the same industry are a firm's capital stock, K_{it}, and the value of the stochastic demand process, x_{it}. Capital stock, K_{it}, is defined as the gross value of property, plant, and equipment (Compustat item PPEGT). In untabulated tests, we define capital stock inclusive of capitalized R&D and obtain similar results. The empirical equivalent of a firm's pre-tax operating profits, $\pi(K_{it}, x_{it}) = x_{it}K_{it}^{\theta}$, is earnings before interest, taxes, depreciation, and amortization (EBITDA), which equals SALE – COGS – XSGA. For a given θ, we back out x_{it} from $\pi(K_{it}, x_{it})$ and K_{it}.

Our model does not accommodate the case when firms have negative cash flows. Therefore, we cannot estimate the model for firms that have negative or missing EBITDA in a given month. Table A2 in the Appendix reports the average number and percentage of firms in each industry for which we are able to estimate the model. There is a large variation of the proportion of firms with positive EBITDA across industries. For example, only 18% of observations in the Biotechnology industry have positive EBITDA. For most industries, however, more than 75% of firm-months have positive EBITDA. The overall proportion of firms for which we can estimate the model is 78%.

In addition to the observable firm-level variables discussed above, our model requires industry-level inputs. It is plausible that there are fundamental economic drivers that determine industry growth rates, which, in turn, drive the growth rates of individual firms. If investors are overly optimistic or pessimistic about individual firms, we are more likely to identify such firms by benchmarking them against comparable firms in the same industry. We, therefore, define the drift and volatility of the demand process, which captures underlying economic drivers in our setup, at the industry level. In our model, a firm's capital depreciates at the rate of λ. In practice, there is a substantial time-series variation in firm-level depreciation rates. To smooth out idiosyncratic fluctuations in depreciation, we use the average industry depreciation rate in the last three years as a proxy for future depreciation rate, λ. Finally, the assumption that the curvature of the production function and the cost of installing new capital are constant across all firms in an industry is required for model identification.

Each industry is characterized by 5 industry-level parameters: the drift of the stochastic process,

μ, its volatility, σ, capital depreciation rate, λ, the curvature of the production function, θ, and the cost of installing new capital, η. Our approach combines calibration and estimation. We calibrate the parameters of the demand process, i.e., μ, σ, and capital depreciation rate, λ, and estimate the curvature of the production function, θ, and the cost of installing new capital, η, directly from the data. The next two subsections discuss our calibration and estimation approaches in detail.

3.1 Calibration

To estimate the drift, μ, of the demand process, we use equity analysts' forecasts of firms' cash flow growth from the Institutional Brokers Estimate System (IBES). The demand process in our model features time-invariant parameters, and the model is set up with an infinite horizon. Therefore, we would have ideally liked to have access to forecasts of terminal growth rates. Unfortunately, equity analysts do not systematically report terminal growth rates, and instead issue so-called long-term growth (LTG) forecasts. These are typically forecasts of growth rates of firms' cash flows over a five-year horizon. We base our proxy for terminal growth rates on LTG rates, and use them to estimate the drift of the demand process at the monthly frequency. We aggregate forecasts issued by all analysts to all firms within an industry. To avoid look-ahead bias, we use all forecasts available within three months prior to the month in which we estimate the model.

Since our model is set under the risk-neutral measure ($\mathbb{Q}$), we convert the estimated growth rates from physical ($\mathbb{P}$) to risk-neutral measure. In doing so, we follow Morellec, Nikolov and Schurhoff (2012), who show that the growth rate under $\mathbb{Q}$, μ_{jt}, equals $g_{jt} - \beta_{jt}\text{MRP}$, where g_{jt} is the growth rate under $\mathbb{P}$ and $\beta_{jt}\text{MRP}$ is the risk premium.[5] Similar to Morellec, Nikolov, and Shuerhoff (2012), we assume that the market risk premium is time-invariant and equals 6% per year.[6] To estimate equity beta for each firm, β_{it}, we run a rolling 36-month regression of the firm's excess returns on market excess returns. To measure expected returns on debt we sort stocks into five distress quintiles based on the "naive" distance-to-default measure of Bharath and Shumway (2008). We then use the the risk-free rate plus the credit spread on AAA bonds, credit spread on

[5] A sufficient condition for this adjustment is that CAPM holds, at least for the tradable asset whose value is perfectly correlated with the stochastic process x_t.

[6] Our results are robust to assuming time-varying risk premium, computed as the difference between realized return on the value-weighted CRSP index and the rate of return on the riskless asset, where the latter is the average of the yields on the short-term Treasury Bill and 10-year Treasury Note.

BAA bonds, and credit spread on BAA bonds plus 2% for the firms in the least distressed, the next two, and the two most distressed quintiles, respectively. We infer debt beta using the CAPM, compute the firm's weighted average beta as the average of debt and equity betas, and compute average β_{jt}MRP across all firms in industry j. The details of the estimation of the drift of the demand process are outlined in Appendix C.

To estimate the volatility of the demand process we use the volatility of firms' quarterly sales. We estimate quarterly sales volatility for each firm using sales data in the last eight quarters. For some industries, sales data are highly seasonal. To take out the seasonal variation in sales, we regress quarterly sales on seasonal dummies for each industry and use residuals as our quarterly time-series of sales. Since our demand process is defined at the industry level, we use sales volatility of the median firm in the industry to approximate the volatility of the demand process.

To estimate annual depreciation rates, we use the ratio of depreciation charges (DP) to the value of property, plant, and equipment (PPEGT) for each firm in an industry over the last three years, and use the median across all industry firms. Finally, when measuring after-tax operating profits, we set corporate annual tax rate τ at 35% for every firm. The results are robust to using marginal tax rates from Graham (1996).

Table 1 summarizes firm-level characteristics that we use in calibration and estimation, as well as industry-level demand growth rates, volatility of demand, and capital depreciation rates. Observations with missing values for the GICS code, the gross capital stock, market value of equity, or negative operating profits are excluded from the final sample. Table A3 reports summary statistics of these characteristics for each industry. Figure 2 shows the timing of data inputs that we use in calibration and estimation of the model.

3.2 Estimation

3.2.1 Estimation procedure

We estimate the model using observed firm market values. Equation (5) gives us the theoretical value of a firm. Once the parameters discussed in the previous section have been calibrated, the firm's theoretical value is the function of the curvature of the production function and capital costs.

For each firm and each month we denote the theoretical firm value as $V_{it}(\theta, \eta)$.

The empirical counterpart of firm i's estimated theoretical value, $\widetilde{V}_{it}$, is the sum of the market values of equity and debt. Market value of equity is defined as end-of-the-month price per share times the number of shares outstanding, PRC×SHROUT. We approximate the market value of debt by its book value, defined as the sum of debt in current liabilities (DLC) and long-term debt (DLTT).

The ratio of the actual firm value, $\widetilde{V}_{it}$, the model-implied value, $V_{it}(\theta, \eta)$, should equal one under the true parameter values, θ and η, if our valuation model is correct and there is no mispricing in the market. Therefore, for each firm and each month we compute the firm's valuation error as the ratio of the observed firm value to model firm value:

$$\epsilon_{it} = \widetilde{V}_{it}/V_{it}(\theta, \eta). \tag{6}$$

We interpret the deviations of $\epsilon_{i,t}$ from one as the market's mispricing of the firm. Values above (below) one imply that the firm's market value is greater (lower) than its model-implied value, and, therefore, we refer to such firms as overvalued (undervalued). It is important to note that valuation errors may capture either true mispricing or model misspecification. Our setup does not allow us to distinguish between the two empirically. In the next section we investigate whether misvaluation gets corrected over time, and therefore the possibility of valuation errors capturing model misspecification works against us finding results.

We estimate the curvature of the production function, θ, and the cost of installing new capital, η, by minimizing aggregate valuation error within industries. For each month and each industry, we pull the industry data for the past 12 months and estimate the model on these data. By doing this, we assume that the production technology is constant within a one-year period.

We define an objective function that is symmetric and does not overweigh either undervalued or overvalued firms. In particular, for each industry j and each month t we minimize:

$$(\hat{\theta}_{jt}, \hat{\eta}_{jt}) = \arg\min \sum_{\tau=t-12}^{t-1} \sum_{\upsilon=1}^{N_{jt}} |\log \epsilon_{\upsilon\tau}|, \tag{7}$$

where N_{jt} is the number of firms in industry j in month t. The minimization is done subject to the upper bound constraint on the curvature of the production function : $\theta < 1 - 1/\beta_1$. Note that the constraint depends on the value of β_1, which, in turn, depends on the value of θ according to equation (4).

The theoretical firm value in equation (5) assumes that firms invest optimally at all times, i.e. that a firm cannot end up outside of the investment boundary. However, in reality it is conceivable that firms may deviate from theoretically optimal investment policy for various reasons, i.e. we can observe a firm outside of the investment boundary, $x_{it} > X(K_{it})$. In situations like this, we assume that a firm makes a lumpy investment to bring itself back to the investment boundary. In other words, if $x_{it} > X(K_{it})$ then we assume that the firm immediately invests $K_{it}^* - K_{it}$, where

$$K_{it}^* = \left(\frac{\beta_1 - 1}{\beta_1} \frac{(1-\tau)\theta x_t}{(r-\mu)\eta} \right)^{\frac{1}{1-\theta}},$$

in which case the firm's theoretical value net of additional investment cost becomes

$$V(K_{it}^*, x_{it}) - (K_{it}^* - K_{it})\eta_{jt}. \tag{8}$$

Notably, in our sample, only about 6% of all firm-months are located outside of the optimal investment boundary and require this adjustment.

At the conclusion of the estimation procedure, we obtain estimates of the curvature of the production function, $\hat{\theta}_{jt}$, and the cost of acquiring capital, $\hat{\eta}_{jt}$, for each industry j and each month t. We define a firm-level misvaluation measure that we use in the subsequent sections as a valuation error at the estimated value of the parameters, i.e., $\widetilde{V}_{it}/V_{it}(\hat{\theta}_{jt}, \hat{\eta}_{jt})$.[7]

3.2.2 Parameter identification

In this subsection we discuss separate identification of the curvature of the production function, θ, and the cost of purchasing and installing new capital, η. Recall that in equation (5) the firm value

[7]Alternatively, it is possible to estimate the curvature of the production function and the costs of installing new capital using firms' revenues and investment, while assuming a specific (e.g., Cobb-Douglas) production technology (e.g., Olley and Pakes (1996) and Levinsohn and Petrin (2003)). However, such procedure would not allow us to identify firm-level shocks, x_{it}, which drive the values of both investment options and assets in place.

consists of two components. The first component is the value of assets in place, AP_{it}:

$$AP_{it} = \frac{(1-\tau)x_{it}K_{it}^{\theta}}{r-\mu+\lambda\theta}, \tag{9}$$

which is increasing in θ, as pictured in Figure 1, and is not affected by η. Therefore, the curvature of the production function, θ, is primarily identified by this component of the firm value. The second part of value equation (5) captures the second component of firm value – the value of its growth options, GO_{it}:

$$GO_{it} = \left[\frac{(1-\tau)\theta}{\beta_1\left(r-\mu+\lambda\theta\right)}\right]^{\beta_1}\left(\frac{\beta_1-1}{\eta}\right)^{\beta_1-1}\frac{x_{it}^{\beta_1}}{(\beta_1(1-\theta)-1)K_{it}^{\beta_1(1-\theta)-1}}, \tag{10}$$

which depends on both θ and η. The value of GO_{it} is increasing in θ and is decreasing in η, as shown in Figure 1. The value of growth options identifies the cost of installing new capital, η. If the value of θ is sufficiently high, this constraint helps identify θ.

3.2.3 Estimation with industry-level valuation errors

Our main estimation approach in (7) assumes that at any point in time, every industry is correctly priced on average. In this section, we relax this assumption and allow for time-varying industry-level misvaluation. We use a modified approach of Rhodes-Kropf, Robinson and Viswanathan (2005) to estimate industry misvaluation. Each year for each industry, we estimate the following regression:

$$\log(\mathrm{ME}_{it}) = \alpha_0 + \alpha_1\log(\mathrm{BE}_{it}) + \alpha_2\log(|\mathrm{NI}_{it}|) + \alpha_3\mathbb{I}(\mathrm{NI}_{it}<0) + \alpha_4\mathrm{Leverage}_{it} + \epsilon_{it}, \tag{11}$$

where the dependent variable is the market value of a firm's equity (ME), and the independent variables are the book value of a firm's equity (BE), the absolute value of net income (NI), the dummy variable for negative net income, and leverage. The predicted value from this regression captures the historical-multiple-based value of the firm obtained by applying annual, industry-average regression multiples to firm-level accounting variables.

Next, we estimate (11) for each industry for each of the years $t-5$ to $t-1$ relative to the year of the observation, and compute five-year averages of coefficient estimates of these annual industry-level regressions. We then multiply these average coefficients by the values of firm i's relevant year-t accounting variables on the right-hand-side of (11) to obtain firm i's year-t historical-multiple-based value.[8]

For each firm each year, we add the book value of debt to both the historical-multiple-based and market values of equity and compute the firm's valuation error as the percentage difference between the resulting firm's pseudo-market value and its multiple-based value. To obtain a measure of industry misvaluation in a given year, we take the misvaluation of a median firm in the industry in that year. To account for industry misvaluation in our estimation procedure, we minimize the following objective function:

$$(\hat{\theta}_{jt}, \hat{\eta}_{jt}) = \arg\min \sum_{\tau=t-12}^{t-1} \left(\sum_{\upsilon=1}^{N_{jt}} |\log \epsilon_{\upsilon\tau}| - \text{median}(\log \epsilon_{j\tau}) \right), \tag{12}$$

where $\text{median}(\log \epsilon_{j\tau})$ denotes the misvaluation of the median firm in industry j in month τ. We use this estimation procedure as a robustness analysis to our main estimation approach in (7).

3.3 Estimation results

3.3.1 Estimates of θ and η

We report estimation results in Table 2. We aggregate the estimates of θ and η at the sector level and report their time-series means in Panel A and at the industry level in Panel B. In addition, the two panels report the distributions of GO/AP ratios across firms in each sector and each industry.

Estimated curvature of production function, θ, ranges between 0.26 and 0.46. On average, capital has the lowest average productivity in the Electrical Equipment industry (which belongs to the Industrials sector) and Household Durables and Textiles, Apparel & Luxury Goods industries (which belong to the Consumer Discretionary sector). Capital is most productive in the Information Technology sector, in particular in the Office Electronics industry. These estimates are in line with the vast literature in macroeconomics and finance that directly or indirectly estimates parameters of the production process. For example, Olley and Pakes (1996) estimate the productivity of capital

[8] The original Rhodes-Kropf, Robinson and Viswanathan (2005) uses the entire time-series to estimate firms' multiple-based values. To avoid look-ahead bias, we do not use any observations that are past the current estimation year.

in the Telecommunications Equipment industry at around 0.34 and Levinsohn and Petrin (2003) obtain estimates in the range of 0.2 to 0.29.[9]

Estimated cost of installing new capital, η, ranges between 1.00 and 1.36. In our model, η captures how much a firm has to pay to purchase \$1 worth of capital. In other words, η captures the wedge between the cost of buying capital and its value on a firm's balance sheet. In addition to the wedge between the purchase price and the book value of capital, this parameter captures capital specificity, installation costs, and other overhead expenses associated with installing new capital in a given industry. The lower bound that we impose on η is 1. This effectively assumes that firms in a given industry on average do not pay for capital less than its book value.

Capital is less firm-specific and is cheaper to install in the Information Technology sector. In particular, Software and Communications Equipment industries have the lowest costs of installing capital. On the other hand, firms in the Materials and Industrials sectors have the highest cost of installing new capital. The two industries with the highest cost of capital are Paper & Forest Products ($\eta = 1.36$) and Transportation Infrastructure ($\eta = 1.34$).

To the best of our knowledge, our paper is the first to estimate the cost of installing new capital for a universe of all publicly traded firms in the U.S. Cooper and Haltiwanger (2006) estimate this cost on a panel of large manufacturing firms. Despite differences in samples used in the two papers, our results are remarkably close to the estimates in Cooper and Haltiwanger (2006) – 1.32 in the Steel industry and 1.25 in the Transportation industry.[10]

Since our estimation is at the monthly level, we can measure the pace at which the production technology and the cost of installing new capital evolve. In particular, we compute absolute changes in the curvature of the production function and the cost of installing capital at 1-month, 3-month, 6-month, 1-year, 2-year, and 3-year horizons. We do this computation for each industry, and then take means across all industries and months. We find that at short horizons the changes are small. For example, at the 3-month horizon, the absolute changes in θ and η are 0.03 and 0.05 respectively, corresponding to 14% and 6% of their respective means. At longer horizons, for example 3 years,

[9] See Ackerberg, Caves and Frazer (2015) for a review and criticism of various approaches to the estimation of the production function.

[10] Note that in Cooper and Haltiwanger (2006) firms can sell capital, and therefore their model features both the buying and selling price of capital. When estimating the model, Cooper and Haltiwanger (2006) fix the buying price of capital at \$1 and estimate the selling price of capital.

the production technology parameter changes by about 28% of its mean value and the cost of installing capital parameter by 14% of its mean value. These results are shown in Table A4 of the Appendix.

3.3.2 Estimates of relative values of growth options

In this subsection we examine the values of firms' growth options (GO) relative to assets in place (AP), which are computed using the model's estimated parameters. The last six columns in Panels A and B of Table 2 show the distribution of estimated GO/AP ratios across sectors (Panel A) and industries (Panel B). There are large differences in the value of growth options relative to the value of existing assets across sectors and industries. On average, firms in Health Care, Information Technology, and Energy sectors tend to have large GO/AP ratios. For example, the average pharmaceutical firm in our sample derives about 40% of its value from investment options and 60% from existing assets, while the proportion of growth options in the value of the average firm in the Information Technology sector is about one third. On the other hand, firms in Materials, Industrials, Consumer Discretionary, and Consumer Staples sectors have the lowest fractions of growth options in their values – less than 10% on average. Note that our inability to estimate model values for unprofitable firms is likely to bias downward our mean estimate of GO/AP, as unprofitable firms are likely to have a smaller proportion of their value represented by existing assets and a larger proportion represented by growth options.

In addition to significant differences in GO/AP ratios across sectors, the variation in GO/AP across firms within industries and sectors is also very significant. For example, in the Software industry (Panel B, GICS 451030), the mean GO/AP ratio is 69.7% while a firm at the 25th percentile has a GO/AP ratio of only 7% and a firm at the 75th percentile has a GO/AP ratio of 76%. Economically, this means that there are some fundamental factors that make some firms more growth-oriented than others, or firms are similar but are at different points in their life cycles.

Our model is specifically designed to decompose firm values into growth options and assets in place components. Therefore, as an indirect validation of the model, we examine the relation between growth option values produced by the model on one hand, and established empirical proxies for growth options on the other. For this purpose, every month we assign firms into ten GO/AP decile portfolios, based on the ratio of growth options and assets in place, as implied by the model, with firms with the least (most) growth options assigned to decile 1 (10). We report in Table 3 the mean characteristics of firms in these decile portfolios (see Table A1 in the Appendix for variable definitions.) Table 3 shows that firms in industries classified by the model as growth-option-intensive have higher market-to-book ratios (2.45 in decile 10 vs 1.49 in decile 1), are typically younger (the average firm in decile 10 firm is half the age of the average firm in decile 1), and invest much more heavily in R&D (R&D expenditures of firms in the top and bottom deciles differ by a factor of 8). They also experience faster asset growth (44% in decile 10 versus 21% in decile 1). Firms with large estimated GO/AP ratios have lower leverage, consistent with the large literature on the negative relation between growth options and leverage.[11] Furthermore, firms with higher GO/AP ratios tend to have higher equity betas (1.38 for firms in the most growth-option-intensive decile versus 1.03 in the least growth-option-intensive decile), are more likely to be listed on NASDAQ (in decile 10, 70% of firms are NASDAQ-listed, versus 39% in decile 1) and have more volatile returns.

Overall, the evidence in Table 3 demonstrates that firms with higher estimated GO/AP ratios exhibit characteristics commonly associated with growth-oriented firms. This suggests that our model is helpful in identifying and valuing investment options.

4 Misvaluation and expected stock returns

4.1 Hypotheses

In this section we empirically test the hypothesis that firms that are overvalued (undervalued) relative to their model values earn lower (higher) subsequent risk-adjusted returns. This hypothesis relies on two assumptions. First, we assume that firm-level misvaluation produced by our model is not random and is positively correlated with unobserved true misvaluation. Second, we assume that this misvaluation is gradually corrected by the market as new, tangible information regarding firm performance arrives. We also hypothesize that the relation between misvaluation and future returns should be stronger for firms whose values are derived to a larger

[11] See, for example, Bradley, Jarrell and Kim (1984), Barclay, Smith and Watts (1995), and Barclay, Morellec and Smith (2006).

degree from investment options. Since investment options are difficult to value, these firms are more likely to be misvalued. To summarize, our two main hypotheses are:

Hypothesis 1 *We expect to find a negative relation between firm misvaluation (relative to the model) and future risk-adjusted equity returns;*

Hypothesis 2 *We expect to find that the negative relation between misvaluation and future risk-adjusted equity returns is stronger for firms with larger proportions of value represented by growth options.*

If the relation between misvaluation and subsequent equity returns is indeed driven by investors' inability to price investment options correctly, then a similar investment strategy based on a version of the model that does not account for growth options should produce weaker returns. Our modelling framework allows us to "shut down" the growth option component in the model. Therefore, we are able to test the following hypothesis:

Hypothesis 3 *The negative relation between firm misvaluation and future risk-adjusted equity returns should be weaker when firms are valued according to a model without investment options.*

4.2 Misvaluation and firm characteristics

We start our empirical analysis by sorting stocks each month by our misvaluation measure, estimated according to equation (7), and assigning stocks into misvaluation deciles, with decile 1 corresponding to most undervalued firms (i.e., those with the lowest market-to-model ratios) and decile 10 corresponding to the most overvalued firms. Before proceeding to the formal tests of Hypotheses 1-3, we report characteristics of firms in decile portfolios sorted on misvaluation in Table 4.

Table 4 shows that the most misvalued stocks (i.e., both undervalued and overvalued) are typically smaller, younger, belong to more growth-oriented industries, invest more in R&D, are less liquid, have lower analyst coverage and higher analysts' forecast dispersion, and have lower institutional ownership than more fairly-valued stocks (i.e., those in middle misvaluation deciles). These results provide an indication that the market has larger difficulties valuing growth-option-rich firms.

Table 4 also demonstrates that more overvalued firms invest more actively than undervalued firms (the investment-to-asset ratios in deciles 10 and 1 are 11.4% and 5.5%, respectively) and are less profitable than undervalued firms. Overvalued firms tend to be past winners and undervalued firms tend to be past losers: the wedge in past 6-month average returns between the top and bottom misvaluation deciles is 23%. This evidence suggests that it is necessary to control for potential exposure to profitability, investment, and momentum factors when examining the relation between misvaluation and future returns. Finally, overvalued firms tend to issue equity more actively, consistent with the hypothesis that their managers time the market in issuing equity to capture the benefits of potential overvaluation.

4.3 Evolution of misvaluation

We posit that the differences between firms' observed market values and estimated model values are attributable to misvaluation. We expect any misvaluation to be corrected over time, since market values eventually converge to true fundamental values as new information arrives and growth options are gradually transformed into assets in place. It is therefore important to examine the dynamics of movement of firms across misvaluation deciles over time. It is reasonable to expect that both highly undervalued and highly overvalued firms would move towards less extreme misvaluation deciles, as fundamental information is gradually incorporated into market prices. The evolution of firms across misvaluation deciles is presented in Table 5. This table reports the average evolution of firms' decile assignments for each of the misvaluation deciles for different time horizons, ranging from 1 month to 3 years.

Panel A of Table 5 demonstrates that there is indeed a tendency of firms in both overvalued and undervalued deciles to drift towards less extreme misvaluation deciles. A large portion of this convergence occurs within one year. For example, firms that belong to the most undervalued decile at a given point in time move to deciles 3-4 on average within one year, while firms in the most overvalued decile move to deciles 7-8 within one year. However, there is still some residual misvaluation that gets further corrected in the following two years. The results in Table 5 further suggest that the correction of misvaluation is symmetric, and both undervalued and overvalued firms tend to drift towards less extreme misvaluation deciles with similar speeds.

Panel B of Table 5 reports the transition probabilities of moving from one misvaluation decile to another. To save space, we only report such probabilities for firms in the two extreme misvaluation deciles. As in Panel A, we report transition probabilities at different horizons, from 1 month to 3 years. Consistent with gradual correction of misvaluation, firms either stay in their original misvaluation decile or move mostly to adjacent deciles at shorter horizons. For example, for most undervalued firms, the probability of moving to the third valuation decile over a one (three, six) month horizon is only 2% (4%, 7%). However, transition probabilities increase substantially for longer horizons. For example, there is a 7% probability of moving from one extreme decile to the opposite decile within three years.

4.4 Misvaluation and future returns: Portfolio sorts

4.4.1 Main tests

We now proceed to the tests of our first hypothesis. For this purpose, for each of the ten misvaluation portfolios, we estimate the regression of value-weighted mean monthly excess return in the month following the assignment to misvaluation deciles, R_{pt}, on monthly returns of factors, defined by various asset pricing models:

$$R_{pt} = \alpha_p + \beta_{\mathbf{p}} \mathbf{R_{Ft}} + \epsilon_{pt}, \tag{13}$$

where α_p is the mean value-weighted risk-adjusted return of portfolio p, $\mathbf{R_{Ft}}$ is a vector of factor returns in month t, and $\beta_{\mathbf{p}}$ is a vector of factor loadings. We do not report the results of estimating (13) using equally-weighted portfolio returns. These results are generally stronger than those using value-weighted portfolio returns. We show one example in the robustness section below.

We use eight benchmarks to estimate risk-adjusted returns:

1) The "naive" benchmark in which the set of factors $\mathbf{R_{F,t}}$ is empty. In this specification, α_p is the mean portfolio return;

2) Capital Asset Pricing Model, in which $\mathbf{R_{Ft}}$ includes MKT, defined as the difference between value-weighted market return and the risk-free rate;

3) Fama and French (1993) model, which includes HML and SMB factors in addition to MKT;

4) Carhart (1997) model, which includes, in addition to Fama and French (1993) three factors, a momentum factor, MOM;

5) Fama and French (2015) model, in which the set $\mathbf{R_{F,t}}$ includes, in addition to MKT, HML, and SMB the following two factors: RMW ("robust minus weak") – the average return on the two robust operating profitability portfolios minus the average return on the two weak operating profitability portfolios, and CMA ("conservative minus aggressive") – the average return on the two conservative investment portfolios minus the average return on the two aggressive investment portfolios;

6) Fama and French (2015) model augmented by the momentum factor, MOM;

7) Hou, Xue and Zhang (2015) model, which includes MKT, and three "q-factors", r_{ME}, $r_{I/A}$, and r_{ROE}, from a triple 2-by-3-by-3 sort by size, investment-to-assets ratio, and return on equity (ROE);

8) Hou, Xue and Zhang (2015) model augmented by the MOM factor.[12]

We report mean risk-adjusted returns in Panel A of Table 6, which includes the 8 specifications discussed above. In all 8 models, the standard errors are Newey-West adjusted with 6 lags. The mean value-weighted return of the portfolio of most undervalued stocks, 1.05% per month, is significantly larger than the mean return of the most overvalued portfolio, 0.15% per month. The annualized difference of 10.8% is significant at the 10 percent level. In general, mean portfolio returns tend to be monotonically decreasing in our misvaluation measure.

Controlling for the exposure to risk factors tends to strengthen the relation between the misvaluation measure and risk-adjusted returns. The differences in alphas from the CAPM, Fama and French (1993), and Carhart (1997) models between the most undervalued and most overvalued decile portfolios are highly statistically significant, with t-statistics of 3.70, 2.79, and 3.40, respectively. They are also highly economically significant: annualized differences between

[12] The majority of factor returns are obtained from Ken French's data library `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`. Monthly returns of the q-factors were provided to us by Lu Zhang.

the alpha of the most undervalued and that of the most overvalued portfolio are between 9.9% and 13.6%. Annualized difference between the alphas of the two extreme portfolios is 10.3% in the case of the Fama and French (2015) model with a t-statistic of 2.89. Augmenting the Fama and French (2015) model by the momentum factor increases the gap between the alphas of the two extreme misvaluation deciles to a highly significant annualized 12.4%. This is consistent with undervalued firms being past losers and overvalued firms being past winners, on average. Even stronger results are obtained when we use the Hou, Xue and Zhang (2015) model, with or without the momentum factor as a benchmark, – the annualized difference between the two extreme misvaluation portfolios' alphas exceeds 16% with a t-statistic exceeding 4.

Panel B of Table 6 reports factor loadings for the most undervalued and overvalued deciles, and for the undervalued-minus-overvalued (UMO) strategy for three of the models in Panel A, which nest other models: Fama and French (1993), Fama and French (2015), and Hou, Xue and Zhang (2015), all augmented by the momentum factor. There are a number of interesting observations from Panel B. First, the loadings of the UMO strategy on the MKT factor are negative and highly significant. In the Fama and French (1993) model, the most undervalued decile loads 1.07 on MKT while the most overvalued decile has a loading of 1.24. Therefore, the UMO strategy is short in stocks with higher exposure to the market risk, which explains improved performance of our UMO strategy after we control for MKT exposure. The differences in the loadings on MKT between the two extreme portfolios are even larger in the other asset pricing models.

Second, UMO is negatively exposed to SMB. The loading of UMO on SMB is -0.33 in the Fama and French (1993) model, with a t-statistic of -3.43. This implies that stocks in the undervalued portfolio tend to be larger than those in the overvalued portfolio, consistent with the evidence in Table 4. Third, UMO is a value strategy. The loadings on the HML factor range from 0.37 to 0.51, and are highly significant. Fourth, the UMO strategy is counter-momentum. UMO loadings on MOM vary from -0.19 to -0.25 and are highly significant in all models. Stocks in the undervalued portfolio tend to be past losers and stocks in the overvalued portfolio tend to be past winners. This explains better performance of the UMO strategy once we control for the exposure to the momentum factor. Fifth, the UMO strategy is not significantly exposed to the profitability factors

RMW or r_{ROE}. Undervalued stocks tend to have slightly higher gross profitability compared to overvalued stocks, but not significantly so. Finally, undervalued stocks have a larger negative exposure to the asset growth factor than overvalued ones, consistent with the evidence on slower growth of the former, reported in Table 3.

4.4.2 Robustness tests

Table 7 presents results of robustness tests of our first hypothesis, in which we change the definition of extreme misvaluation groups, the method of computing portfolio returns, as well as the horizon of returns following assignment to misvaluation groups; impose restrictions on the sample composition; and change the way in which we estimate the model's parameters and resulting misvaluation. Table 7 has 8 columns, similar to Table 6. To save space, in each robustness test, we only report risk-adjusted returns of the two extreme misvaluation portfolios and the differences between them.

In Panel A, instead of sorting stocks into deciles based on our misvaluation measure, we sort them into quintiles. The results are consistent with the baseline estimation. For example, when we use the Fama and French (2015) model, the annualized risk-adjusted return on undervalued-minus-overvalued (UMO) strategy becomes lower, 9.7%, but the t-statistic increases to 3.56. This result alleviates a concern that the difference in risk-adjusted returns between the most undervalued and most overvalued stocks is driven by a small number of stocks in undervalued and overvalued deciles.

In Panel B, we sort stocks into two portfolios: the undervalued portfolio contains firms with misvaluation measure lower than one, while the overvalued portfolio contains firms with misvaluation measure greater than one. The results become weaker compared to Table 6 and to Panel A of this table, but not significantly so. This is expected because instead of sorting firms into 10 or 5 portfolios, we sort them into 2 portfolios only. Nevertheless, all of the differences between risk-adjusted returns of undervalued and overvalued portfolios are still economically large and statistically significant. The lowest annualized alpha is from the Fama and French (2015) model and equals 6.4%, with a t-statistic of 1.91.

Panel C reports results for equally-weighted portfolio returns. The results are more economically and statistically significant than the baseline results using value-weighted returns. The monthly mean return in the undervalued portfolio is 1.45% while in the overvalued portfolio it is 0.19%, an annualized difference of 15%, with a t-statistic of 2.66. Annualized risk-adjusted returns of the UMO strategy from the seven asset pricing models vary from 11.8% (Fama and French (2015) model) to 16.4% (Fama and French (1993) model augmented by the momentum factor). The lowest t-statistic is 4.42.

In Panel D, instead of estimating both parameters θ and η simultaneously, we structurally estimate only one parameter of the model. We fix the cost of installing new capital, η, at one, and estimate only the curvature of the production function, θ. The reason is the potential concern that θ and η are not separately identified. The difference between risk-adjusted returns of undervalued and overvalued portfolios declines, but only marginally. For example, annualized alpha of the UMO strategy within Fama and French (2015) model decreases from 9.8% to 9.5% with an associated decrease in the t-statistic from 2.89 to 2.81.

In Panel E, we estimate the model parameters on a subset of all firms that have relatively high analyst coverage. In particular, for each industry-month, we estimate the model using firms with above-industry-median analyst coverage. There are two motivations for performing this test. First, firms that are followed by more analysts are more likely to be correctly priced. Second, since one of our key model inputs – the drift of industry profit – is estimated using analyst projections, it is likely to be estimated more precisely within firms with relatively strong analyst coverage. Risk-adjusted returns of the UMO strategy decrease for all asset pricing models relative to the baseline results, but they remain economically large and statistically significant.

In Panel F, we estimate firm-level misvaluation while accounting for estimated industry-level misvaluation, as described in Section 3.2.3. The differences between risk-adjusted returns of the most undervalued and the most overvalued firms are typically 20%–30% lower than in the baseline specification without industry misvaluation. However, they are still economically large – ranging from 7% to 13% per year – and are highly statistically significant.

In Panels G and H, we replace the one-month return on the left-hand side of (13) by 3-month and 12-month returns following the month of assignment into misvaluation deciles, respectively. The differences in the 3-month risk-adjusted returns between the two extreme misvaluation deciles are statistically significant at the 10% level in all 8 models. Annualized alphas of the UMO strategy range from 3.9% to 8.4%. The annualized risk-adjusted returns of the UMO strategy are similar in the case of 12-month returns, suggesting that while the performance of the UMO strategy is strongest at the one-month horizon, there is little decay in the performance of the strategy between months 4 and 12. We interpret this as evidence that investors learn slowly about firm mispricing and correct it over time, consistent with the gradual movement of firms across misvaluation deciles documented in Table 5.

In addition to the robustness tests reported in Table 7, the differences in risk-adjusted returns between undervalued and overvalued deciles remain significant after making the following changes to the estimation procedure:

1) Defining industries based on 3-digit SIC and 2-digit NAICS classifications and estimating firm-level misvaluation relative to these industries. While GICS classification may be more appropriate for our estimation, as argued above, SIC and NAICS classifications are standard in corporate finance.

2) Using marginal corporate tax rates from Graham (1996) instead of assuming a fixed tax rate of 35%. Using marginal tax rates has its benefits and drawbacks. On one hand, assuming a constant tax rate for all firms introduces noise into our estimation. On the other hand, the difference between the marginal and average tax rates may be correlated with firm profitability, which depends on the demand shock. In this case, using marginal tax rates will introduce bias into the estimation.

3) Including capitalized R&D, estimated following Hirshleifer, Hsu and Li (2013), in the measure of capital stock. While many of the most misvalued firms belong to R&D-intensive industries, there are drawbacks of directly including capitalized R&D in the measure of installed capital. First, unlike CAPEX, capitalized R&D needs to be estimated, which may introduce measurement error into model parameter estimation. Second, many firms include R&D in their SG&A expenses. If the level of R&D expenses is correlated with firms' choices of the way R&D is reported, inclusion of capitalized R&D in the measure of capital may bias the estimation of model parameters.

4.5 Cross-sectional firm-level tests

4.5.1 Main tests

In this section we estimate monthly cross-sectional Fama and MacBeth (1973) regressions of individual firm excess returns R_{it} on a measure of misvaluation, and a vector of firm characteristics known at the beginning of month t:

$$R_{it} = \alpha_t + \beta_t MISVAL_{it} + \delta_{\mathbf{t}} \mathbf{X_{it}} + \epsilon_{it}, \tag{14}$$

where $MISVAL_{it}$ is firm i's measure of misvaluation in month $t-1$. Because of skewness in our misvaluation measure, we use the natural logarithm of firm-level misvaluation, estimated as in equation (7). $\mathbf{X_{it}}$ is a vector of firm characteristics that were identified in past literature to be related to future returns. These characteristics are log market equity (log(ME)), log equity book-to-market (log(B/M)), investment-to-assets ratio, profitability, and past returns at one-month and one-year horizons (e.g., Novy-Marx (2013) and Ball et al. (2016)). See Table A1 in the Appendix for definitions of these variables.

We report average coefficient estimates of the regression in (14) in Table 8, which has 3 columns. In the first column, $\mathbf{X_{it}}$ includes all aforementioned explanatory variables except for log misvaluation. Consistent with past studies, returns are positively related to log(B/M), profitability, and one-month past return, and are negatively correlated with log(ME), investment-to-assets ratio, and one-year past return.

In the second column, the only explanatory variable is log misvaluation. The estimate on log misvaluation is negative and highly statistically significant with a t-statistic of almost -7. Moreover, the effect of misvaluation on returns is also economically large. The standard deviation of log misvaluation is 0.85. Multiplying it by the coefficient estimate on log misvaluation implies that a one-standard-deviation increase in misvaluation is associated with a 0.37% reduction in monthly return.

In Column 3, $\mathbf{X_{it}}$ includes both traditional characteristics and log misvaluation. Augmenting the traditional model by log misvaluation reduces the economic significance of the coefficients on log(B/M) and profitability. Traditional characteristics reduce the economic significance of the coefficient on log misvaluation only marginally, and they do not affect its statistical significance.

4.5.2 Robustness tests

In Table 9, we examine robustness of the cross-sectional relation between returns and log-misvaluation. In the first column of Table 9, we compute misvaluation relative to values obtained from a model in which we estimate the curvature of the production function, θ, only, while assuming that the cost of installing new capital, η, equals one. In Column 2, we estimate the model only for firms that have above-industry-median analyst coverage and compute misvaluation of all firms relative to model values obtained from that estimation. In the third column, we introduce industry-level misvaluation, estimated using the methodology of Rhodes-Kropf, Robinson and Viswanathan (2005), and compute firm-level misvaluation while incorporating industry-level misvaluation. In Columns 4 and 5, we examine 3-month and 12-month returns respectively following the month in which misvaluation is estimated.

Consistent with Table 8, the coefficients on log misvaluation are negative and highly statistically significant in all specifications. The economic significance in the first three columns is similar to the last column in Table 8 – a one-standard-deviation increase in log misvaluation is associated with 0.28%-0.30% reduction in next-month return, and, as follows from the last two columns, with 0.9% (2%) reduction in 3-month (12-month) return.

The results of cross-sectional tests in Tables 8 and 9 are fully consistent with the results of time-series portfolio tests reported in Tables 6 and 7. Overall, the evidence strongly supports our first hypothesis: firms that our model considers undervalued significantly outperform overvalued firms.

4.6 Misvaluation and growth options

Our second hypothesis states that the differences between risk-adjusted returns of most undervalued stocks and those of most overvalued ones should be larger within subsamples of firms with abundant growth options than within subsamples of assets-in-place-based firms. The evidence in Table 4 that most misvalued (i.e., both undervalued and overvalued) firms have characteristics usually associated with growth-options firms supports this conjecture.

To test this hypothesis, we first examine the performance of the UMO strategy in the subsamples of stocks sorted based on our estimated measure of importance of firms' growth options in firm values. Second, we analyze the performance of the UMO strategy in the subsamples sorted on industry-level market-to-book ratios.

4.6.1 Industry GO/AP sorts

If the performance of the UMO strategy documented in Tables 6 and 7 is driven by investors' inability to correctly value firms' investment options, we should expect stronger performance of the strategy in the subsample of more growth-oriented firms. We calculate median GO/AP ratio in each industry and sort firms into terciles based on the median GO/AP value in the industry to which each firm belongs. Consistent with the evidence in Table 4 that growth-option-rich firms are more mispriced on average than assets-in-place-based firms, the mean absolute misvaluation is about 50% higher within the high GO/AP tercile than within the low GO/AP tercile. After assigning firms into GO/AP terciles, we sort them into misvaluation deciles, creating 3-by-10 portfolios. We then compute the difference between risk-adjusted returns of the two extreme misvaluation portfolios within two extreme GO/AP terciles.

The results of estimating the regressions for double-sorted portfolios are presented in Panel A of Table 10. This panel demonstrates that the performance of the UMO strategy is largely driven by the tercile of highest GO/AP firms. In the lowest GO/AP tercile, the mean annualized value-weighted return of the portfolio of most undervalued stocks is 12.4% and the mean annualized return of the most overvalued portfolio is 8.6%. The annualized difference of 3.8% is not statistically significant. On the other hand, in the highest GO/AP tercile, the mean annualized value-weighted return of the portfolio of most undervalued stocks is 13.3% per month and the mean return of the most overvalued portfolio is 2.5% per month. The annualized difference of 10.8% is statistically significant at the 10 percent level.

Adjusting the returns to the exposure to risk factors strengthens the conclusion that the relation between misvaluation and returns is present only within the growth-options-intensive tercile. Annualized differences between the alpha of the most undervalued and that of the most overvalued portfolio range between 9.4% and 16.9% in the highest GO/AP tercile, all highly statistically significant with t-statistics ranging from 2.29 to 3.82. None of the differences in risk-adjusted returns beween undervalued and overvalued portfolios are significant in the lowest GO/AP tercile.

4.6.2 Industry M/B sorts

Since the GO/AP ratio used to sort firms into terciles of growth options in the previous section comes from a structural model, it may pick up some unobservable model misspecification that could be correlated with GO/AP ratios but not with the true value of firms' growth options. To alleviate this concern, we provide additional model-free evidence using an alternative, well accepted measure of the proportion of firm value represented by growth options – market-to-book (M/B) ratio (e.g., Smith and Watts (1992), Barclay, Smith and Watts (1997)). Importantly, in addition to being a proxy for growth options, market-to-book is also directly related to misvaluation as discussed in Rhodes-Kropf, Robinson and Viswanathan (2005). This creates a potential problem if we double-sort on firm-level M/B and on the misvaluation measure. To avoid this, instead of sorting on firm-level M/B we use industry-level M/B, which is orthogonal to firm-level M/B by construction and, as a result, orthogonal to our misvaluation measure. We first sort firms into terciles based on the median market-to-book (M/B) ratio within a firm's industry. Within each industry-level M/B tercile, we sort firms into misvaluation deciles.

The results of double-sorting firms by industry M/B and estimated misvaluation are reported in Panel B of Table 10. Moving from the lowest to the highest industry M/B tercile, mean annualized return of the UMO portfolio increases from 6.7% to 11.9%. The t-statistic in the lowest industry M/B tercile is 0.91 while in the highest industry M/B tercile it is 1.93. Annualized risk-adjusted returns of the UMO strategy in the low-industry-M/B tercile range between 3.6% and 10.7% and are statistically insignificant in all specifications. In the high-industry-M/B tercile, on the other hand, the annualized UMO strategy alphas range between 10% and 15.5%, and are significant at the 1 percent level in all specifications.

4.6.3 Shutting down growth options: Counterfactual analysis

To test Hypothesis 3, we shut down growth options in the valuation equation and estimate the model while assuming that each firm's value is derived solely from its existing assets. If we disallow expansion (by assuming that the cost of adding new capital, η, is infinite), firm value becomes the present value of a perpetuity of after-tax EBIT:

$$V(K_t, x_t) = \frac{(1-\tau)x_t K_t^{\theta}}{r - \mu + \lambda\theta}. \tag{15}$$

Firm value in equation (15) is the function of the curvature of the production function, θ, only. Similar to the base-case analysis, we define valuation errors as

$$\epsilon_{it} = \tilde{V}_{it}/V_{it}(\theta),$$

and re-estimate the model.

The results are presented in Table 11, whose structure is identical to that of Table 6. The mean value-weighted return of the portfolio of most undervalued stocks, 1.15% per month, is larger than the mean return of the most overvalued portfolio, 0.63% per month, however the annualized difference of 6.4% is not statistically significant with a t-statistic of 0.67. The differences in alphas between the most undervalued and most overvalued portfolios remain insignificant after we control for the UMO portfolio's exposure to risk factors. Importantly, the relation between risk-adjusted returns and misvaluation is not monotonic at all in all 8 specifications, when misvaluation is computed relative to the model without growth options. Overall, the model that does not account for the value of growth options fails to produce significant differences in risk-adjusted returns between the most undervalued and the most overvalued firms. We interpret the results in Table 11 as evidence that misvaluation of firms' growth options is an important part of misvaluation of firms by investors.

Overall, the evidence in Tables 10 and 11 strongly supports our second hypothesis: the relation between misvaluation and future returns is stronger among firms with more growth options, potentially because of investors' difficulties in adequately assessing the value of investment options, and gradual realization of this value by the market over time.

4.7 Misvaluation in high and low investor sentiment environments

We proceed by examining the performance of undervalued and overvalued stocks during times of high and low investor sentiment. Numerous papers in behavioral finance and economics show that stock prices are more likely to deviate from their fundamental values in a high investor sentiment environment.[13] Consistent with these findings, Stambaugh, Yu and Yuan (2012) show that many asset pricing anomalies are more pronounced when investor sentiment is high. If the differences between returns of undervalued and overvalued stocks are driven by deviations from fundamentals, we should expect these differences to be larger during times of high investor sentiment.

We test this hypothesis in two ways. First, we examine risk-adjusted returns of portfolios of most undervalued and most overvalued stocks in times of high and low investor sentiment using Baker and Wurgler (2006) sentiment index.[14] Second, we test whether the effect of investor sentiment on the relation between misvaluation and future returns is stronger among growth-oriented firms. Table 10 demonstrates that the performance of the UMO portfolio is stronger among growth firms. If investors are more likely to misprice growth firms and they are more likely to do so when sentiment is high, we expect better risk-adjusted returns of the UMO portfolio in the subset of growth firms during times of high investor sentiment.

The results in Panel A of Table 12 show that undervalued stocks significantly outperform overvalued ones when investor sentiment is high. For example, UMO strategy generates 1.55% abnormal monthly returns relative to Fama and French (2015) model. UMO risk-adjusted returns in periods of high investor sentiment are even larger when other asset pricing models are used as benchmarks. During times of low sentiment, UMO risk-adjusted returns are positive, but their economic magnitude is smaller and statistical significance is weaker.

In Panel B of Table 12, we examine the performance of UMO strategy during periods of high and low investor sentiment within subsets of high and low growth-options firms. As in Table 10, we

[13] See, for example, Shiller (2003) and Baker and Wurgler (2007).

[14] This index is based on the first principle component of the following proxies for investor sentiment: NYSE trading volume, the dividend premium, the closed-end fund discount, the number and average first-day returns on IPOs, and the equity share in new issues. The results reported below are robust to using the orthogonalized version of the sentiment index, which excludes macroeconomic conditions from the index.

independently sort firms into subsets of high and low growth options and into misvaluation deciles, but we now perform this double sorting separately within periods of high and low sentiment. Similar to Table 10, we use two growth option measures: industry-level GO/AP ratio and industry-level M/B ratio. Firms in industries with above-median (below-median) GO/AP or M/B ratios are classified as high-growth-option (low-growth-option) firms.

The results in Panel B show that the strong performance of undervalued stocks relative to overvalued ones during times of high sentiment is driven by the subset of high GO/AP and high M/B firms. For example, when we use Fama and French (2015) model as the benchmark, most undervalued high GO/AP (M/B) firms outperform most overvalued high GO/AP (M/B) firms by highly significant 1.16% (1.47%) per month. On the other hand, within low GO/AP (M/B) subsamples, most undervalued firms outperform most overvalued ones by insignificant 0.8% (0.47%) per month. Overall, the results in Table 12 provide additional evidence consistent with investors' inability to correctly price investment options, which leads to misvaluation in equity markets.

5 Conclusion

Traditional valuation techniques, such as discounted cash flow or valuation by multiples, are not well suited for valuing real options in general and investment options in particular. In this paper we propose that market participants' inability to correctly estimate values of firms with embedded investment options may lead to misvaluation of such firms, which is subsequently partially corrected.

To test this hypothesis, we first construct a real options model of a firm's optimal investment in the presence of demand uncertainty. We estimate the model at the industry level by matching firms' model-implied values to their market values. We then compute a measure of misvaluation and empirically study the relation between misvaluation and future returns.

Our empirical results are consistent with our hypothesis. We find, using both time-series portfolio tests and cross-sectional regressions, that our misvaluation measure is negatively related to future returns – a relation that is highly significant, both economically and statistically.

In addition, consistent with the hypothesis that misvaluation is driven by the difficulties that market participants face when valuing investment options, the relation between misvaluation and future returns is substantially stronger among firms for which growth options constitute a relatively large proportion of value. It is also stronger during times of high investor sentiment, when stock market valuations are more likely to diverge from fundamental values. Overall, our findings suggest that investors have difficulties appropriately incorporating the values of growth options into firm valuation, resulting in equity misvaluation.

References

Abel, Andrew, and Janice Eberly. 1996. "Optimal Investment with Costly Reversibility." *Review of Economic Studies*, 63(4): 581–593.

Ackerberg, Daniel, Kevin Caves, and Garth Frazer. 2015. "Identification Properties of Recent Production Function Estimators." *Econometrica*, 86(6): 2411–2451.

Aguerrevere, Felipe. 2009. "Real Options, Product Market Competition, and Asset Returns." *The Journal of Finance*, 64(2): 957–983.

Babenko, Ilona, Oliver Boguth, and Yuri Tserlukevich. 2016. "Idiosyncratic Cash Flows and Systematic Risk." *The Journal of Finance*, 71(1): 425–456.

Baker, Malcolm, and Jeffrey Wurgler. 2006. "Investor Sentiment and the Cross-Section of Stock Returns." *Journal of Finance*, 61(4): 1645–1680.

Baker, Malcolm, and Jeffrey Wurgler. 2007. "Investor Sentiment in the Stock Market." *Journal of Economic Perspectives*, 21(2): 129–151.

Ball, Ray, Joseph Gerakos, Juhani Linnainmaa, and Valeri Nikolaev. 2016. "Accruals, cash flows, and operating profitability in the cross section of stock returns." *Journal of Financial Economics*, 121(1): 28–45.

Barclay, Michael, Clifford Smith, and Ross Watts. 1995. "The Determinants of Corporate Leverage and Dividend Policies." *Journal of Applied Corporate Finance*, 7(4): 4–19.

Barclay, Michael, Clifford Smith, and Ross Watts. 1997. "The Determinants of Corporate Leverage and Dividend Policies." *Journal of Financial Education*, 23: 1–15.

Barclay, Michael, Erwan Morellec, and Clifford Smith. 2006. "On the Debt Capacity of Growth Options." *Journal of Business*, 79(1): 37–60.

Berk, Jonathan, Richard Green, and Vasant Naik. 1999. "Optimal Investment, Growth Options, and Security Returns." *Journal of Finance*, 54(5): 1553–1607.

Bharath, Sreedhar, and Tyler Shumway. 2008. "Forecasting Default with the Merton Distance to Default Model." *Review of Financial Studies*, 21(3): 1339–1369.

Bhojraj, Sanjeev, Charles Lee, and Derek Oler. 2003. "What's My Line? A Comparison of Industry Classification Schemes for Capital Market Research." *Journal of Accounting Research*, 41(5): 745–774.

Bradley, Michael, Gregg Jarrell, and E. Han Kim. 1984. "On the Existence of an Optimal Capital Structure: Theory and Evidence." *Journal of Finance*, 39(3): 857–878.

Carhart, Mark. 1997. "On persistence in mutual fund performance." *Journal of Finance*, 52(1): 57–82.

Carlson, Murray, Adlai Fisher, and Ron Giammarino. 2004. "Corporate Investment and Asset Price Dynamics: Implications for the Cross-section of Returns." *The Journal of Finance*, 59(6): 2577–2603.

Chan, Louis, Jason Karceski, and Josef Lakonishok. 2003. "The Level and Persistence of Growth Rates." *Journal of Finance*, 58(2): 643–684.

Claus, James, and Jacob Thomas. 2001. "Equity Premia as Low as Three Percent? Evidence from Analysts' Earnings Forecasts for Domestic and International Stock Markets." *Journal of Finance*, 56(5): 1629–1666.

Cooper, Russell, and John Haltiwanger. 2006. "On the Nature of Capital Adjustment Costs." *Review of Economic Studies*, 73(3): 611–633.

Easton, Peter. 2004. "PE Ratios, PEG Ratios, and Estimating the Implied Expected Rate of Return on Equity Capital." *The Accounting Review*, 79(1): 73–95.

Eisdorfer, Assaf, Amit Goyal, and Alexei Zhdanov. 2016. "Default Option and the Cross-Section of Stock Returns." Pennsylvania State University Working Paper.

Fama, Eugene, and James MacBeth. 1973. "Risk, Return, and Equilibrium: Empirical Tests." *Journal of Political Economy*, 81(3): 607–636.

Fama, Eugene, and Kenneth French. 1993. "Common risk factors in the returns on stocks and bonds." *Journal of Financial Economics*, 33(1): 3–56.

Fama, Eugene, and Kenneth French. 2015. "A five-factor asset pricing model." *Journal of Financial Economics*, 116(1): 1–22.

Gebhardt, William, Charles M. C. Lee, and Bhaskaran Swaminathan. 2001. "Toward an Implied Cost of Capital." *Journal of Accounting Research*, 39(1): 135–176.

Graham, John. 1996. "Proxies for the corporate marginal tax rate." *Journal of Financial Economics*, 42(2): 187–221.

Grenadier, Steven. 2002. "Option Exercise Games: An Application to the Equilibrium Investment Strategies of Firms." *Review of Financial Studies*, 15(3): 691–721.

Hackbarth, Dirk, and Erwan Morellec. 2008. "Stock Returns in Mergers and Acquisitions." *The Journal of Finance*, 63(3): 1213–1252.

Hirshleifer, David. 2015. "Behavioral Finance." *Annual Review of Financial Economics*, 7: 133–159.

Hirshleifer, David, Po-Hsuan Hsu, and Dongmei Li. 2013. "Innovative efficiency and stock returns." *Journal of Financial Economics*, 107(3): 632–654.

Hou, Kewei, Chen Xue, and Lu Zhang. 2015. "Digesting Anomalies: An Investment Approach." *Review of Financial Studies*, 28(3): 650–705.

Hou, Kewei, Mathijs A. van Dijk, and Yinglei Zhang. 2012. "The implied cost of capital: A new approach." *Journal of Accounting and Economics*, 53(3): 504–526.

Levinsohn, James, and Amil Petrin. 2003. "Estimating Production Functions Using Inputs to Control for Unobservables." *Review of Economic Studies*, 70(2): 317–341.

Liu, Laura Xiaolei, Toni M. Whited, and Lu Zhang. 2009. "Investment-Based Expected Stock Returns." *Journal of Political Economy*, 117(6): 1105–1139.

Morellec, Erwan. 2001. "Asset liquidity, capital structure, and secured debt." *Journal of Financial Economics*, 61(2): 173–206.

Morellec, Erwan, Boris Nikolov, and Norman Schurhoff. 2012. "Corporate Governance and Capital Structure Dynamics." *Journal of Finance*, 67(3): 803–848.

Novy-Marx, Robert. 2007. "An Equilibrium Model of Investment Under Uncertainty." *Review of Financial Studies*, 20(5): 1461–1502.

Novy-Marx, Robert. 2013. "The other side of value: The gross profitability premium." *Journal of Financial Economics*, 108(1): 1–28.

Olley, G. Steven, and Ariel Pakes. 1996. "The Dynamics of Productivity in the Telecommunications Equipment Industry." *Econometrica*, 64(6): 1263–1297.

Pindyck, Robert. 1988. "Irreversible Investment, Capacity Choice, and the Value of the Firm." *American Economic Review*, 78(5): 969–985.

Pindyck, Robert. 1991. "Irreversibility, Uncertainty, and Investment." *Journal of Economic Literature*, 29(3): 1110–1148.

Rhodes-Kropf, Matthew, David Robinson, and S. Viswanathan. 2005. "Valuation waves and merger activity: The empirical evidence." *Journal of Financial Economics*, 77(3): 561–603.

Sagi, Jacob, and Mark Seasholes. 2007. "Firm-specific Attributes and the Cross-section of Momentum." *Journal of Financial Economics*, 84(2): 389–434.

Shiller, Robert. 2003. "From Efficient Markets Theory to Behavioral Finance." *Journal of Economic Perspectives*, 17(1): 83–104.

Smith, Clifford, and Ross Watts. 1992. "The investment opportunity set and corporate financing, dividend, and compensation policies." *Journal of Financial Economics*, 32(3): 263–292.

Stambaugh, Robert, Jianfeng Yu, and Yu Yuan. 2012. "The short of it: Investor sentiment and anomalies." *Journal of Financial Economics*, 104(2): 288–302.

Strebulaev, Ilya, and Toni Whited. 2012. "Dynamic Models and Structural Estimation in Corporate Finance." *Foundations and Trends in Finance*, 6(1-2): 1–163.

Warusawitharana, Missaka, and Toni Whited. 2016. "Equity Market Misvaluation, Financing, and Investment." *Review of Financial Studies*, 29(3): 603–654.

Appendix to “Misvaluation of Investment Options”

This Appendix contains 3 sections. Section A explains our choice of the profit function in the model. Section B contains the derivation of firm value in the model. Section C presents details of estimation of the drift parameter of the demand process.

A Profit function

Consider a firm with a Cobb-Douglas production function

$$q = \kappa L^{\phi} K^{1-\phi},$$

where L is labor, κ is a constant, and $0 < \phi < 1$ is the share of labor in the production process. The cost of labor is constant and given by w, and the output price is given by the inverse demand function

$$p = y^{1-\gamma} q^{\gamma},$$

where y is an uncertainty parameter and $0 < \gamma < 1$. The firm maximizes its instantaneous profit by optimally choosing the amount of labor:

$$L^* = \arg\max_L [y^{1-\gamma} \left(\kappa L^{\phi} K^{1-\phi}\right)^{\gamma} - wL].$$

Solving this yields

$$L = \left[\frac{wK^{(\phi-1)\gamma} y^{\gamma-1}}{\kappa^{\gamma}\phi\gamma}\right]^{\frac{1}{\phi\gamma-1}},$$

and

$$\pi = C y^{\frac{\gamma-1}{\phi\gamma-1}} K^{\frac{\gamma(\phi-1)}{\phi\gamma-1}},$$

where $C > 0$ is a constant:

$$C = \kappa^{\frac{\gamma}{1-\phi\gamma}} w^{\frac{\phi\gamma}{\phi\gamma-1}} \left[(\phi\gamma)^{\frac{\phi\gamma}{1-\phi\gamma}} - (\phi\gamma)^{\frac{1}{1-\phi\gamma}}\right].$$

This profit function is equivalent to the one in our model, equation (1), with $x_t = Cy^{\frac{\gamma-1}{\phi\gamma-1}}$ and $\theta = \frac{\gamma(\phi-1)}{\phi\gamma-1}$.

B Derivation of firm value

In what follows, we prove that the value of the firm is indeed given by equation (5). The value of assets in place equals the present value of all cash flows generated by the firm's (depreciating) capital:

$$AP(K_t, x_t) = (1-\tau)K_t^\theta E \int_0^\infty x_t e^{-\lambda\theta t} e^{-rt} dt = \frac{(1-\tau)x_t K_t^\theta}{r-\mu+\lambda\theta}.$$

Using standard arguments, it can be shown that the value of growth options $F(K_t, x_t)$ follows the following PDE:

$$\frac{1}{2}\sigma^2 x_t^2 F_{xx} + \mu x_t F_x - \lambda K_t F_K - rF = 0.$$

The value of a marginal unit of capital to be installed is $f(K_t, x_t)dK$, where $f(K_t, x_t) = F_K(K_t, x_t)$. $f(K_t, x_t)$ follows the following PDE:

$$\frac{1}{2}\sigma^2 x_t^2 f_{xx} + \mu x_t f_x - \lambda K_t f_K - (r+\lambda)f = 0.$$

The expected increase in the PV of one additional unit of capital is

$$v(K_t, x_t) = \frac{(1-\tau)x_t\theta}{(r-\mu+\lambda\theta)K_t^{1-\theta}}.$$

Now, denote $y = \frac{x_t}{K_t^{1-\theta}}$. Assume that $f(K_t, x_t)$ is a function of $y : f(K_t, x_t) = g(y)$. Substituting $f_x(x_t, K_t) = K_t^{\theta-1} g'(y)$ yields:

$$\frac{1}{2}\sigma^2 y^2 g''(y) + \mu y g'(y) - \lambda(\theta-1)yg'(y) - (r+\lambda)g(y) = 0,$$

or, equivalently

$$\frac{1}{2}\sigma^2 y^2 g''(y) + [\mu - \lambda(\theta-1)]\, yg'(y) - (r+\lambda)g(y) = 0.$$

This ODE has the following solution:

$$g(y) = By^{\beta_1}$$

where β_1 is the positive root of the equation

$$\frac{1}{2}\sigma^2\beta(\beta-1) + [\mu - \lambda(\theta-1)]\,\beta - (r+\lambda) = 0.$$

The optimal investment threshold together with the constant B are determined by the following boundary conditions:

1) Value matching condition:

$$B\left(\frac{X}{K_t^{1-\theta}}\right)^{\beta_1} = \frac{BX^{\beta_1}}{K_t^{\beta_1(1-\theta)}} = \frac{(1-\tau)X\theta}{(r-\mu+\lambda\theta)K_t^{1-\theta}} - \eta;$$

2) Smooth pasting condition:

$$B\beta_1 X(K)^{\beta_1-1}K_t^{\beta_1(\theta-1)} = \frac{(1-\tau)X\theta}{(r-\mu+\lambda\theta)K_t^{1-\theta}}.$$

It follows that the optimal investment threshold is

$$X(K_t) = \frac{\beta_1}{\beta_1-1}\frac{(r-\mu+\lambda\theta)\eta K_t^{1-\theta}}{(1-\tau)\theta},$$

and the constant B is

$$B = \frac{1}{\beta_1}\left[\frac{(1-\tau)\theta}{r-\mu+\lambda\theta}\right]^{\beta_1}\eta^{1-\beta_1}\left(\frac{\beta_1}{\beta_1-1}\right)^{1-\beta_1} = \left[\frac{(1-\tau)\theta}{\beta_1\left(r-\mu+\lambda\theta\right)}\right]^{\beta_1}\left(\frac{\beta_1-1}{\eta}\right)^{\beta_1-1}.$$

The value of the growth options is

$$F(K_t, x_t) = Bx_t^{\beta_1}\int_{K_t}^{\infty}K^{(\theta-1)\beta_1}dK = \frac{Bx_t^{\beta_1}}{(\theta-1)\beta_1+1}K_t^{(\theta-1)\beta_1+1},$$

and the total value of the firm is given in equation (5).

C Estimation of the long-term growth rate of industry demand

We estimate the drift of the demand process using equity analysts' forecasts of long-term growth (LTG) rates. We use the Institutional Brokers Estimate System (IBES) to obtain LTG rates, which are typically coded by "0" in IBES.

We aggregate LTG at the industry level each month. In particular, for a given month for which we estimate the model, we take all LTG forecasts issued in the last 90 days by every analyst. Out of these forecasts, we take the latest forecast issued by each analyst for a given firm. This approach avoids biasing our results in favor of analysts who issue frequent updates to their forecasts during the quarter. This gives us a cross-section of unique analyst forecasts for each firm in the industry. We then average these forecasts using a two-step procedure. First, we take the median forecast across all analysts for a given firm. Second, we take the median forecast across all firms, which is our estimate of the industry long-term growth rate.

LTG forecasts tend to be overly optimistic. We follow the approach of Chan, Karceski and Lakonishok (2003) and Morellec, Nikolov and Schurhoff (2012) and adjust these forecasts downwards using the following linear function: Adjusted LTG $= 0.007264043 + 0.408605737 \times$Median LTG, where the median LTG is our estimate of the industry growth rate forecast. If, for example, a given industry has a long-term estimate of a growth rate of 10%, the adjusted growth rate becomes 4.15%.[15]

A typical LTG forecast issued by an analyst is a forecast of growth rate over the next five years. In our model, however, firms are infinitely lived, and the demand process drifts with rate μ into infinity. Therefore, we need to back out the terminal growth rate out of the five-year LTG forecasts. We employ the following procedure. We assume that a firm's cash flow will grow at the "Adjusted LTG" rate as specified above over the first five years, after which point the cash flows will grow at 2%.[16] From these two rates we back out a single perpetual growth rate that corresponds to the same firm value as the combination of the two growth rates above, as we explain below.

Consider a firm that has a cash flow of \$1 today. This cash flow will grow at rate LTG over the next five years and from year 6 onwards it will grow at 2% in perpetuity. Assume that this firm's cost of capital is r_A. The value of this firm today is

$$\frac{1}{r_A - \mathrm{LTG}}\left(1 + \frac{(1+\mathrm{LTG})^5}{(1+r_A)^5}\right) + \frac{1}{(1+r_A)^5}\frac{1}{r_A - 0.02}.$$

Our goal is to find an equivalent perpetual growth rate of cash flows that starts now and gives the same firm value. In other words, LTG' is given as an implicit solution to the following equation:

$$\frac{1}{r_A - \mathrm{LTG'}} = \frac{1}{r_A - \mathrm{LTG}}\left(1 + \frac{(1+\mathrm{LTG})^5}{(1+r_A)^5}\right) + \frac{1}{(1+r_A)^5}\frac{1}{r_A - 0.02}.$$

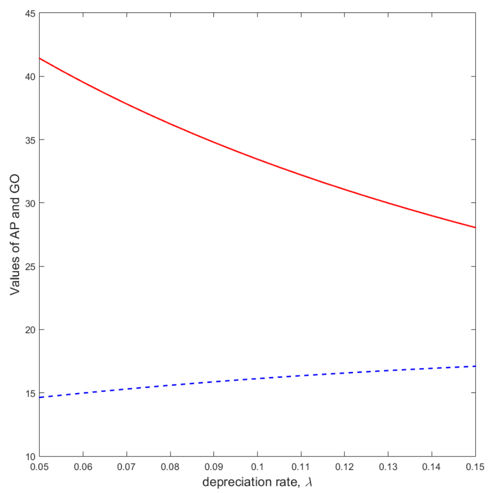
Note that the rate LTG' is a function of the discount rate, r_A. To obtain r_A, we estimate each firm's cost of capital every month. Since we do this adjustment at the industry level, we use the cost of capital of the median firm in the industry. Our final step is to risk-adjust the above growth rate. We discuss our risk-adjusting procedure in Section 3.

[15] The result in the original Chan, Karceski and Lakonishok (2003) is obtained by regressing LTG forecasts on realized growth rates.

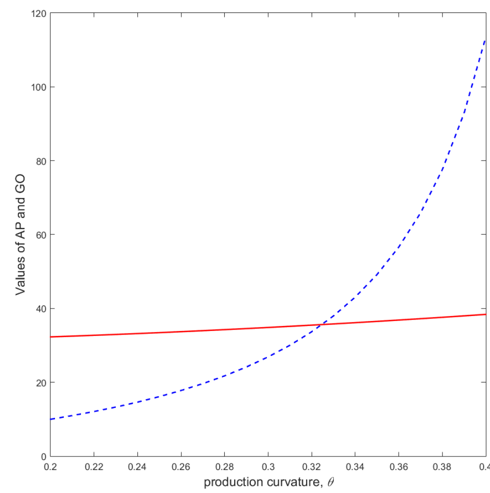
[16] Although our choice of 2% terminal growth rate is ad-hoc, all our results hold if we assume terminal growth rate ranging between 0.75% and 4.5%.

Figure 1: Comparative Statics

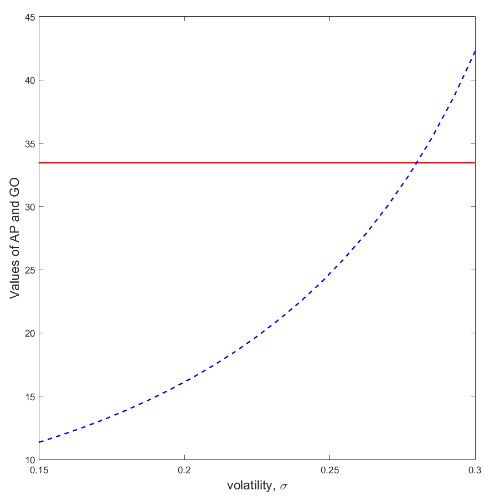
This figure presents the values of assets in place (AP, solid red lines) and growth options (GO, dashed blue lines) as functions of model parameters λ, σ, θ, η. λ is the depreciation rate. σ is the volatility of the demand shock. θ is the curvature of the production function. η is the cost of purchasing and installing new capital. The remaining parameter values are set as follows: $\lambda = 0.1$, $\sigma = 0.2$, $\theta = 0.25$, $\eta = 1$, $r = 0.05$, $\mu = 0.01$.



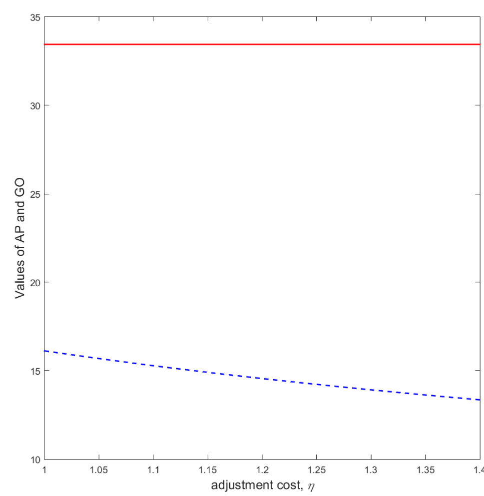
(a) Values of AP and GO as functions of λ



(b) Values of AP and GO as functions of θ



(c) Values of AP and GO as functions of σ



(d) Values of AP and GO as functions of η

Figure 2: Timeline of data inputs

This figure shows the timing of data inputs into portfolio formation in time-series tests. Portfolios are formed in month t. The market value of a firm is measured as of the end of month t. The model value is measured at a time that depends on the firm's fiscal yearend. We require the date of release of accounting data (Compustat DATADATE) to be at least six months before month t and use the closest month to $t-6$. Analyst forecasts that we use are released in months $[t-3, t-1]$.

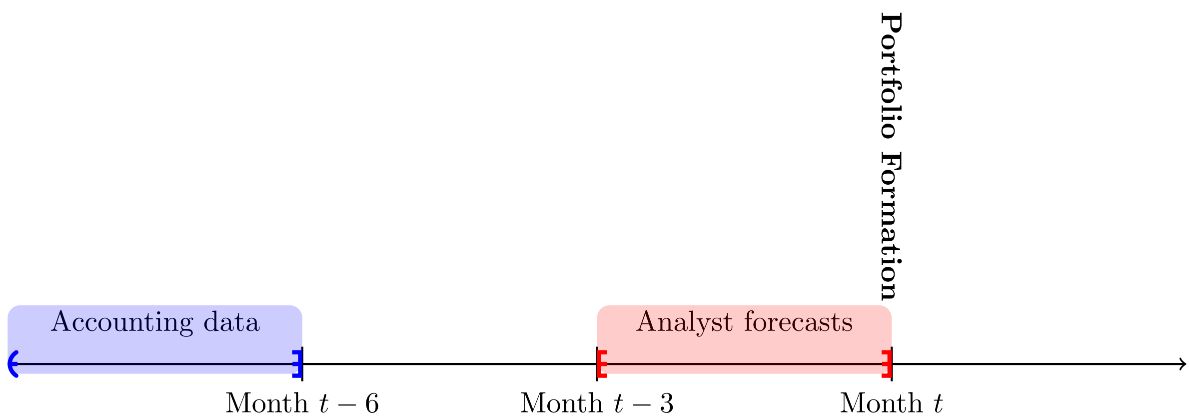


Table 1: Descriptive statistics of variables used in misvaluation estimation

This table provides summary statistics for the main variables used in estimation. The sample is based on 2014 Industrial Compustat Annual and Quarterly Fundamental Files, Center for Research in Security Prices (CRSP), and Institutional Brokers Estimate System (IBES). The sample covers the period from January 1980 to December 2014. Variables are either at the monthly or annual frequency. The last column in the table indicates the frequency with which a given variable is observed. Market value of equity, capital, revenue, COGS, SG&A, EBITDA, long-term and short-term debt are in millions of dollars. Variables are defined in Table A1.

	Mean	St. Dev.	10%	25%	50%	75%	90%	# Obs.	Frequency
Firm Characteristics									
Market value of equity	1,890.88	11,272.75	8.49	27.13	127.04	649.82	2,650.45	1,506,719	Monthly
Capital	1,002.26	6,533.98	3.99	13.52	61.67	323.14	1,466.51	129,876	Annual
Revenue	1,602.59	8,610.16	15.25	45.41	169.01	706.34	2,620.35	129,876	Annual
Cost of goods sold	1,079.73	6,388.69	7.65	25.09	100.89	444.00	1,727.48	129,876	Annual
SG&A	280.79	1,457.47	2.96	8.56	31.15	127.03	463.01	129,876	Annual
EBITDA	242.07	1,409.62	1.22	4.32	18.69	91.10	360.87	129,876	Annual
Long-term debt	386.10	3,386.84	0.00	0.96	13.62	121.10	596.45	129,645	Annual
Short-term debt	127.97	2,315.13	0.00	0.20	2.23	13.04	69.60	129,742	Annual
Equity beta	1.13	0.75	0.25	0.66	1.08	1.56	2.11	1,323,780	Monthly
Industry Characteristics									
Demand growth rate	0.03	0.01	0.02	0.03	0.03	0.04	0.04	25,260	Monthly
Demand volatility	0.19	0.27	0.08	0.11	0.15	0.20	0.28	27,447	Monthly
Capital depreciation rate	0.09	0.04	0.05	0.07	0.08	0.11	0.14	27,650	Monthly

Table 2: Estimation Results

This table reports estimation results aggregated at the sector level (Panel A) and industry level (Panel B). For each sector and industry, we report average curvature of the production function, θ, in Column 3, and average cost of installing capital, η, in Column 4. Columns 5 through 10 report the distribution of GO/AP ratios across firms in each industry/sector. To compute GO/AP ratios, we calculate the value of growth options (GO) and the value of assets in place (AP) for each firm at the estimated value of industry-level parameters (θ, η).

Panel A: Sector Level

Sector	GICS Code	θ	η	Value of GO / Value of AP					
				Mean	10P	25P	Median	75P	90P
Energy	10	0.30	1.25	32.2%	0.2%	1.3%	5.4%	21.0%	54.7%
Materials	15	0.33	1.30	3.4%	0.0%	0.1%	0.7%	2.7%	7.5%
Industrials	20	0.32	1.21	11.7%	0.3%	1.2%	4.0%	11.9%	28.8%
Consumer Discretionary	25	0.30	1.19	8.3%	0.2%	1.1%	3.8%	10.3%	20.2%
Consumer Staples	30	0.31	1.26	9.3%	0.2%	0.9%	3.6%	11.1%	25.1%
Health Care	35	0.34	1.07	73.1%	1.7%	6.7%	22.5%	59.8%	153.7%
Information Technology	45	0.36	1.04	53.6%	1.4%	4.1%	12.2%	35.6%	92.3%
Telecommunication Services	50	0.34	1.17	26.1%	0.1%	0.4%	2.5%	8.4%	29.8%
Mean		0.32	1.18	24.5%	0.6%	2.1%	6.9%	19.8%	49.2%
Standard Deviation		0.04	0.10	35.9%	1.2%	3.5%	9.8%	25.3%	73.2%

Table 2: Estimation results – continued

Panel B: Industry Level

Industry	GICS Code	θ	η	Value of Growth Options / Value of Assets in Place					
				Mean	10P	25P	Median	75P	90P
Energy Equipment & Services	101010	0.33	1.22	52.1%	0.4%	2.0%	7.9%	33.0%	88.9%
Oil, Gas & Consumable Fuels	101020	0.27	1.28	12.4%	0.1%	0.7%	3.0%	9.0%	20.5%
Chemicals	151010	0.29	1.19	2.1%	0.0%	0.1%	0.4%	1.5%	4.5%
Construction Materials	151020	0.35	1.31	4.5%	0.0%	0.1%	0.7%	3.6%	9.4%
Containers & Packaging	151030	0.41	1.34	2.8%	0.1%	0.2%	0.9%	2.9%	7.4%
Metals & Mining	151040	0.29	1.30	6.1%	0.0%	0.3%	1.2%	4.6%	13.2%
Paper & Forest Products	151050	0.33	1.36	1.5%	0.0%	0.0%	0.1%	0.6%	3.3%
Aerospace & Defense	201010	0.27	1.19	7.3%	0.1%	0.4%	2.1%	7.0%	17.2%
Building Products	201020	0.32	1.33	5.1%	0.1%	0.4%	1.6%	5.1%	13.5%
Construction & Engineering	201030	0.29	1.14	9.9%	0.2%	1.0%	4.0%	11.4%	26.1%
Electrical Equipment	201040	0.26	1.13	2.6%	0.1%	0.4%	1.3%	3.3%	6.6%
Industrial Conglomerates	201050	0.36	1.21	9.9%	0.1%	1.0%	4.9%	12.2%	24.8%
Machinery	201060	0.29	1.13	5.2%	0.1%	0.6%	1.9%	5.0%	13.3%
Trading Companies & Distributors	201070	0.31	1.26	9.5%	0.5%	1.7%	5.5%	14.1%	24.1%
Commercial Services & Supplies	202010	0.28	1.18	7.5%	0.2%	1.0%	3.5%	9.9%	20.4%
Professional Services	202020	0.33	1.15	25.5%	2.6%	8.0%	19.1%	32.6%	55.9%
Air Freight & Logistics	203010	0.36	1.13	12.6%	0.2%	1.0%	5.5%	13.3%	33.4%
Airlines	203020	0.29	1.26	3.5%	0.0%	0.0%	0.2%	1.3%	9.3%
Marine	203030	0.37	1.31	21.8%	0.1%	0.5%	1.9%	9.6%	38.2%
Road & Rail	203040	0.38	1.20	4.6%	0.0%	0.1%	1.0%	4.1%	12.2%
Transportation Infrastructure	203050	0.31	1.34	39.3%	0.1%	0.7%	3.6%	37.8%	107.7%
Auto Components	251010	0.34	1.21	6.3%	0.1%	0.4%	1.9%	7.4%	17.7%
Automobiles	251020	0.32	1.17	13.3%	0.1%	0.5%	2.1%	7.0%	17.8%
Household Durables	252010	0.26	1.26	6.4%	0.2%	0.8%	3.4%	8.8%	16.9%
Leisure Equipment & Products	252020	0.27	1.17	8.2%	0.3%	1.2%	4.4%	11.6%	21.9%
Textiles, Apparel & Luxury Goods	252030	0.26	1.32	6.8%	0.1%	0.8%	3.1%	9.1%	18.2%
Hotels, Restaurants & Leisure	253010	0.29	1.24	4.2%	0.0%	0.1%	0.8%	4.0%	12.8%
Diversified Consumer Services	253020	0.38	1.17	11.7%	0.4%	1.9%	6.7%	16.2%	30.1%
Media	254010	0.27	1.17	9.0%	0.4%	1.8%	5.6%	13.2%	23.0%
Distributors	255010	0.27	1.23	13.7%	0.2%	1.6%	7.0%	19.8%	35.4%
Internet & Catalog Retail	255020	0.31	1.09	16.5%	0.6%	3.2%	9.5%	22.2%	39.4%
Multiline Retail	255030	0.36	1.20	1.1%	0.0%	0.0%	0.1%	0.6%	2.6%
Specialty Retail	255040	0.28	1.10	2.7%	0.1%	0.4%	1.4%	3.7%	7.1%
Food & Staples Retailing	301010	0.38	1.21	3.6%	0.0%	0.2%	0.7%	2.8%	9.8%
Beverages	302010	0.29	1.33	10.3%	0.2%	1.0%	3.9%	12.6%	28.2%
Food Products	302020	0.31	1.21	6.1%	0.1%	0.4%	1.4%	5.2%	17.7%
Tobacco	302030	0.29	1.28	17.5%	0.8%	3.1%	9.9%	24.1%	39.9%
Household Products	303010	0.29	1.33	10.2%	0.0%	0.1%	1.9%	11.5%	34.7%
Personal Products	303020	0.27	1.21	7.8%	0.2%	0.9%	3.8%	10.3%	20.1%
Health Care Equipment & Supplies	351010	0.35	1.01	31.6%	0.4%	2.7%	9.5%	26.4%	83.7%
Health Care Providers & Services	351020	0.32	1.13	22.3%	1.6%	5.2%	14.8%	28.1%	50.6%
Health Care Technology	351030	0.34	1.06	103.4%	5.1%	14.0%	39.4%	80.7%	147.9%
Biotechnology	352010	0.34	1.14	209.0%	3.0%	16.3%	52.7%	147.6%	480.9%
Pharmaceuticals	352020	0.35	1.08	51.8%	0.0%	0.7%	13.4%	62.8%	125.8%
Life Sciences Tools & Services	352030	0.33	1.01	20.7%	0.3%	1.4%	5.5%	13.2%	33.5%
Internet Software & Services	451010	0.33	1.04	49.4%	6.7%	14.9%	30.1%	52.3%	77.4%
IT Services	451020	0.28	1.02	18.7%	1.3%	4.8%	12.3%	24.3%	37.7%
Software	451030	0.40	1.00	69.7%	1.7%	6.8%	27.6%	76.1%	173.4%
Communications Equipment	452010	0.39	1.00	76.5%	0.8%	2.7%	8.7%	31.7%	96.4%
Computers & Peripherals	452020	0.33	1.01	13.6%	0.3%	1.3%	4.7%	13.2%	40.7%
Electronic Equipment & Instruments	452030	0.28	1.05	6.9%	0.3%	1.3%	3.7%	8.9%	17.6%
Office Electronics	452040	0.46	1.18	100.1%	0.6%	2.2%	9.8%	47.5%	171.1%
Semiconductor Equipment & Products	452050	0.35	1.04	44.1%	0.5%	1.3%	3.9%	12.4%	45.2%
Semiconductors & Semiconductor Equipment	453010	0.39	1.02	103.8%	0.4%	1.7%	9.4%	54.2%	171.0%
Diversified Telecommunication Services	501010	0.35	1.23	21.1%	0.0%	0.1%	1.2%	6.4%	27.8%
Wireless Telecommunication Services	501020	0.34	1.12	31.0%	0.1%	0.7%	3.9%	10.4%	31.9%

Table 3: Estimated GO/AP ratio and growth-option-related firm characteristics

This table presents mean growth-options-related characteristics for each decile of firms sorted by the industry median estimated ratio of growth options (GO) to assets in place (AP). To compute GO/AP ratios, we calculate the value of growth options (GO) and the value of assets in place (AP) for each firm at the estimated value of industry-level parameters (θ, η). All variables are defined in Table A1.

Industry GO/AP decile	Industry M/B	Age	R&D	Asset growth	Leverage	Equity beta	St. Dev. returns	NASDAQ
1 (Least growth options)	1.49	33.55	0.010	0.21	0.29	1.03	2.74	0.39
2	1.53	29.38	0.010	0.27	0.28	1.05	3.01	0.40
3	1.59	33.32	0.009	0.21	0.23	1.11	3.03	0.44
4	1.42	34.14	0.020	0.19	0.25	1.03	2.90	0.34
5	1.50	29.50	0.013	0.23	0.25	1.09	3.18	0.46
6	1.71	24.21	0.030	0.26	0.23	1.18	3.28	0.55
7	1.76	23.16	0.034	0.28	0.24	1.21	3.24	0.54
8	1.98	21.83	0.063	0.36	0.19	1.34	3.32	0.63
9	2.06	20.80	0.045	0.34	0.20	1.19	3.27	0.59
10 (Most growth options)	2.45	16.89	0.083	0.44	0.15	1.38	3.52	0.70

Table 4: Misvaluation and firm characteristics

This table presents mean characteristics of firms in each misvaluation decile. Valuation decile "U" denotes the most undervalued decile. Decile "O" denotes the most overvalued. Misvaluation is computed according to (7). All variables are defined in Table A1.

Misvaluation decile	Assets	MV equity	Age	Industry M/B	R&D	Asset growth	Invest-ment	Profit-ability	Net issuance	6-month return	St. Dev. of returns	Amihud measure	Institutional ownership	Number of analysts	Forecast dispersion
U	1,901	1,853	20.09	1.83	0.039	0.296	0.055	0.122	0.027	-5.51	3.56	0.49	0.44	3.90	0.122
2	2,427	2,241	26.70	1.70	0.026	0.206	0.053	0.104	0.016	-1.32	3.18	0.40	0.51	4.17	0.084
3	2,420	2,242	29.45	1.67	0.023	0.190	0.053	0.099	0.016	2.04	2.96	0.32	0.53	4.44	0.075
4	2,408	2,376	31.72	1.66	0.023	0.197	0.054	0.098	0.015	4.58	2.84	0.28	0.56	4.47	0.063
5	2,395	2,382	32.46	1.66	0.024	0.206	0.057	0.098	0.017	6.96	2.81	0.26	0.57	4.67	0.067
6	2,307	2,390	32.57	1.68	0.025	0.225	0.063	0.098	0.020	8.97	2.81	0.24	0.58	4.76	0.064
7	2,297	2,513	30.63	1.70	0.027	0.261	0.075	0.096	0.028	10.87	2.89	0.25	0.57	4.79	0.055
8	2,165	2,473	26.95	1.74	0.030	0.314	0.089	0.091	0.037	13.47	3.05	0.26	0.57	4.75	0.054
9	1,610	2,072	21.59	1.80	0.039	0.388	0.104	0.075	0.052	16.28	3.38	0.31	0.54	4.69	0.056
O	651	1,114	15.25	1.89	0.052	0.474	0.114	0.018	0.074	17.27	3.93	0.44	0.47	3.91	0.099

Table 5: Evolution of misvaluation

This table reports mean forward misvaluation decile over the 1-month, 3-month, 6-month, 1-year, 2-year, and 3-year horizons for firms belonging to each misvaluation decile (Panel A) and transition probabilities over the same horizons for two extreme misvaluation deciles (Panel B). Misvaluation is computed according to (7).

Panel A: Summary of movements across misvaluation deciles

Horizon	1 month	3 months	6 months	1 year	2 years	3 years
Misvaluation decile	Mean forward misvaluation decile					
1 (Most undervalued)	1.42	1.99	2.68	3.57	4.22	4.51
2	2.27	2.69	3.25	4.00	4.54	4.77
3	3.18	3.47	3.86	4.38	4.79	4.94
4	4.10	4.25	4.48	4.77	5.03	5.14
5	5.01	5.05	5.10	5.16	5.29	5.32
6	5.92	5.82	5.73	5.60	5.55	5.54
7	6.85	6.65	6.43	6.09	5.84	5.72
8	7.79	7.49	7.12	6.55	6.12	5.95
9	8.75	8.36	7.83	7.05	6.46	6.19
10 (Most overvalued)	9.71	9.23	8.50	7.59	6.74	6.31

Table 5: EVOLUTION OF MISVALUATION – CONTINUED

Panel B: Transitions of firms in extreme misvaluation deciles

		Forward misvaluation decile									
Horizon	Misvaluation decile	1	2	3	4	5	6	7	8	9	10
1 month	1 (Most undervalued)	0.82	0.12	0.02	0.01	0.01	0.01	0.00	0.01	0.01	0.01
	10 (Most overvalued)	0.00	0.00	0.00	0.00	0.00	0.01	0.01	0.01	0.08	0.87
3 months	1 (Most undervalued)	0.67	0.16	0.04	0.03	0.02	0.02	0.01	0.01	0.02	0.02
	10 (Most overvalued)	0.01	0.01	0.01	0.01	0.01	0.02	0.02	0.04	0.14	0.73
6 months	1 (Most undervalued)	0.53	0.17	0.07	0.04	0.03	0.03	0.03	0.03	0.03	0.04
	10 (Most overvalued)	0.03	0.02	0.02	0.02	0.03	0.03	0.04	0.07	0.18	0.54
1 year	1 (Most undervalued)	0.37	0.17	0.09	0.07	0.05	0.05	0.05	0.05	0.05	0.06
	10 (Most overvalued)	0.04	0.04	0.04	0.04	0.05	0.06	0.08	0.12	0.21	0.32
2 years	1 (Most undervalued)	0.26	0.15	0.10	0.08	0.07	0.06	0.06	0.06	0.07	0.07
	10 (Most overvalued)	0.06	0.06	0.06	0.06	0.07	0.08	0.09	0.13	0.18	0.20
3 years	1 (Most undervalued)	0.22	0.14	0.11	0.09	0.08	0.07	0.07	0.07	0.08	0.07
	10 (Most overvalued)	0.07	0.08	0.07	0.08	0.08	0.09	0.10	0.12	0.16	0.15

Table 6: Excess returns to portfolios sorted on misvaluation

Panel A presents monthly value-weighted average risk-adjusted returns to portfolios sorted on misvaluation measure (7). The sample covers 1980-2014. Column 1 presents mean returns. Column 2 presents alphas relative to the MKT factor. Column 3 presents alphas relative to MKT, HML, and SMB factors, following Fama and French (1993). Column 4 presents alphas relative to MKT, HML, SMB, and MOM factors, following Carhart (1997). Column 5 presents alphas relative to MKT, HML, SMB, RMW, and CMA, following Fama and French (2015). Column 6 presents alphas relative to MKT, HML, SMB, RMW, CMA, and MOM. Column 7 presents alphas relative to MKT, r_{ME}, $r_{I/A}$, and r_{ROE} factors, following Hou, Xue and Zhang (2015). Column 8 presents alphas relative to MKT, r_{ME}, $r_{I/A}$, and r_{ROE}, and MOM factors. The description of factors is found in Section 4. Standard errors are Newey-West adjusted, with 6 lags. T-statistics are reported in parentheses. The difference row presents the difference in mean risk-adjusted returns between two extreme misvaluation deciles with the t-statistics for the difference in parentheses. Panel B reports mean loadings on factors used in the 3 asset pricing models in columns 4, 6, and 8 of Panel A, for the most undervalued and most overvalued deciles of stocks. T-statistics of the factor loadings are reported in parentheses. The difference row presents the difference in mean factor loadings between the two extreme misvaluation deciles with the t-statistics for the difference reported in parentheses.

Panel A: Mean excess returns and alphas

Misvaluation decile	Mean	CAPM	FF3	FF3 +MOM	FF5	FF5 +MOM	Q	Q +MOM
1 (Most undervalued)	1.05	0.38	0.36	0.44	0.63	0.69	0.90	0.93
	(3.45)	(1.96)	(1.85)	(2.25)	(3.24)	(3.54)	(4.67)	(4.57)
2	0.82	0.15	0.18	0.28	0.26	0.35	0.46	0.66
	(2.92)	(0.98)	(1.20)	(1.91)	(1.75)	(2.35)	(3.02)	(4.33)
3	0.81	0.24	0.22	0.31	0.14	0.22	0.26	0.31
	(3.50)	(2.10)	(1.95)	(2.76)	(1.19)	(1.93)	(2.15)	(2.67)
4	0.80	0.23	0.24	0.37	0.08	0.20	0.24	0.24
	(3.48)	(2.05)	(2.08)	(3.45)	(0.68)	(1.93)	(2.05)	(2.10)
5	0.65	0.03	-0.01	0.04	-0.15	-0.10	-0.08	-0.14
	(2.65)	(0.24)	(-0.06)	(0.35)	(-1.38)	(-0.96)	(-0.66)	(-1.34)
6	0.70	0.08	0.05	0.11	-0.13	-0.08	-0.08	-0.04
	(2.87)	(0.76)	(0.49)	(0.96)	(-1.23)	(-0.73)	(-0.64)	(-0.40)
7	0.58	-0.04	0.01	0.01	-0.19	-0.19	-0.24	-0.25
	(2.37)	(-0.32)	(0.09)	(0.05)	(-1.78)	(-1.73)	(-2.03)	(-2.37)
8	0.48	-0.19	-0.13	-0.13	-0.32	-0.31	-0.37	-0.30
	(1.82)	(-1.69)	(-1.12)	(-1.06)	(-2.87)	(-2.74)	(-3.03)	(-2.91)
9	0.35	-0.35	-0.22	-0.37	-0.30	-0.43	-0.53	-0.43
	(1.21)	(-2.27)	(-1.42)	(-2.51)	(-1.90)	(-2.86)	(-3.35)	(-3.71)
10 (Most overvalued)	0.15	-0.69	-0.43	-0.53	-0.19	-0.28	-0.41	-0.19
	(0.40)	(-3.22)	(-2.08)	(-2.55)	(-0.92)	(-1.39)	(-1.85)	(-1.13)
Difference 1-10	0.90	1.07	0.79	0.97	0.82	0.98	1.31	1.12
	(1.91)	(3.70)	(2.79)	(3.40)	(2.89)	(3.45)	(4.46)	()

Table 6: Excess returns to portfolios sorted on misvaluation – Continued

Panel B: Factor Loadings

Model	Misvaluation Decile	MKT	SMB	HML	RMW	CMA	r_{ME}	$r_{I/A}$	r_{ROE}	MOM
FF3+MOM	Most undervalued	1.07	-0.11	-0.02						-0.11
		(23.85)	(-1.68)	(-0.27)						(-2.64)
	Most overvalued	1.24	0.22	-0.39						0.13
		(26.37)	(3.14)	(-5.35)						(3.03)
	Difference	-0.17	-0.33	0.37						-0.25
		(-2.60)	(-3.43)	(3.68)						(-4.01)
FF5+MOM	Most undervalued	0.97	-0.10	0.17	-0.35	-0.46				-0.10
		(20.78)	(-1.58)	(1.96)	(-4.22)	(-3.58)				(-2.34)
	Most overvalued	1.16	0.18	-0.33	-0.47	-0.16				0.15
		(23.58)	(2.67)	(-3.65)	(-5.40)	(-1.17)				(3.42)
	Difference	-0.18	-0.29	0.51	0.12	-0.30				-0.24
		(-2.69)	(-3.02)	(3.99)	(0.99)	(-1.63)				(-4.09)
Q+MOM	Most undervalued	0.92					0.10	-0.50	-0.42	0.00
		(20.95)					(1.50)	(-4.95)	(-4.93)	(0.03)
	Most overvalued	1.21					0.18	-0.50	-0.25	0.19
		(24.40)					(2.45)	(-4.39)	(-2.62)	(3.56)
	Difference	-0.29					-0.08	0.00	-0.17	-0.19
		(-4.44)					(-0.85)	(0.02)	(-1.29)	(-2.65)

Table 7: Excess returns to portfolios sorted on misvaluation: robustness

This table reports mean monthly risk-adjusted returns of portfolios of firms belonging to extreme misvaluation groups using various asset pricing models. The sample period is 1980-2014. See Table 6 for the description of the columns. Variables are defined in Table A1. In Panel A, we report mean risk-adjusted returns for the extreme misvaluation quintiles instead of deciles. In Panel B, we report mean risk-adjusted returns for subsamples of firms that have misvaluation below and above one (i.e. subsamples of undervalued and overvalued firms). In Panel C we use equally-weighted monthly portfolio returns when computing mean risk-adjusted returns. In Panel D, we compute theoretical firm values and resulting misvaluation while estimating the model with only one free parameter, θ, while fixing η at one. In Panel E, we compute theoretical firm values and resulting misvaluation while estimating θ and η using only firms with above-median analyst coverage in their industries. In Panel F, we compute firm-level misvaluation while estimating industry-level misvaluation based on Rhodes-Kropf, Robinson and Viswanathan (2005) procedure. In Panel G, the dependent variable is 3-month return following the formation of decile portfolios. In Panel H, the dependent variable is 12-month return following the formation of decile portfolios. In all regressions, standard errors are Newey-West adjusted (with 6 lags). T-statistics are reported in parentheses. The difference row in each panel presents the difference in mean risk-adjusted returns between two extreme misvaluation groups with the t-statistics for the difference reported in parentheses.

Misvaluation decile	Mean	CAPM	FF3	FF3 +MOM	FF5	FF5 +MOM	Q	Q +MOM
Panel A: Quintiles of misvaluation								
Most undervalued	0.93	0.26	0.28	0.36	0.49	0.55	0.73	0.73
	(3.36)	(1.79)	(1.90)	(2.44)	(3.27)	(3.71)	(5.10)	(5.09)
Most overvalued	0.23	-0.52	-0.33	-0.47	-0.29	-0.42	-0.51	-0.51
	(0.75)	(-3.24)	(-2.13)	(-3.13)	(-1.81)	(-2.70)	(-3.09)	(-3.14)
Difference	0.70	0.78	0.61	0.83	0.78	0.97	1.24	1.24
	(1.69)	(3.60)	(2.85)	(3.94)	(3.56)	(4.52)	(5.67)	(5.73)
Panel B: Undervalued vs overvalued								
Undervalued	0.97	0.30	0.30	0.35	0.47	0.51	0.64	0.64
	(3.26)	(1.64)	(1.60)	(1.91)	(2.49)	(2.73)	(3.41)	(3.41)
Overvalued	0.26	-0.57	-0.32	-0.39	-0.07	-0.14	-0.26	-0.25
	(0.72)	(-2.68)	(-1.54)	(-1.86)	(-0.32)	(-0.66)	(-1.17)	(-1.16)
Difference	0.71	0.87	0.61	0.74	0.53	0.65	0.89	0.89
	(1.51)	(3.10)	(2.21)	(2.66)	(1.91)	(2.33)	(3.09)	(3.10)
Panel C: Equally-weighted returns								
Most undervalued	1.45	0.72	0.68	0.87	0.98	1.13	1.12	1.11
	(4.42)	(3.49)	(3.92)	(5.18)	(5.78)	(6.99)	(7.20)	(7.77)
Most overvalued	0.19	-0.61	-0.48	-0.51	0.00	-0.04	-0.16	-0.15
	(0.54)	(-3.12)	(-2.82)	(-2.96)	(0.00)	(-0.31)	(-1.15)	(-1.15)
Difference	1.26	1.34	1.17	1.38	0.98	1.18	1.27	1.27
	(2.66)	(4.68)	(4.77)	(5.73)	(4.42)	(5.44)	(6.19)	(6.47)
Panel D: Estimating one parameter								
Most undervalued	1.13	0.46	0.50	0.62	0.69	0.79	0.95	0.95
	(3.77)	(2.47)	(2.60)	(3.24)	(3.57)	(4.11)	(4.97)	(4.98)
Most overvalued	0.21	-0.62	-0.36	-0.45	-0.10	-0.19	-0.31	-0.31
	(0.59)	(-2.91)	(-1.76)	(-2.20)	(-0.48)	(-0.93)	(-1.40)	(-1.41)
Difference	0.91	1.08	0.86	1.07	0.79	0.98	1.26	1.25
	(1.94)	(3.81)	(3.06)	(3.81)	(2.81)	(3.49)	(4.32)	(4.34)

Table 7: Excess returns to portfolios sorted on misvaluation: robustness – continued

Misvaluation decile	Mean	CAPM	FF3	FF3 +MOM	FF5	FF5 +MOM	Q	Q +MOM
Panel E: Estimation based on firms with above-median analyst coverage								
Most undervalued	0.97	0.30	0.30	0.35	0.47	0.51	0.64	0.64
	(3.26)	(1.64)	(1.60)	(1.91)	(2.49)	(2.73)	(3.41)	(3.41)
Most overvalued	0.26	-0.57	-0.32	-0.39	-0.07	-0.14	-0.26	-0.25
	(0.72)	(-2.68)	(-1.54)	(-1.86)	(-0.32)	(-0.66)	(-1.17)	(-1.16)
Difference	0.71	0.87	0.61	0.74	0.53	0.65	0.89	0.89
	(1.51)	(3.10)	(2.21)	(2.66)	(1.91)	(2.33)	(3.09)	(3.10)
Panel F: Estimation with industry misvaluation								
Most undervalued	0.95	0.28	0.26	0.38	0.52	0.62	0.79	0.79
	(3.24)	(1.55)	(1.44)	(2.09)	(2.90)	(3.46)	(4.45)	(4.49)
Most overvalued	0.27	-0.57	-0.34	-0.45	-0.08	-0.19	-0.29	-0.29
	(0.74)	(-2.69)	(-1.66)	(-2.20)	(-0.37)	(-0.91)	(-1.34)	(-1.35)
Difference	0.68	0.84	0.60	0.84	0.60	0.81	1.08	1.08
	(1.46)	(3.06)	(2.19)	(3.03)	(2.20)	(2.97)	(3.85)	(3.90)
Panel G: 3-month returns								
Most undervalued	2.92	0.85	0.90	0.92	1.16	1.20	1.73	1.69
	(5.52)	(2.67)	(2.71)	(2.66)	(3.29)	(3.31)	(4.84)	(4.71)
Most overvalued	1.34	-1.24	-0.53	-0.71	0.18	-0.01	-0.23	-0.33
	(2.00)	(-2.96)	(-1.26)	(-1.63)	(0.41)	(-0.02)	(-0.48)	(-0.68)
Difference	1.58	2.09	1.43	1.64	0.97	1.21	1.97	2.02
	(1.85)	(3.97)	(2.67)	(2.93)	(1.72)	(2.08)	(3.26)	(3.35)
Panel H: 12-month returns								
Most undervalued	10.88	2.03	2.45	4.91	2.11	4.63	5.64	5.89
	(9.21)	(2.70)	(2.93)	(5.68)	(2.11)	(4.62)	(5.30)	(5.60)
Most overvalued	7.10	-5.61	-4.72	-5.62	-2.93	-3.68	-5.52	-5.63
	(2.41)	(-1.96)	(-1.49)	(-1.62)	(-0.77)	(-0.91)	(-1.27)	(-1.29)
Difference	3.78	7.64	7.17	10.53	5.03	8.31	11.16	11.52
	(1.19)	(2.58)	(2.19)	(2.94)	(1.28)	(1.99)	(2.50)	(2.57)

Table 8: Fama-MacBeth regressions of returns on misvaluation measure

This table presents results of estimating cross-sectional regressions of returns on the natural logarithm of misvaluation measure and the following characteristics: the natural logarithm of equity book to market ratio, the natural logarithm of market value, investment-to-assets ratio, gross profitability-to-assets ratio, 1-month past return, and return over the months [-12,-1). All independent variables are winsorized at the 1st and 99th percentiles. See Table A1 for variable definitions. The regressions are estimated monthly. The table reports means of coefficient estimates and their Newey-West-adjusted t-statistics in parentheses. In Column 1, the set of explanatory variables exclude the logarithm of misvaluation. In Column 2, the only explanatory variable is the logarithm of misvaluation. In Column 3, the set of explanatory variables includes both the characteristics above and logarithm of misvaluation.

	(1)	(2)	(3)
Intercept	0.90	0.90	1.06
	(2.69)	(3.35)	(3.24)
log(B/M)	0.36		0.23
	(5.53)		(3.59)
log(ME)	-0.05		-0.07
	(-1.42)		(-2.06)
Investment	-0.99		-0.88
	(-6.76)		(-6.02)
Profitability	0.53		0.35
	(4.37)		(2.70)
Return [-1,0)	0.01		0.01
	(5.13)		(5.47)
Return [-12,-1)	-0.04		-0.04
	(-8.60)		(-8.48)
Log (misvaluation)		-0.45	-0.38
		(-6.95)	(-7.34)
R squared	3.22%	0.42%	3.45%

Table 9: Fama-MacBeth regressions: robustness

This table presents results of estimating cross-sectional regressions of returns on the natural logarithm of misvaluation measure and the following characteristics: the natural logarithm of equity book to market ratio, the natural logarithm of market value, investment-to-assets ratio, gross profitability-to-assets ratio, 1-month past return, and return over the months [-12,-1). All independent variables are winsorized at the 1st and 99th percentiles. See Table A1 for variable definitions. The regressions are estimated monthly. The table reports means of coefficient estimates and their Newey-West-adjusted t-statistics in parentheses. In Column 1, we compute theoretical firm values and resulting misvaluation while estimating the model with only one free parameter, θ, while fixing η at one. In Column 2, we compute theoretical firm values and resulting misvaluation while estimating θ and η using only firms with above-median analyst coverage in their industries. In Column 3, firm-level misvaluation is computed while estimating industry-level misvaluation based on Rhodes-Kropf, Robinson and Viswanathan (2005) procedure. In Column 4, the dependent variable is 3-month return following the formation of decile portfolios. In Column 5, the dependent variable is 12-month return following the formation of decile portfolios.

	Estimating 1 parameter	Above median coverage	Industry misvaluation	3-month returns	12-month returns
Intercept	1.06	1.04	1.05	3.44	17.02
	(3.24)	(3.18)	(3.20)	(4.88)	(10.23)
Log(B/M)	0.24	0.25	0.24	0.64	3.26
	(3.80)	(3.89)	(3.79)	(4.92)	(10.00)
log(ME)	-0.07	-0.07	-0.07	-0.25	-1.32
	(-2.05)	(-1.96)	(-2.06)	(-3.58)	(-8.45)
Investment	-0.89	-0.94	-0.83	-2.12	-6.91
	(-6.48)	(-6.40)	(-6.06)	(-8.00)	(-11.82)
Profitability	0.34	0.35	0.36	0.81	3.89
	(2.61)	(2.73)	(2.77)	(3.46)	(7.01)
Return [-1,0)	0.01	0.01	0.01	0.02	0.02
	(5.47)	(5.42)	(5.61)	(6.74)	(2.05)
Return [-12,-1)	-0.04	-0.03	-0.04	-0.01	0.08
	(-8.58)	(-8.20)	(-8.28)	(-1.56)	(5.77)
Log (misvaluation)	-0.35	-0.33	-0.37	-1.07	-2.41
	(-6.73)	(-6.79)	(-6.79)	(-11.39)	(-9.71)
R squared	3.43%	3.43%	3.48%	3.96%	4.19%

Table 10: DOUBLE SORTS ON MISVALUATION AND THE VALUE OF INVESTMENT OPTIONS

This table reports value-weighted risk-adjusted mean returns of portfolios sorted into misvaluation deciles and independently into terciles of industry median GO/AP ratios (Panel A) and industry median market-to-book ratios (Panel B). GO/AP is the ratio of a firm's value of growth options (GO) to the value of its assets in place (AP). The table reports mean risk-adjusted returns only for the extreme terciles of industry median GO/AP ratio (in Panel A) and industry median M/B ratio (in Panel B), and extreme misvaluation deciles.

Tercile	Misvaluation decile	Mean	CAPM	FF3	FF3 +MOM	FF5	FF5 +MOM	Q	Q +MOM
Panel A: GO/AP Terciles									
Lowest	Most undervalued	1.03	0.36	0.16	0.34	0.17	0.33	0.50	0.49
		(3.08)	(1.51)	(0.68)	(1.50)	(0.73)	(1.44)	(1.97)	(2.02)
	Most overvalued	0.72	-0.06	0.12	-0.03	0.17	0.04	-0.20	-0.19
		(1.31)	(-0.12)	(0.25)	(-0.06)	(0.35)	(0.07)	(-0.38)	(-0.38)
	Difference	0.32	0.42	0.04	0.37	0.00	0.30	0.69	0.69
		(0.49)	(0.79)	(0.07)	(0.69)	(-0.00)	(0.54)	(1.21)	(1.21)
Highest	Most undervalued	1.11	0.42	0.51	0.59	0.83	0.88	1.05	1.05
		(3.29)	(1.78)	(2.16)	(2.46)	(3.48)	(3.69)	(4.48)	(4.48)
	Most overvalued	0.21	-0.62	-0.27	-0.34	0.07	0.01	-0.26	-0.26
		(0.54)	(-2.51)	(-1.18)	(-1.44)	(0.32)	(0.02)	(-1.04)	(-1.04)
	Difference	0.91	1.04	0.79	0.93	0.75	0.88	1.31	1.31
		(1.78)	(3.04)	(2.37)	(2.76)	(2.29)	(2.65)	(3.82)	(3.82)
Panel B: M/B Terciles									
Lowest	Most undervalued	1.17	0.58	0.41	0.53	0.34	0.45	0.58	0.57
		(3.99)	(2.78)	(2.12)	(2.75)	(1.73)	(2.29)	(2.65)	(2.70)
	Most overvalued	0.61	-0.16	0.01	-0.18	0.04	-0.13	-0.31	-0.31
		(1.12)	(-0.33)	(0.03)	(-0.37)	(0.08)	(-0.27)	(-0.62)	(-0.61)
	Difference	0.56	0.74	0.40	0.70	0.30	0.58	0.89	0.88
		(0.91)	(1.42)	(0.78)	(1.38)	(0.58)	(1.11)	(1.61)	(1.61)
Highest	Most undervalued	1.01	0.33	0.43	0.55	0.74	0.84	0.97	0.97
		(3.00)	(1.38)	(1.81)	(2.30)	(3.12)	(3.51)	(4.11)	(4.10)
	Most overvalued	0.02	-0.85	-0.44	-0.48	-0.09	-0.14	-0.32	-0.32
		(0.05)	(-3.53)	(-1.98)	(-2.17)	(-0.40)	(-0.64)	(-1.33)	(-1.33)
	Difference	0.99	1.17	0.87	1.03	0.83	0.98	1.29	1.29
		(1.93)	(3.48)	(2.68)	(3.16)	(2.58)	(3.03)	(3.84)	(3.84)

Table 11: Returns to portfolios sorted on misvaluation of assets in place

This table reports mean monthly risk-adjusted returns of portfolios of firms belonging to misvaluation deciles. The sample period is 1980-2014. Misvaluation is computed according to (15). Portfolio returns are computed as value-weighted mean returns of stocks belonging to a portfolio. Column 1 presents mean returns. Column 2 presents alphas relative to the MKT factor. Column 3 presents alphas relative to MKT, HML, and SMB factors, following Fama and French (1993). Column 4 presents alphas relative to MKT, HML, SMB, and MOM factors, following Carhart (1997). Column 5 presents alphas relative to MKT, HML, SMB, RMW, and CMA, following Fama and French (2015). Column 6 presents alphas relative to MKT, HML, SMB, RMW, CMA, and MOM. Column 7 presents alphas relative to MKT, r_{ME}, $r_{I/A}$, and r_{ROE} factors, following Hou, Xue and Zhang (2015). Column 8 presents alphas relative to MKT, r_{ME}, $r_{I/A}$, and r_{ROE}, and MOM factors. The description of factors is found in Section 4. In all regressions, standard errors are Newey-West adjusted (with 6 lags). T-statistics are reported in parentheses. The difference row presents the difference in mean risk-adjusted returns between two extreme misvaluation deciles with the t-statistics for the difference in parentheses.

Misvaluation decile	Mean	CAPM	FF3	FF3 +MOM	FF5	FF5 +MOM	Q	Q +MOM
1 (Most undervalued)	1.15	0.52	0.23	0.43	0.13	0.31	0.27	0.30
	(2.28)	(1.11)	(0.52)	(0.94)	(0.29)	(0.67)	(0.55)	(0.64)
2	-0.10	-0.82	-1.04	-0.93	-0.88	-0.78	-0.86	-0.85
	(-0.19)	(-1.87)	(-2.41)	(-2.13)	(-1.98)	(-1.74)	(-1.88)	(-1.88)
3	0.47	0.01	-0.12	-0.14	-0.16	-0.17	-0.13	-0.12
	(1.17)	(0.02)	(-0.34)	(-0.37)	(-0.41)	(-0.45)	(-0.33)	(-0.31)
4	1.06	0.50	0.27	0.36	0.19	0.27	0.33	0.33
	(2.71)	(1.46)	(0.79)	(1.05)	(0.52)	(0.76)	(0.89)	(0.91)
5	1.33	0.59	0.43	0.51	0.32	0.40	0.36	0.36
	(3.00)	(1.60)	(1.14)	(1.35)	(0.82)	(1.02)	(0.90)	(0.91)
6	1.04	0.38	0.26	0.36	0.06	0.16	0.07	0.07
	(2.56)	(1.13)	(0.76)	(1.06)	(0.16)	(0.45)	(0.18)	(0.21)
7	0.65	-0.09	-0.18	-0.18	-0.28	-0.28	-0.34	-0.33
	(1.67)	(-0.29)	(-0.57)	(-0.58)	(-0.88)	(-0.87)	(-1.03)	(-1.02)
8	1.04	0.34	0.36	0.33	0.31	0.28	0.13	0.14
	(2.57)	(1.02)	(1.10)	(0.97)	(0.92)	(0.80)	(0.37)	(0.40)
9	0.44	-0.37	-0.27	-0.25	-0.20	-0.18	-0.43	-0.43
	(1.02)	(-1.13)	(-0.82)	(-0.76)	(-0.58)	(-0.53)	(-1.23)	(-1.23)
10 (Most overvalued)	0.63	-0.28	-0.16	-0.21	0.48	0.42	0.27	0.26
	(1.06)	(-0.55)	(-0.32)	(-0.39)	(0.93)	(0.80)	(0.50)	(0.49)
Difference 1-10	0.52	0.80	0.39	0.64	-0.35	-0.10	0.00	0.04
	(0.67)	(1.16)	(0.58)	(0.92)	(-0.50)	(-0.15)	(-0.00)	(0.05)

Table 12: Excess returns in low and high sentiment environment

This table shows the performance of portfolios sorted on misvaluation (Panel A), and misvaluation and growth options (Panel B) during times of low and high sentiment. Sentiment is a non-orthogonalized index of Baker and Wurgler (2006). Panel A reports value-weighted returns of portfolios of most undervalued (lowest misvaluation decile) and most overvalued (top misvaluation decile) stocks and the difference between them in periods of low (below-median) and high (above-median) investor sentiment. Panel B reports value-weighted returns of UMO portfolios (i.e. the difference in returns of the most undervalued decile minus the most overvalued decile), constructed using high-growth-option firms (those with above-median measure of growth options) and low-growth-option firms (those with below-median measure of growth options), during times when our measure of investor sentiment is above and below median. The value of growth options is measured as either industry-median GO/AP ratio or industry-median M/B ratio. A firm's GO/AP is the ratio of a firm's value of growth options (GO) to the value of its assets in place (AP). M/B is the market-to-book ratio. Standard errors are Newey-West adjusted with 6 lags. t-statistics are reported in parentheses.

Panel A: Returns to portfolios sorted on sentiment and misvaluation

Misvaluation decile	Mean	CAPM	FF3	FF3 +MOM	FF5	FF5 +MOM	Q	Q +MOM
Low sentiment								
Most undervalued	1.21	0.36	0.34	0.44	0.46	0.52	0.69	0.64
	(2.89)	(1.43)	(1.38)	(1.80)	(1.84)	(2.11)	(2.70)	(2.54)
Most overvalued	1.09	0.05	-0.03	-0.05	0.11	0.08	0.16	0.23
	(2.25)	(0.21)	(-0.11)	(-0.22)	(0.48)	(0.35)	(0.58)	(0.86)
Difference	0.12	0.30	0.37	0.49	0.35	0.44	0.53	0.41
	(0.18)	(0.85)	(1.09)	(1.46)	(1.05)	(1.32)	(1.43)	(1.13)
High sentiment								
Most undervalued	0.97	0.64	1.07	1.20	1.16	1.27	1.39	1.39
	(1.80)	(1.78)	(2.96)	(3.24)	(3.13)	(3.41)	(3.78)	(3.78)
Most overvalued	-0.71	-1.12	-0.54	-0.55	-0.39	-0.43	-0.55	-0.55
	(-1.24)	(-4.24)	(-2.28)	(-2.29)	(-1.62)	(-1.76)	(-2.13)	(-2.17)
Difference	1.68	1.76	1.61	1.75	1.55	1.70	1.94	1.94
	(2.13)	(3.95)	(3.73)	(3.97)	(3.51)	(3.82)	(4.32)	(4.35)

Table 12: Excess returns in low and high sentiment environment – continued

Panel B: Returns to UMO portfolios sorted on sentiment and growth options

Investment options subsample	Mean	CAPM	FF3	FF3 +MOM	FF5	FF5 +MOM	Q	Q +MOM
Low sentiment								
GO/AP halves								
Low GO/AP	0.25	0.30	0.34	0.53	0.38	0.49	0.46	0.36
	(0.38)	(0.68)	(0.78)	(1.22)	(0.85)	(1.10)	(0.99)	(0.79)
High GO/AP	-0.55	-0.23	-0.16	0.03	-0.38	-0.25	0.14	0.01
	(-0.76)	(-0.52)	(-0.39)	(0.08)	(-0.93)	(-0.62)	(0.31)	(0.02)
M/B halves								
Low M/B	0.15	0.22	0.23	0.40	0.22	0.35	0.33	0.23
	(0.22)	(0.45)	(0.51)	(0.89)	(0.45)	(0.77)	(0.67)	(0.50)
High M/B	-0.22	0.04	0.08	0.25	-0.16	-0.01	0.28	0.19
	(-0.31)	(0.08)	(0.21)	(0.65)	(-0.40)	(-0.03)	(0.63)	(0.43)
High sentiment								
GO/AP halves								
Low GO/AP	0.63	0.64	0.18	0.79	0.80	1.17	1.46	1.46
	(0.86)	(1.20)	(0.33)	(1.45)	(1.43)	(2.15)	(2.62)	(2.68)
High GO/AP	1.59	1.69	1.48	1.54	1.16	1.27	1.61	1.61
	(1.78)	(3.21)	(3.10)	(3.16)	(2.35)	(2.57)	(3.16)	(3.16)
M/B halves								
Low M/B	0.96	0.98	0.42	0.68	0.47	0.67	1.12	1.11
	(1.35)	(2.08)	(0.88)	(1.47)	(0.96)	(1.41)	(2.24)	(2.33)
High M/B	1.78	1.88	1.74	1.72	1.47	1.51	1.80	1.81
	(2.05)	(3.53)	(3.71)	(3.60)	(3.05)	(3.11)	(3.64)	(3.64)

Table A1: Variable definitions

This table defines all variables used in the calibration, estimation, and empirical analysis. The main data sources are Compustat, CRSP, and IBES.

Variable	**Definition**	**Source**
Panel A: Variables used in calibration and estimation		
Physical capital	Gross property, plant and equipment (PP&E), item PPEGT.	Compustat
Pre-tax operating profit	Earnings before interest, taxes, depreciation, and amortization (EBITDA), defined as SALE-COGS-XSGA.	Compustat
Drift of demand process	Estimated perpetual growth rate of demand in an industry. We start with forecasts of the long-term growth (LTG) rates issued by analysts and make various adjustments to it, discussed in Appendix C.	IBES, CRSP, Compustat
Volatility of demand process	Industry-median volatility of residuals of regressions of sales, item SALE, on quarterly dummies over 8 quarters.	Compustat
Depreciation	Industry median ratio of depreciation expense, DP, to lagged gross PP&E, item PPEGT.	Compustat
Panel B: Characteristics related to growth options		
Industry M/B	Industry-median firm pseudo-M/B ratio, which is the ratio of the firm's pseudo-market value and its book assets. The firm's pseudo-market value is the sum of market value of equity, computed as the stock price, PRCC_C, times the number of shares outstanding, CSHO, and book debt. Book debt is defined as the difference between book assets, AT, and book value of equity. Book equity is stockholders' equity, Compustat item TEQ, minus preferred stock plus balance sheet deferred taxes and investment tax credit, TXDITC, if available. If stockholders' equity is missing, we use common equity plus preferred stock par value, CEQ+PSTK. If these variables are missing, we use book assets less liabilities, AT-(DLTT+DLC). Preferred stock is preferred stock liquidating value, PSTKRV, preferred stock redemption value, PSTKL, or preferred stock par value, PSTK, in that order of availability.	CRSP and Compustat
Age	The difference between current year and the year of founding, the year of incorporation, or the year the firm first appears in CRSP files, in that order of availability.	CRSP and Compustat
R&D expenditures	The ratio of R&D expenditures, XRD, to lagged assets, AT.	Compustat
Asset growth	The ratio of book assets, AT, and lagged book assets, minus one.	Compustat
Leverage	The ratio of the sum of long-term debt, DLTT, and short-term debt, DLC, to book assets, AT.	Compustat
Equity beta	Estimated using CAPM for the 60-month period preceding the month of the observation.	CRSP
Standard deviation of returns	Computed using daily returns over a month preceding the month of the observation.	CRSP
NASDAQ dummy	Equals one if the stock exchange indicator, HEXCD, equals three.	CSRP

Table A1: Variable definitions – continued

Variable	Definition	Source
Panel C: Characteristics used in cross-sectional tests		
log (ME)	The natural logarithm of the market value of equity. Market equity is defined as the stock price, PRCC_C at the end of June preceding the month of the observation, times the number of shares outstanding, CSHO.	CSRP
log (equity B/M)	The natural logarithm of the equity book-to-market ratio. Book equity is stockholders' equity, TEQ, minus preferred stock plus balance sheet deferred taxes and investment tax credit, TXDITC, if available. If stockholders' equity is missing, we use common equity plus preferred stock par value, CEQ+PSTK. If these variables are missing, we use book assets less liabilities, AT-(DLTT+DLC). Preferred stock is preferred stock liquidating value, PSTKRV, preferred stock redemption value, PSTKL, or preferred stock par value, PSTK, in that order of availability. Market equity is the stock price at the end of June, PRCC_C,times the number of shares outstanding, CSHO. We match returns from January to June of year t with Compustat-based variables of year $t-2$, while the returns from July until December are matched with Compustat variables of year $t-1$.	CRSP and Compustat
Investment	The difference between gross property plant and equipment, PPEGT, and its lagged value, divided by lagged total assets, AT.	Compustat
Profitabilty	The difference between revenue, SALE, and cost of goods sold, COGS, divided by lagged total assets, AT.	Compustat
Return [-1,0)	Return in month $t-1$ prior to the month of the observation, t.	CRSP
Return [-12,-1)	Return over the 11-month period $[t-12, t-2]$ prior to the month of the observation, t.	CRSP
Panel D: Characteristics used in Rhodes-Kropf, Robinson and Viswanathan (2005) regressions		
log (ME)	Same as above.	CRSP
log (BE)	The natural logarithm of the book value of equity. Book equity is stockholders' equity, TEQ, minus preferred stock plus balance sheet deferred taxes and investment tax credit, TXDITC, if available. If stockholders' equity is missing, we use common equity plus preferred stock par value, CEQ+PSTK. If these variables are missing, we use book assets less liabilities, AT-(DLTT+DLC). Preferred stock is preferred stock liquidating value, PSTKRV, preferred stock redemption value, PSTKL, or preferred stock par value, PSTK, in that order of availability.	Compustat
log (|NI|)	The natural logarithm of the absolute value of net income, NI.	Compustat
$\mathbb{I}$(NI<0)	The indicator variable that equals one if net income is below zero, and equals zero, otherwise.	Compustat
Leverage	The ratio of the sum of long-term debt, DLTT, and short-term debt, DLC, to book assets, AT.	Compustat

Table A1: Variable definitions – continued

Variable	Definition	Source
	Panel E: Other characteristics	
Assets	Book assets, item AT.	Compustat
Market value of equity	Stock price, PRCC_C, times the number of shares outstanding, CSHO.	CRSP
Capital investment	The difference between net PP&E, PPENT, and lagged PPENT, divided by lagged book assets, AT.	Compustat
Profitability	The ratio of operating income after depreciation, OIADP, and sales, SALE.	Compustat
Net issue	The difference between equity issues, SSTK, and repurchases, PRSTK, divided by lagged market value of equity, defined as above.	CRSP and Compustat
Amihud's illiquidity measure	Monthly average of daily ratios of absolute return to dollar trading volume. Measured in dollars.	CRSP
Institutional ownership	The number of all shares held by institutions divided by total shares outstanding.	
Number of analysts	Number of analysts that provide at least one forecast for the firm in the period $[t-3, t-1]$ prior to the month of the observation, t.	IBES
Forecast dispersion	Coefficient of variation of last forecast provided by each analyst in the period $[t-3, t-1]$ prior to the month of the observation, t.	IBES

Table A2: Number of firms used in estimation

This table shows the number of firms we use in estimation of the model and in portfolio formation. Column 3 reports the median number of all firms in a given industry each month from 1980 until 2014. Column 4 reports the median number of firms with positive values of EBITDA, which we use in estimation. The last column reports the average proportion of firms for which we are able to compute the misvaluation measure.

Industry	GICS code	Total number of firms	Number of firms used in estimation	Percent of firms used in estimation
Energy Equipment & Services	101010	74	68	86.1
Oil, Gas & Consumable Fuels	101020	217	148	68.7
Chemicals	151010	104	87	87.4
Construction Materials	151020	18	16	91.7
Containers & Packaging	151030	33	30	91.8
Metals & Mining	151040	129	88	62.8
Paper & Forest Products	151050	30	27	92.1
Aerospace & Defense	201010	84	64	82.4
Building Products	201020	53	43	87.1
Construction & Engineering	201030	44	34	81.7
Electrical Equipment	201040	82	68	75.5
Industrial Conglomerates	201050	10	9	90.6
Machinery	201060	178	151	86.8
Trading Companies & Distributors	201070	27	22	81.0
Commercial Services & Supplies	202010	217	161	78.0
Professional Services	202020	36	32	75.6
Air Freight & Logistics	203010	15	11	70.1
Airlines	203020	23	10	51.6
Marine	203030	9	8	84.4
Road & Rail	203040	47	16	34.0
Transportation Infrastructure	203050	6	4	78.2
Auto Components	251010	59	50	85.7
Automobiles	251020	11	8	79.6
Household Durables	252010	136	102	78.2
Leisure Equipment & Products	252020	49	39	80.7
Textiles, Apparel & Luxury Goods	252030	109	91	85.6
Hotels, Restaurants & Leisure	253010	141	105	76.7
Diversified Consumer Services	253020	25	19	79.1
Media	254010	160	108	68.9
Distributors	255010	66	50	83.1
Internet & Catalog Retail	255020	23	15	68.2
Multiline Retail	255030	36	30	92.5
Specialty Retail	255040	141	125	88.5
Food & Staples Retailing	301010	69	62	92.9
Beverages	302010	25	22	85.3
Food Products	302020	89	76	87.9
Tobacco	302030	7	7	98.4
Household Products	303010	18	16	85.8
Personal Products	303020	40	30	76.3
Health Care Equipment & Supplies	351010	185	108	61.2
Health Care Providers & Services	351020	142	96	64.7
Health Care Technology	351030	19	10	54.5
Biotechnology	352010	162	17	17.7
Pharmaceuticals	352020	85	40	50.5
Life Sciences Tools & Services	352030	32	22	71.3
Internet Software & Services	451010	86	26	50.7
IT Services	451020	85	68	74.7
Software	451030	165	113	63.3
Communications Equipment	452010	129	84	69.5
Computers & Peripherals	452020	96	66	68.7
Electronic Equipment & Instruments	452030	184	136	77.1
Office Electronics	452040	5	2	56.0
Semiconductor Equipment & Products – Discontinued	452050	26	21	72.7
Semiconductors & Semiconductor Equipment	453010	114	83	78.0
Diversified Telecommunication Services	501010	49	24	46.1
Wireless Telecommunication Services	501020	24	13	52.2

Table A3: Industry characteristics

This table shows the composition of industries. Mean growth rate and growth rate volatility are time-series averages of the parameters of industry-specific processes. Depreciation rate is the time-series average of capital depreciation rate in a given industry.

Industry	GICS Code	Number of firms in 1980	Number of firms in 2014	Mean growth rate	Growth rate volatility	Depreciation rate
Energy Equipment & Services	101010	38	70	3.65%	26.45%	7.46%
Oil, Gas & Consumable Fuels	101020	121	143	2.83%	28.78%	6.17%
Chemicals	151010	77	71	2.85%	11.50%	6.36%
Construction Materials	151020	26	7	3.22%	14.78%	5.68%
Containers & Packaging	151030	28	20	2.77%	11.46%	6.93%
Metals & Mining	151040	93	62	2.99%	21.96%	5.41%
Paper & Forest Products	151050	35	15	2.81%	13.24%	4.93%
Aerospace & Defense	201010	67	53	2.95%	15.80%	7.95%
Building Products	201020	57	31	3.24%	14.31%	7.15%
Construction & Engineering	201030	32	25	3.26%	18.57%	9.43%
Electrical Equipment	201040	67	36	3.11%	14.89%	7.71%
Industrial Conglomerates	201050	9	6	2.89%	10.50%	7.96%
Machinery	201060	139	111	3.04%	15.02%	7.35%
Trading Companies & Distributors	201070	13	33	3.14%	11.66%	7.92%
Commercial Services & Supplies	202010	141	65	3.28%	13.68%	8.89%
Professional Services	202020	8	43	3.50%	14.62%	13.82%
Air Freight & Logistics	203010	9	11	3.21%	14.33%	10.08%
Airlines	203020	18	10	3.08%	15.11%	6.93%
Marine	203030	2	22	2.95%	20.41%	5.13%
Road & Rail	203040	21	10	3.00%	12.76%	8.17%
Transportation Infrastructure	203050	4	3	3.04%	22.47%	7.09%
Auto Components	251010	46	35	3.05%	13.93%	7.68%
Automobiles	251020	3	7	2.84%	17.26%	8.62%
Household Durables	252010	113	49	3.17%	15.14%	7.89%
Leisure Equipment & Products	252020	38	16	3.22%	16.09%	9.90%
Textiles, Apparel & Luxury Goods	252030	91	40	3.19%	12.22%	8.84%
Hotels, Restaurants & Leisure	253010	54	88	3.41%	12.06%	6.35%
Diversified Consumer Services	253020	4	27	3.25%	10.27%	9.80%
Media	254010	63	80	3.11%	14.20%	11.31%
Distributors	255010	60	8	3.24%	13.61%	10.30%
Internet & Catalog Retail	255020	3	19	4.43%	12.46%	13.07%
Multiline Retail	255030	37	14	3.05%	5.69%	6.73%
Specialty Retail	255040	73	97	3.53%	8.95%	9.06%
Food & Staples Retailing	301010	68	26	2.99%	5.83%	7.05%
Beverages	302010	12	21	2.78%	10.50%	7.03%
Food Products	302020	73	61	2.67%	9.76%	6.59%
Tobacco	302030	7	7	2.84%	13.68%	6.68%
Household Products	303010	17	12	2.68%	8.20%	6.57%
Personal Products	303020	22	21	2.98%	10.93%	8.72%
Health Care Equipment & Supplies	351010	51	76	3.45%	13.72%	10.63%
Health Care Providers & Services	351020	28	70	3.53%	15.42%	12.36%
Health Care Technology	351030	3	15	4.07%	29.63%	19.92%
Biotechnology	352010	9	21	5.33%	27.44%	9.74%
Pharmaceuticals	352020	31	36	3.16%	14.99%	9.69%
Life Sciences Tools & Services	352030	7	23	3.58%	12.32%	10.20%
Internet Software & Services	451010	6	68	4.70%	39.18%	19.11%
IT Services	451020	20	72	3.42%	14.72%	15.42%
Software	451030	20	88	4.56%	17.81%	17.85%
Communications Equipment	452010	50	61	4.25%	20.60%	12.47%
Computers & Peripherals	452020	45	28	3.93%	18.42%	13.12%
Electronic Equipment & Instruments	452030	80	92	3.79%	16.77%	9.71%
Office Electronics	452040	8	3	4.80%	16.91%	9.97%
Semiconductor Equipment & Products	452050	20	4	4.86%	21.06%	10.93%
Semiconductors & Semiconductor Equipment	453010	19	88	4.58%	23.83%	10.81%
Diversified Telecommunication Services	501010	5	26	2.50%	20.06%	9.75%
Wireless Telecommunication Services	501020	3	9	3.27%	16.06%	11.75%

Table A4: Persistence of estimated parameters θ and η

This table reports means of absolute changes in the estimated curvature of the production function, θ, and the cost of purchasing and installing new capital, η, at 1-month, 3-month, 6-month, 1-year, 2-year, and 3-year horizons. The means are pooled across industries and across months. Column 2 shows the mean of absolute changes in θ at the same horizons. Column 3 reports the mean of absolute changes in η. The last two columns report the ratios of absolute changes in θ and η respectively, and their corresponding mean values.

Lag	Absolute change in θ	Absolute change in η	Percentage absolute change in θ	Percentage absolute change in η
1	0.015	0.027	4.8%	2.3%
3	0.030	0.050	9.5%	4.3%
6	0.046	0.076	14.4%	6.5%
12	0.067	0.111	21.0%	9.4%
24	0.087	0.148	27.0%	12.6%
36	0.090	0.163	28.3%	13.8%