Finding a Good Price in Opaque Over-the-Counter Markets*

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Abstract

This paper offers a dynamic model of opaque over-the-counter markets. A seller searches for an attractive price by visiting multiple buyers one at a time. The buyers do not observe negotiations elsewhere in the market, including the order of contacts. Under stated conditions, a repeat contact with a buyer reveals the seller’s reduced outside options and worsens the price quote. When the value of the asset is uncertain, search exacerbates adverse selection, leading to potential market breakdown. Market fragmentation mitigates search-induced adverse selection by concentrating it elsewhere. Inferences of asset values are less sensitive to information in OTC market than in centralized auctions.

Keywords: over-the-counter market, transparency, search, adverse selection, market breakdown, outside options, bargaining

JEL Classification: G14, C78, D82, D83

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1 Introduction

Trading in many segments of financial markets is over-the-counter (OTC). Compared to exchanges, OTC markets are opaque. For example, in markets for corporate bonds, municipal bonds, mortgage-backed securities, asset-backed securities, and exotic derivatives, firm (executable) prices are usually not quoted publicly, and traders often search for attractive prices by contacting multiple counterparties in sequence. Once a quote is provided, the opportunity to accept it lapses quickly. In corporate bond markets, for example, “Telephone quotations indicate a firm price but are only good ‘as long as the breath is warm,’ which limits one’s ability to obtain multiple quotations before committing to trade” (Bessembinder and Maxwell, 2008). Even when quotes are displayed on electronic systems, they are often indicative and can differ from actual transaction prices.\footnote{For example, Froot (2008) finds large and persistent disparity between quoted prices on Reuters and actual transaction prices. For TRACE-ineligible securities, which include the majority of MBS and ABS, transaction-quote disparity is 200bp for the bottom third of trade under the quote and 100bp for the top third of trades over the quote. The disparity shrinks by only about a half ten days after the trade. For TRACE-eligible securities, the corresponding numbers go down to about 100bp, 50bp, and 0.2, which are still substantial.} Electronic trading, which makes it easier to obtain multiple quotes quickly, is also limited in the markets for many fixed-income securities and derivatives.\footnote{For example, SIFMA (2009) finds that electronic trading accounts for less than 20% of European sell-side trading volume for credit and sovereigns. For interest-rate swaps, credit default swaps and asset-backed securities, the fractions are lower than 10%. Barclay, Hendershott, and Kotz (2006) finds that the market share of electronic intermediation falls from 81% to 12% when U.S. Treasury securities go off-the-run.}

In this paper, I develop a model of opaque OTC markets. A seller, say a distressed investor, wishes to sell an indivisible asset to one of $N > 1$ buyers, say quote-providing dealer banks. There is no pre-trade transparency. The seller must visit the buyers one at a time. When visited, a buyer makes a quote for the asset. The seller may sell the asset to the currently contacted potential buyer, or may turn down the offer and contact another buyer. Because a buyer does not observe negotiations elsewhere in the market, he faces contact-order uncertainty – uncertainty regarding the order in which the competing buyers are visited by the seller. The seller may also make a repeat contact with a previously rejected buyer when a new buyer’s quote is sufficiently unattractive.

I show that the likelihood of repeat contact creates strategic pricing behavior by quote providers. When the seller and buyers have independent private values for owning the asset beyond its commonly known fundamental value, a returning
seller faces no adverse price movement due to fundamental news, but invites adverse inference about the price quotes available elsewhere in the market. For example, a seller may initially refuse an unattractive bid from one buyer, only to learn that other buyers’ bids are even worse. In this case, the seller takes into account the likely inference of the original buyer if she contacts him for a second time. Upon a second contact by the seller, the original buyer infers that the seller’s outside options are sufficiently unattractive to warrant the repeat contact, despite the adverse inference. He revises his bid downward accordingly. The intuition that a repeat contact signals a reduced outside option, and results in a lower offer, is confirmed as the first main result of this paper.

As the second main result of this paper, I show that when the fundamental value of the asset is uncertain, search induces an additional source of adverse selection. To see the intuition, suppose that the seller observes the fundamental value $v$ of the asset, but that buyers only observe noisy signals of $v$. The seller randomly chooses the order of contacts with the buyers. I also assume that buyers have higher private values for owning the asset than the seller, so potential gain from trade is positive.

I show that a buyer’s expected fundamental value of the asset conditional on his own signal and on being visited, $E(v|\text{signal, visit})$, is strictly lower than the expected fundamental value of the asset conditional only on his own signal, $E(v|\text{signal})$, provided $N \geq 2$. Intuitively, the fact that the asset is currently offered for sale means that nobody has bought it so far, which in turn suggests that other buyers may have received pessimistic signals of its fundamental value. Anticipating such adverse selection, a buyer may quote a low price for the asset, even when his own signal indicates that the fundamental value of the asset is high.

As the number of buyers becomes larger, search-induced adverse selection can become sufficiently severe to cause a market breakdown. This result suggests, perhaps surprisingly, that an opaque OTC market could in some cases benefit from fragmentation. That is, a limited number of available counterparties could alleviates adverse selection created by opacity. However, fragmentation does not eliminate adverse selection, but concentrates it in other parts of the market.

Compared to winner’s curse in centralized auctions, search-induced adverse selection is less sensitive to payoff-relevant information. Intuitively, in a first-price auction a higher signal of a buyer translates into a higher bid, and thus a
higher probability of winning. In OTC market, because the seller cannot solicit all buyers’ quotes simultaneously, a higher signal of a particular buyer does not change the search path of the seller nor the inference of it.

This paper seems to offer the first model that captures the joint implications of uncertain contact order, bargaining power, adverse selection, and market opacity. The results of this paper have implications for the price behavior in opaque markets, the dynamics of bargaining power, the value of trading relationships, the role of information in forming inferences, and the regulation and design of market structure, among others.

**Relation to the literature**

The ability of quote providers to revise their quotes upon repeat contacts distinguishes this paper from the literature of sequential search with recall. For example, in the dealer-market models of Biais (1993), de Frutos and Manzano (2002), Yin (2005), and Green (2007), the quote providers commit to their original quotes when the quote seeker returns. In this paper, as in functioning OTC market for financial securities, a rejected quote lapses immediately. Repeat contacts are absent in models based on the “random matching” of an infinite number of buyers and sellers, as in Duffie, Gârleanu, and Pedersen (2005, 2007), Vayanos and Wang (2007), and Vayanos and Weill (2008), among others.

Although search and bargaining have long been studied in the real-estate literature, the real estate markets behave quite differently from the OTC financial markets in several important respects. First, rejected offers in real estates markets can often be recalled (Quan and Quigley, 1991; Cheng, Lin, and Liu, 2008), whereas rejected offers in markets for OTC financial securities cannot (Bessembinder and Maxwell, 2008). Second, while real estate brokers match buyers and sellers as agents (Yinger, 1981; Yavas, 1992), dealers in OTC financial markets trade primarily as principals on their own accounts. Third, outside options in real estate markets depend much on the exogenous arrivals of new buyers or sellers, whereas outside options of quote-seekers in OTC financial markets rely on endogenous, bilateral contacts to a commonly known group of dealers. Finally, although private-value models well capture the idea that each real estate has different appeal to different investors, they may not be suitable to study those OTC financial securities for which adverse selection is a major concern.
Related to this paper, Lauermann and Wolinsky (2010) study the interaction between search and adverse selection. Although we ask a similar question, this paper differs much from theirs in both approach and emphasis. First, in their model the searcher gives the quotes, whereas in my model the searcher receive quotes. Besides realism, this specification also allows me to characterize equilibriums in closed form. Second, while Lauermann and Wolinsky focus on how search affects information aggregation as the market grows large, I emphasize the role of endogenous market fragmentation as a natural response to search-induced adverse selection. Third, while their model is set up only with common value, I also study a private-value setting and characterize how uncertain contact order can affect the bargaining power between counterparties. In matching-based search models with common values, such as Duffie, Malamud, and Manso (2010) and Chiu and Koepl (2010), search-induced adverse selection is absent because contacts are exogenous.

Finally, this paper sheds light on bargaining models with outside options. Relevant papers include Chatterjee and Lee (1998), de Fraja and Muthoo (2000), Gantner (2008), and Fuchs and Skrzypacz (2008), among others. In those models, outside options are often exogenous. A buyer’s quote improves if the seller delays trade and returns to the same buyer, as the delay signals a high valuation of the seller. In contrast, when outside options are endogenous, this paper has the opposite prediction: repeat contact reveals a reduced outside option and worsens the price quotes.

2 Dynamic Search with Repeat Contacts

There is one quote seeker and $N \geq 2$ ex-ante identical quote providers. Everyone is risk neutral. Without loss of generality, suppose that the quote seeker is a seller and the quote providers are buyers. The seller has one unit of an indivisible asset she wishes sell. The seller’s valuation $v_0$ and the buyers’ valuations $v^i$, $i = 1, 2, \ldots, N$ are jointly independent and privately held information. The buyers’ values have identical cumulative distribution function $F_b : [0, \bar{v}_b] \to [0, 1]$ and the seller’s value has cumulative distribution function $F_s : [0, \bar{v}_s] \to [0, 1]$.

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3Private valuations can be interpreted as stemming from inventory positions, hedging needs, margin requirements, leverage constraints, or benefits of control, all of which are likely to be private. A common-value setting is considered in Section 3.
where $F_b$ and $F_s$ are continuous and differentiable.

The seller hires an agent (such as a broker) who contacts buyers on the seller’s behalf. Each contact to a buyer is instantaneous, but incurs a small fixed cost of $c > 0$ that is paid by the seller’s agent. This arrangement can naturally arise from, for example, a long-term brokerage contract in which the seller compensates her agent’s search costs by committing a brokerage fee in advance. In return, the agent’s top priority is to maximize the seller’s expected *gross* surplus. Only after that does the agent try to minimize her own search costs, if possible.\(^4\) Because the interests of the seller and her agents are aligned, for simplicity I refer to them collectively as “the seller.”

The market is over-the-counter. The seller contacts buyers one at a time. Upon a contact, the selected buyer makes a quote for the asset. The seller cannot counteroffer, but can accept or reject the quote. If she accepts the quote, then transaction occurs at the quoted price and the game ends. If she rejects it, then the buyer’s quote lapses immediately and cannot be recalled later. After rejecting a quote, the seller may subsequently contact a new buyer, randomly chosen with equal probability and independently of everything else, or contact an already visited buyer. Upon the next contact, the same negotiation is repeated, and so on. Any contacts between two counterparties is unobservable to anyone else. For simplicity, I will refer to the $n$th buyer visited for the first time by the seller as “the $n$th buyer.”

Figure 1 shows two possible game paths, for $N = 2$. Along these paths, the seller switches buyers after each contact. As shown, the buyers do not observe the order of contacts.

In selecting an equilibrium, I focus on symmetric, perfectly-revealing equilibrium, in which buyers use the same quoting strategies and the first quote of a buyer perfectly reveals his valuation. Without loss of generality, we can restrict our modeling attention to offers that are accepted with strictly positive probability.

Let $\beta_1, \beta_2 : [0, \bar{v}_b] \to \mathbb{R}$ be the symmetric quoting strategies of a buyer with value $v$ upon the first and second contact, respectively. (As we will see, there are

\(^4\)Because brokerage fees are often set at a much lower frequency (say annually) than the frequency of trading (say daily), for any given trade it is a good approximation that the seller does not internalize the search costs. Technically, this separation of gross revenue and search costs bypasses the Diamond paradox (Diamond, 1971).
at most two contacts in equilibrium.) As a starting point, we observe that no buyer strictly increases his offer upon a repeat contact, for otherwise the earlier, lower offer is rejected with probability 1. So for all \( v \), we have

\[
\beta_1(v) \geq \beta_2(v). \tag{1}
\]

Now I proceed to an equilibrium analysis of this market, first for \( N = 2 \) and then for a general \( N \). In both equilibriums, inequality (1) is in fact strict: whenever the second contact is made, \( \beta_1(v) > \beta_2(v) \).
2.1 A market with \( N = 2 \) buyers

Let’s first consider the case of \( N = 2 \). I start from \( \beta_1(v) \geq \beta_2(v) \) and characterize an equilibrium in which \( \beta_2(v) < \beta_1(v) \) whenever a repeat contact is made.

Before a formal analysis, I qualitatively describe the equilibrium and its intuition. The equilibrium has three paths on which transaction occurs, as illustrated in Figures 2 and 3. On the first equilibrium path (Figure 2, top), the seller sells the asset upon the first contact with the first buyer. On the second equilibrium path (Figure 2, bottom), the seller sells the asset upon the first contact with the second buyer. On both paths, a visited buyer cannot perfectly distinguish between the two equilibrium paths, and quotes the same price on both.

On the third equilibrium path (Figure 3), the seller sells the asset upon a repeat contact with the first buyer. A repeat contact occurs when the first buyer’s quote is “bad” and the second buyer’s quote is sufficiently worse. Upon this repeat contact, not only does the first buyer infers the entire payoff-relevant history of the game, he also infers that the values of the seller and the other buyer are sufficiently low to guarantee the return, despite the adverse inference. The revisited buyer lowers his quote accordingly. The seller, now in a worse bargaining position, accepts this lower price.

Formally, let \( v \) be the value of a generic buyer whose strategy is of concern, and \( v' \) the value of the other buyer. For given strategies \((\beta_1,\beta_2)\) of buyers, the seller’s strategy is straightforward.

**Lemma 1.** Given the buyers’ strategies \((\beta_1,\beta_2)\), the seller accepts the first buyer’s first quote of \( b_1 \) if and only if \( b_1 \geq R(b_1,v_0) \), where

\[
R(b_1,v_0) = \mathbb{E}[\max(\beta_2(\beta_1^{-1}(b_1)), \beta_1(v'), v_0)].
\]  

(2)

Conditional on rejecting \( b_1 \), the seller accepts the second buyer’s first quote \( \beta_1(v') \) if \( \beta_1(v') \geq \beta_2(\beta_1^{-1}(b_1)) \) and \( \beta_1(v') \geq v_0 \). Otherwise she returns to the first buyer and accepts his second quote if it is no lower than her value \( v_0 \).

Now we turn to the buyers’ strategies. Observe that \( R(b_1,v_0) \) depends only on \( b_1 \) if \( v_0 \leq \beta_2(\beta_1^{-1}(b_1)) \). Let

\[
\tilde{R}(b_1) = \mathbb{E}[\max(\beta_2(\beta_1^{-1}(b_1)), \beta_1(v'))].
\]

(3)
Figure 2: Equilibrium path 1 and path 2. Top: The seller sells the asset upon the first contact with the first buyer. Bottom: The seller sells the asset upon the first contact with the second buyer. On both paths, the buyer is uncertain of the contact order.
Figure 3: Equilibrium path 3. The seller sells the asset upon the second contact with the first buyer. The buyer infers the contact order.

Because $\hat{R}(0) > 0$ and $\hat{R}(\beta_1(\bar{v}_b)) \leq \beta_1(\bar{v}_b)$, there exists some $b_1 \in (0, \beta_1(\bar{v}_b)]$ such that

$$b_1 = \hat{R}(b_1).$$

In the equilibrium proposed shortly, I will require that $b_1$ be the unique solution of $\hat{R}(b) = b$. If $b_1 \geq b_1$, it is easy to show that there exists some value $U(b_1) \in (\beta_2(\beta_1^{-1}(b_1)), b_1]$ of the seller such that

$$b_1 = R(b_1, U(b_1)).$$

In the equilibrium I will also require that $U(b_1)$ be unique and differentiable. Intuitively, $U(b_1)$ is the value of the “boundary seller” given the first quote of $b_1$: $b_1$ is rejected for $v_0 > U(b_1)$ and accepted for $v_0 \leq U(b_1)$. In particular, $U(b_1) = \beta_2(\beta_1^{-1}(b_1))$ and $U(\beta_1(\bar{v}_b)) = \beta_1(\bar{v}_b)$.

If $b_1 < b_1$, then a first quote of $b_1$ is rejected by all sellers, because $R(b_1, v_0) \geq \hat{R}(b_1) > b_1$ for all $v_0$. We can thus write $U(b) \equiv 0$ for all $b < b_1$. The discontinuity
of $U$ at $b_1$ implies that $\beta_1$ may be discontinuous when a buyer’s value is

$$v \equiv \beta_1^{-1}(b_1).$$

(6)

The first quote $\beta_1(v)$ is nonetheless right continuous at $v$ with left limit.

We further define the following probabilities:

$$\alpha \equiv P(R(\beta_1(v'), v_0) > \beta_1(v')),$$

(7)

$$H(b) \equiv P(R(b, v_0) \leq b),$$

(8)

$$G(b) \equiv P(v_0 \leq b, \beta_2(v') \leq b | R(\beta_1(v'), v_0) > \beta_1(v')),$$

(9)

where $v'$ and $v_0$ are unknown. In equilibrium, $\alpha$ is the probability that the seller rejects the first buyer’s first quote; $H(b)$ is the probability that the seller accepts the first buyer’s first quote $b$; and $G(b)$ is the probability that the seller accepts the second buyer’s first quote of $b$ after rejecting the first buyer.

Before stating the equilibrium, we first prove the following lemma.

**Lemma 2.** For all $x > 0$,

$$\frac{G'(x)}{G(x)} > \frac{f_s(x)}{F_s(x)}.$$

(10)

**Proof.** See Appendix A. $\square$

**Proposition 1** (Repeat contacts in OTC market). The buyers’ quoting strategies $(\beta_1, \beta_2)$ and the seller’s strategy given in Lemma 1 consist of an equilibrium if:

1. A buyer’s second quote $\beta_2(v)$ upon a repeat contact satisfies

$$(v - \beta_2(v))f_s(\beta_2(v)) - F_s(\beta_2(v)) = 0.$$  

(11)

2. A buyer’s first quote $\beta_1(v)$ satisfies

$$(v - \beta_1(v))G'(\beta_1(v)) - G(\beta_1(v)) = 0, \text{ if } v < v,$$

(12)

$$(v - \beta_1(v))\left[\alpha G'(\beta_1(v)) + H'(\beta_1(v))\right] - \left[\alpha G(\beta_1(v)) + H(\beta_1(v))\right] = 0, \text{ if } v \geq v$$

(13)

3. Technical conditions. (i) For all $v > 0$, the functions $(v - b)F_s(b)$, $(v - b)G(b)$, and $(v - b)H(b)$ are strictly concave in $b \in [0, v]$; (ii) $\hat{R}(b) - b$ is
strictly decreasing in \( b \); (iii) \( R(b_1, v_0) - b_1 \) is strictly increasing in \( v_0 \) for \( b \geq b_1 \); and (iv)

\[
(v - b_1)(\alpha G(b_1) + H(b_1)) = (v - \lim_{v \uparrow} \beta_1(v))\alpha G(\lim_{v \uparrow} \beta_1(v)). \tag{14}
\]

Moreover, when this equilibrium exists, repeat contact occurs only if the first quote of the first buyer is strictly below \( b_1 \). Upon a repeat contact, \( \beta_2(v) < \beta_1(v) \) for all \( v > 0 \).

The proof of Proposition 1 is delegated to Appendix A but we outline its main intuition here. First, a revisited buyer quotes as if he is a monopolist because he infers that the other buyer’s value is “too low to compete.” Upon the first contact, however, a buyer faces the potential competition from the other buyers, so he quotes a higher price \( \beta_1(v) \). The technical conditions stated in the proposition guarantee that the first-order approach to profit maximization is valid and that the two thresholds \( b_1 \) and \( U(\cdot) \) are well defined. Equation 14 guarantees profit continuity: a buyer with value \( v \) is indifferent between quoting \( b_1 \) and quoting slightly below it.

Implications of the equilibrium are discussed in Section 2.3.

2.2 Committing to search all \( N \geq 2 \) buyers

I now allow for \( N \geq 2 \) buyers. For tractability, I analyze the equilibrium with one additional institutional assumption: the seller commits to contact all \( N \) buyers before accepting any offer. This commitment is reminiscent of best-execution regulations of broker-dealers operating in OTC markets. For example, the Financial Industry Regulatory Authority (FINRA) of the United States requires that “... in any transaction for or with a customer pertaining to the execution of an order in a non-exchange-listed security, a member or person associated with a member shall contact and obtain quotations from three dealers (or all dealers if three or less) to determine the best inter-dealer market for the subject security.”

Because the seller commits to search all buyers, any quote accepted by the seller comes either from the \( N \)th buyer upon the first contact or from any other buyer upon a repeat contact. We let \( b_j^k \) be the first quote from the \( j \)th buyer upon the \( k \)th contact, where \( 1 \leq j \leq N \) and \( 1 \leq k \leq 2 \).

\[\text{Rule 2320 (f), Best Execution and Interpositioning, to become effective on May 9, 2011.}\]
Proposition 2 (Repeat contact with search commitment). For all \( v > 0 \), let function \( \beta_2(v) \) be implicitly defined by the differential equation

\[
0 = [(v-\beta_2(v))f_s(\beta_2(v)) - F_s(\beta_2(v))]F_b(\beta_2(v))\beta_2'(v) + (v-\beta_2(v))(N-2)f_b(v)F_s(\beta_2(v)),
\]

with boundary condition \( \beta_2(0) = 0 \). Given any \( v \) and \( \beta_2(v) \), let \( \beta_1(v) \) be implicitly defined by

\[
0 = [(v-\beta_1(v))f_s(\beta_1(v)) - F_s(\beta_1(v))]F_b(\beta_1(v))\beta_1'(v) + (v-\beta_1(v))(N-1)f_b(\beta_1(v))F_s(\beta_1(v)) + (v-\beta_1(v))(N-2)f_b(\beta_1(v))F_s(\beta_1(v)).
\]

If \( \beta_2(v) \) and \( \beta_1(v) \) are unique solutions of (15) and (16) and they are strictly increasing in \( v \), then there exists an equilibrium in which:

1. Upon the \( k \)-th contact, the buyers use pricing schedule \( \beta_k, k = 1, 2 \), which satisfies \( \beta_1(v) > \beta_2(v) \) for all \( v > 0 \).

2. The seller searches through \( N \) buyers one by one. After visiting all buyers, the seller leaves the market if \( v_0 > \max(b_N^1, \max_{1 \leq j \leq N-1} \beta_2(\beta_1^{-1}(b^j_1))) \). Otherwise, if \( b_N^1 \geq \max_{1 \leq j \leq N-1} \beta_2(\beta_1^{-1}(b^j_1)) \), then the seller accepts the first quote from the \( N \)th buyer. Otherwise the seller returns to the buyer with the highest first quote and accepts his second quote.

Proof. See Appendix A.

The intuition of Proposition 2 is similar to that of Proposition 1. Upon the first contact, a buyer faces competition from all other \( N - 1 \) buyers. However, once the seller returns, the revisited buyer infers that the value of the \( N \)th buyer is too low to be competitive, leaving \( N - 2 \) competitive buyers (hence the \( N - 2 \) in (15)). As a result, a buyer lowers his quote upon a repeat contact, to exploit the reduced outside option of the seller.

One interesting special case of Proposition 2 is when \( N \geq 3 \) and the seller’s value \( v_0 \) is commonly known to be zero. This case allows explicit analytical form of the buyers’ second quotes, as shown the following corollary.

Corollary 1. Suppose that \( N \geq 3 \) and the seller’s value is commonly known to
be zero. For all \( v > 0 \), a buyer’s second quote is given by
\[
\beta_2(v) = \int_{u=0}^{v} udF_b(u)^{N-2} = \mathbb{E}\left[Z^{(1)}|Z^{(1)} < v\right]. \tag{17}
\]
where \( Z^{(1)} \) be the first order statistic of the values of the other \( N - 2 \) buyers (excluding the \( N \)th buyer). Moreover, \( \beta_2(v) \) is strictly increasing in \( v \). For all \( v \geq 0 \), the first quote \( \beta_1(v) \) satisfies
\[
0 = -F_b(\beta_2^{-1}(\beta_1(v)))\beta_2'(\beta_1(v)) + (v - \beta_1(v))(N - 1)f_b(\beta_2^{-1}(\beta_1(v))).
\]

Proof. See Appendix A. \qed

With \( v_0 = 0 \), we can explicitly compare the equilibrium of Corollary 1 to a first price auction in which the seller obtains quotes from all buyers simultaneously. By standard result in auction theory, a buyer’s bid in a first-price auction is
\[
\beta_a(v) = \mathbb{E}\left[Y^{(1)}|Y^{(1)} < v\right], \quad \forall v > 0,
\]
where \( Y^{(1)} \) is the first order statistic of the values of the other \( N - 1 \) buyers.\(^6\) It is straightforward to verify that \( \beta_2(v) < \beta_a(v) \) for all \( v > 0 \). Intuitively, because \( Z^{(1)} < Y^{(1)} \) with positive probability, less competition leads to less competitive quotes.

Example. Suppose that the distribution of buyers’ values has the form \( F_b(v) = v^\gamma, v \in [0, 1] \), for some \( \gamma > 0 \). The seller’s value is commonly known to be zero. By (17), the buyer’s second quote is
\[
\beta_2(v) = \frac{v^{(N-2)\gamma+1} - \int_{u=0}^{v} u^{(N-2)\gamma}du}{v^{(N-2)\gamma}} = \frac{(N - 2)\gamma}{(N - 2)\gamma + 1} v.
\]
A similar calculation gives the auction bid
\[
\beta_a(v) = \frac{(N - 1)\gamma}{(N - 1)\gamma + 1} v.
\]
\(^6\)We can derive this result as follows. Suppose that the symmetric bidding strategy is \( \beta_a(v) \). Maximizing \((v - b)F_b(\beta_a^{-1}(b))^{N-1}\) gives the first-order condition \( 0 = -F_b(v)^{N-1}\beta_a'_{\beta_a}(v) + (v - \beta_a(v))(N - 1)f_b(\beta_a^{-1}(\beta_a(v))) \), which is rearranged to \( d[F_b(v)^{N-1}\beta_a(v)]/dv = vd(F_b(v)^{N-1})/dv \). Integrating both sides from 0 to \( v \), we have \( \beta_a(v) = F_b(v)^{-(N-1)}\int_{u=0}^{v} udF_b(u)^{N-1} = \mathbb{E}\left[Y^{(1)}|Y^{(1)} < v\right] \).
Given $\beta_2$, the first quote $b_1$ satisfies

\[
0 = - \left( \frac{(N - 2)\gamma + 1}{(N - 2)\gamma} b_1 \right)^\gamma \left( \frac{(N - 2)\gamma + 1}{(N - 2)\gamma + 1} + (v - b_1)(N - 1)\gamma \right) \left( \frac{(N - 2)\gamma + 1}{(N - 2)\gamma} b_1 \right)^{\gamma - 1},
\]

which reduces to

\[
b_1 = \beta_1(v) = \frac{(N - 1)\gamma}{(N - 1)\gamma + 1} v = \beta_a(v).
\]

2.3 Discussion and implications

In the equilibriums of Proposition 1 and Proposition 2, a repeat contact leads to a worse price. When the trading direction of the quote seeker is unobservable to the quote providers, a worse price is reflected in a wider bid-ask spread. This effect is not captured by existing models of auctions and bargaining. In auctions, repeat contacts are absent because contacts are simultaneous. Although bargaining allows for repeat contacts, existing models of bargaining emphasize the role of time discount in screening valuations, whereby a delay signals a “strong” valuation and thus a repeat contact improves the price offered to the quote seeker. When time discount is infinitesimal, these models predict that quotes barely move upon a repeat contact. In the model of this paper, however, outside options are endogenous and a repeat contact implies a strictly worse quote.

Because repeat contacts reveal valuable information regarding outside options, a financial institution may benefit by keeping a complete history of its interactions with clients. Indeed, many broker-dealers organize their traders by specialization, whereby all transactions of a particular security are handled by one trader. This specialization reduces the opportunity for customers to avoid repeated contacts by contacting a different person in the same firm.

The results here also predict positive correlations between the quotes of different counterparties, even when these counterparties have independent valuations and have no means of direct communication. A sufficiently unattractive quote from a new counterparty is likely to be followed by another unattractive quote from the original counterparty, due to the inference drawn from a repeat contact. In effect, traders on one side of the market learn something about each other’s valuations by interacting with the other side of the market. This learning generates positive correlation of the prices offered across the markets.

The sequential nature of search implies that allocation may not always be
efficient in OTC market. In the case with two buyers, for example, the seller may sell the asset to the first buyer without visiting the second one, who happens to have a higher valuation. It is also possible that the seller learns that the second buyer’s value is lower but nonetheless trades with him, because she does not want to suffer the lower second quote from the first buyer upon a revisit. The equilibrium with \( N \) buyers has a similar efficiency loss. In contrast, a standard auction is allocative efficient, as it awards the asset to the buyer with the highest bid.

Because the seller’s asset does not always end up with the buyer who values it most, buyers can re-trade among themselves after dealing with the seller. For example, a dealer who enters a swap or foreign-exchange trade with a corporate customer may enter an offsetting trade in the inter-dealer market. The more efficient the initial trade with customer, the less inter-dealer trading we expect to see. In the foreign exchange market, for example, the percentage of inter-dealer trading in total turnover has declined steadily from 63\% in 1998 to 53\% in 2004 to 39\% in 2010, according to Bank for International Settlements (2010). Perhaps this trend is not that surprising, given that the growth of electronic trading during the past decade makes it easier to access multiple quotes simultaneously, at least in liquid currencies.

The efficiency gain associated with easier access to multiple dealer quotes has implications for the design of electronic trading systems of (standard) OTC derivatives, as required by the Dodd-Frank Bill of United States and a proposed rule by the European Commission. In the U.S., the Commodity Futures Trading Commission (2011) states that “to ensure that multiple participants have the ability to reach multiple counterparties, the Commission proposes to require SEFs [Swap Execution Facilities] to provide that market participants transmit a request for quote to at least five potential counterparties in the trading system or platform.” The model of this paper suggests that these requirements can enhance competition, leading to more efficient allocations and a lower percentage of inter-dealer trading in the swap contracts concerned.
3 Adverse Selection with Common Value

So far, I have analyzed a model of an opaque OTC market with private values. In this section, I incorporate a common value in the model, and examine the interplay between uncertain contact order, market opacity, and adverse selection.

The market structure is as the same as before. The seller searches through \( N \) buyers one by one at a random order, and each contact is instantaneous and unobservable to anyone except the two counterparties involved. The value \( v \) of the asset has a binomial distribution

\[
P(v = V_H) = p_H, \quad P(v = V_L) = p_L = 1 - p_H,
\]

for \( V_H > V_L \geq 0 \). The binomial distribution of asset value is quite standard in models of adverse selection and is tractable. The seller observes \( v \) perfectly but the buyers do not. Instead, conditional on \( v \), buyers receive i.i.d. signals \( s \) with a continuous distribution \( F(\cdot|v) : [0, \bar{s}] \to [0, 1] \), where \( 0 < \bar{s} \leq \infty \). The probability densities satisfy the monotone likelihood ratio property (MLRP)

\[
\frac{d}{ds} \left( \frac{f(s|v = V_H)}{f(s|v = V_L)} \right) > 0, \quad \text{for all } s \in [0, \bar{s}].
\]

Intuitively, higher signals are more likely to occur if the asset value is higher. For simplicity, I write \( F_\theta(s) \equiv F(s|v = V_\theta) \) and \( f_\theta(s) \equiv f(s|v = V_\theta) \), for \( \theta = H, L \). A standard result is that MLRP implies first-order stochastic dominance:

\[
F_H(s) < F_L(s), \quad \text{for all } s \in (0, \bar{s}),
\]

a proof of which is provided in Appendix A. Furthermore, all buyers have the same value \( v \) for the asset (hence common value) and the seller values the asset at \( Dv \) for some commonly know \( D < 1 \), where \( D \) can be viewed as a measure of the potential gain from trade.

Conditional on receiving a signal of \( s \), the distribution of \( v \) is uniquely determined by the likelihood ratio

\[
\frac{P(v = V_H|s)}{P(v = v_L|s)} = \frac{p_H}{p_L} \cdot \frac{f_H(s)}{f_L(s)},
\]

where the first fraction is the prior, and the second is the information embodied
in the signal $s$. To rule out some trivialities, I further assume that
\[
p_H V_H + p_L V_L < D V_H < \frac{p_H f_H(\bar{s}) V_H + p_L f_L(\bar{s}) V_L}{p_H f_H(\bar{s}) + p_L f_L(\bar{s})}.
\]
(22)
That is, adverse selection is sufficiently severe so that the ex-ante expected asset value is lower than the high-value seller’s reservation price. However, a monopolist buyer who receives the most optimistic signal can nonetheless purchase the asset from a high-value buyer at a price of $D V_H$.

**Proposition 3** (Search-induced adverse selection). Under condition (22), there exists a signal $s^* \in (0, \bar{s})$, implicitly defined by
\[
J(s^*, N) = \frac{p_H}{p_L} \cdot \frac{f_H(s^*)}{f_L(s^*)} \cdot \sum_{k=0}^{N-1} F_H(s^*)^k \sum_{k=0}^{N-1} F_L(s^*)^k = \frac{D V_H - V_L}{(1 - D)V_H}.
\]
(23)
If such cutoff signal $s^*$ is unique, then there exists an equilibrium in which:

1. If a buyer receives a signal of $s \geq s^*$ then he quotes a price of $D V_H$, otherwise he quotes a price of $V_L$.

2. A seller searches through $N$ buyers one by one and accepts the first quote that is at least $D V_H$. If no buyer quotes $D V_H$ or above, then a seller with a high-value asset leaves the market, whereas a seller with a low-value asset accepts a quote of $V_L$ from the last buyer.

In this equilibrium, conditional on being visited by the seller, a buyer with a signal of $s \in (0, \bar{s})$ assigns the likelihood ratio
\[
I_{OTC}(s, N) = \frac{\mathbb{P}(v = V_H|s, \text{visit})}{\mathbb{P}(v = V_L|s, \text{visit})} = \frac{p_H}{p_L} \cdot \frac{f_H(s)}{f_L(s)} \cdot \frac{\sum_{k=0}^{N-1} F_H(s^*)^k}{\sum_{k=0}^{N-1} F_L(s^*)^k}.
\]
(24)
Moreover, if $\partial J(s^*, N)/\partial s^* > 0$ for all $N$, then the equilibrium cutoff signal $s^*$ is strictly increasing in $N$ and the inference $I_{OTC}(s, N)$ is strictly decreasing in $N$.

**Proof.** See Appendix A.

The intuition of the equilibrium of Proposition 3 is simple. The first fraction on the right hand side of (24) is the prior belief, the second is the informative update from the signal, and the third is search-induced adverse selection. Because $F_H(s^*) < F_L(s^*)$, $\sum_{k=0}^{N-1} F_H(s^*)^k / \sum_{k=0}^{N-1} F_L(s^*)^k < 1$. Intuitively, because
a buyer is more likely to be searched when quotes elsewhere are low, he puts a higher weight on event \(\{v = V_L\}\) than on event \(\{v = V_H\}\). Put simply, being searched is bad news for the value of the asset.

Search-induced adverse selection can be particularly acute when the number of buyers \(N\) becomes large. As the market becomes larger, a visiting seller could have contacted more buyers, which in turn suggests that more buyers have received low signals. To mitigate this adverse selection, buyers impose an ever higher cutoff signal \(s^*\). In the limit, search-induced adverse selection dominates any informative signal except the most optimistic one \(\bar{s}\). With probability 1, therefore, no buyer will quote the high price of \(DV_H\) and the market breaks down.

**Proposition 4** (Breakdown of large opaque markets). Suppose that (22) holds and \(\partial J(s, N)/\partial s > 0\) for all \(N\). Then in the equilibrium of Proposition 3:

1. As \(N \to \infty\), \(s^* \to \bar{s}\).
2. For all \(s < \bar{s}\), \(\lim_{N \to \infty} \mathbb{E}[v|s, \text{visit}] < DV_H\).
3. If \(f_H(\bar{s})/f_L(\bar{s}) < \infty\), then \(\lim_{N \to \infty} \mathbb{E}[v|\bar{s}, \text{visit}] = DV_H\).

**Proof.** See Appendix A. □

**Example.** Let \(F_H(s) = s^2\) and \(F_L(s) = s\) for \(s \in [0, 1]\). Also let \(V_H = 1\), \(V_L = 0\), \(D = 0.6\), and \(p_H = p_L = 0.5\). We leave \(N\) as a free parameter. It is easy to check that (22) holds. For these parameters, (23) reduces to

\[
\frac{2s^*(1 + s^*N)}{1 + s^*} = \frac{3}{2},
\]

the left hand side of which is strictly increasing in \(s^*\) for all \(N\). So for each \(N\), there exists a unique cutoff signal \(s^*\), increasing in \(N\), to pin down the equilibrium strategy of Proposition 3. Figure 4 plots the cutoff signal \(s^*\) as a function of \(N\). As \(N\) becomes larger, an ever higher cutoff signal is required for a buyer to quote the high price of \(DV_H\). With a relatively optimistic signal of \(s = 0.9\), this buyer finds it unprofitable to purchase the asset at a price of \(DV_H\) as long as \(N > 5\).

Proposition 4 suggests that restricting the size of the market mitigates search-induced adverse selection. For example, the seller can commit to visit a “favored” group of \(K\) buyers before visiting the “disfavored” group of the other \(N - K\)
Figure 4: Cutoff signal $s^*$ in the equilibrium of Proposition 3, for $F_H(s) = s^2$ and $F_L(s) = s$, $s \in [0, 1]$. Model parameters: $V_H = 1$, $V_L = 0$, $D = 0.6$, and $p_H = p_L = 0.5$.

buyers. In each group, the seller assigns a random contact order. Now the favored group lowers the cutoff signal, due to a shorter expected search path of the seller. The disfavored group, however, is now certain that the seller has visited at least $K$ buyers before visiting them. They thus assign a strictly higher cutoff signal. Fragmentation of opaque market concentrates adverse selection, rather than eliminates it.

**Proposition 5 (Concentrating adverse selection by fragmentation).** Suppose that (22) holds and $\partial J(s^*, N)/\partial s^* > 0$ for all $N$. Also suppose that all $N$ buyers are partitioned into a favored group of $K$ buyers and a disfavored group of $N - K$ buyers. If

$$\frac{p_H}{p_L} \cdot \frac{f_H(s)}{f_L(s)} \cdot \frac{\sum_{k=0}^{N-K-1} F_H(s)^k}{\sum_{k=0}^{N-K-1} F_L(s)^k} \cdot \frac{F_H(s_K^*)^K}{F_L(s_K^*)^K} \geq \frac{D V_H - V_L}{(1 - D) V_H},$$

then there exists a cutoff signal $s_{N-K}^* \in [0, s]$ for the disfavored group, implicitly defined by

$$\frac{p_H}{p_L} \cdot \frac{f_H(s_{N-K}^*)}{f_L(s_{N-K}^*)} \cdot \frac{\sum_{k=0}^{N-K-1} F_H(s_{N-K}^*)^k}{\sum_{k=0}^{N-K-1} F_L(s_{N-K}^*)^k} \cdot \frac{F_H(s_K^*)^K}{F_L(s_K^*)^K} = \frac{D V_H - V_L}{(1 - D) V_H}.$$

If $s_{N-K}^*$ exists, it satisfies

$$s_K^* < s_N^* < s_{N-K}^*.$$

20
where $s^*_N$ is the original cutoff signal for $N$ buyers and $s^*_K$ is the cutoff signal for the favored group of $K$ buyers.

Proof. See Appendix A.

The last term on the left hand side of (26) reflects a disfavored buyer’s inference: The seller visits only because she has searched through all the favored group and they all have received signals below $s^*_K$. Because of this additional source of adverse selection, the disfavored group of buyers require a higher cutoff signal $s^*_{N-K}$. When (25) does not hold, even the most optimistic signal $ar{s}$ cannot persuade a disfavored buyer to buy the asset at a price of $DV_H$.

I now compare search-induced adverse selection in OTC market to winner’s curse in centralized first-price auctions.

3.1 Search versus first-price auction

Among centralized trading mechanisms, a first-price auction is the most natural counterpart to the OTC market considered in this paper. Both markets are opaque in the sense that quotes (respectively, bids) are not observed publicly. The difference is, of course, that buyers compete simultaneously with each other in the auction but only intertemporarily in an OTC market with search.

Given the binomial distribution of the asset value, a natural measure of a buyer’s inference regarding the asset value in a first-price auction is a likelihood ratio, analogous to (24). Suppose that buyer use symmetric bidding strategy $\beta_A(s)$ that is strictly increasing in signal $s$. Then in equilibrium, the winning buyer with signal $s$ assigns a likelihood ratio

$$I_A(s, N) = \frac{\mathbb{P}(v = V_H | s, \text{win})}{\mathbb{P}(v = V_L | s, \text{win})} = \frac{p_H}{p_L} \cdot \frac{f_H(s)}{f_L(s)} \cdot \frac{F_H(s)^{N-1}}{F_L(s)^{N-1}}. \quad (28)$$

The first two fractions on the right hand side of (28) are, as in the OTC market, the prior and signal update, whereas the last fraction $F_H(s)^{N-1}/F_L(s)^{N-1}$, strictly lower than 1, represents the winning bidder’s adverse inference that all other $N-1$ buyers’ signals are strictly lower – the familiar “winner’s curse.”

How does winner’s curse compare with search-induced adverse selection in OTC market? A natural measure of this comparison is the difference between
the logs of the respective likelihood ratios

\[
\log I_A(s, N) - \log I_{OTC}(s, N) = \log \left( \frac{\sum_{k=0}^{N-1} F_H(s)^k}{\sum_{k=0}^{N-1} F_L(s)^k} \right).
\]

By taking the log, we focus on the adverse selection part of the inferences, as the prior and the signal are the same between the two market structures.

**Proposition 6** (OTC versus first-price auction). *Buyers’ inferences on asset value are more sensitive to signals in the first-price auction than in the OTC market, that is,*

\[
\frac{d}{ds} \log I_A(s, N) - \frac{d}{ds} \log I_{OTC}(s, N) > 0.
\]  

(29)

*Moreover, for any cutoff signal \(s^*\) given by Proposition 3, there exists a unique auction cutoff signal \(s^A \in (s^*, \bar{s})\), such that the expected asset value conditional on receiving \(s^A\) and winning the auction is equal to \(DV_H\). That is, \(\mathbb{E}(v|s^A, \text{win}) = DV_H\).*

**Proof.** See Appendix A. \(\square\)

**Figure 5** plots the log likelihood ratios \(\log I_A(s, N)\) and \(\log I_{OTC}(s, N)\) as a function of \(s\), for \(N = 4\) buyers. Distribution functions and model parameters are the same as in **Figure 4**. For any given signal \(s\) (the plot only shows \(s \geq 0.7\)), the log likelihood ratio in auction market is steeper than that in OTC market. Since the two likelihood ratios cross each other, there is no simple ranking of adverse selection between the two market structures. In addition, for a buyer to be willing to pay \(DV_H\) for an asset, it takes a higher cutoff signal in the auction than in the OTC market. As \(N \to \infty\), a buyer’s expected asset value condition on winning the first-price auction is lower than \(DV_H\) unless he receives the highest possible signal \(\bar{s}\) – an outcome similar to search in OTC market.

The information sensitivity result of **Proposition 6** has a simple intuition. Due to the lack of simultaneous contacts in the OTC market, the probability of being visited is solely determined by quotes from a subset of buyers (those already visited). This search-induced adverse selection, captured by the last fraction of (24), applies regardless of a buyer’s signal \(s\). By comparison, winning an auction unambiguously tells that the winner has the highest signal. When a buyer’s signal is very high, say close to \(\bar{s}\), he wins with probability close to 1 regardless of the true asset value, so adverse selection is not as severe. However, when a
buyer’s signal is very low, say close to 0, winning is much more likely when the asset value is low, so adverse selection is very severe. Put simply, search-induced adverse selection in the OTC market has a large “overhead charge” but a small “incremental punishment” for lower signals, whereas winner’s curse in auction markets has no overhead charge but a large incremental punishment for lower signals.

3.2 Discussion and implications

Proposition 6 suggests that payoff-relevant information has a larger impact on quoted prices in first-price auctions than in OTC market. Increasing the number of counterparties that can be accessed simultaneously will, the model predict, increase the cross-section dispersion of quoted prices. In addition, because larger price sensitivity to information leads to larger potential profits or losses, simultaneous contacts also encourage more information acquisition by the quote providers. These predictions have implications on the potential impact of new regulations of OTC derivatives, such as the Commodity Futures Trading Commission (2011) proposal to move bilateral OTC trading of standard OTC derivatives onto a more competitive request-for-quote system.

Although information aggregation in large market (as \( N \to \infty \)) is a key issue in centralized auctions (Wilson, 1977; Milgrom, 1979; Kremer, 2002), I put less
emphasis on it in the OTC market considered here for two reasons. First, results on information aggregation often rely on restrictive conditions on the conditional distribution of informative signals. For example, in the context of the model of this section, to completely distinguish the $H$ state from the $L$ state, there must be some signal that is infinitely more likely to happen under the $H$ state than under the $L$ state. With MLRP, this restriction amounts to $f_H(\bar{s})/f_H(\bar{s}) = \infty$, a rather delicate condition. Second, when search cost is positive (no matter how small), searching for infinitely many counterparties (in expectation) may not always be practical.

Instead, I interpret the breakdown of large opaque markets (Proposition 4) as a potential explanation for endogenous market fragmentation (Proposition 5). Fragmentation overcomes search-induced adverse selection by effectively limiting the potential number of counterparties on any path of search. For the same quote seeker, Proposition 5 predicts that committing to a favored counterparty improves the prices offered by that counterparty, but worsens prices offered by other disfavored counterparties in the event that the favored one receives sufficiently pessimistic signals of asset quality. This prediction is testable in security trading, bank lending, and other OTC markets for which adverse selection is of concern. For example, Bharath, Dahiya, Saunders, and Srinivasan (2009) finds that repeated borrowing from the same lender lowers the loan spread by 10-17 basis points. Although this finding is also consistent with costly information acquisition, information acquisition does not predict that other banks give a worse loan spread to a customer who always go to a competitor, compared to a customer who selects a bank randomly on each loan. The model of this paper makes the latter prediction, which can be potentially tested against the hypothesis of information acquisition.

This fragmented structure of an OTC market looks quite similar to the structure of many OTC-traded assets, such as mortgage-backed securities (MBS), asset-backed securities (ABS), and collateralized debt obligations (CDOs). Both market structures involve creating liquidity by “pooling and tranching” (DeMarzo, 2005). In the fragmented market considered here, the seller “tranches” the pool of counterparties, just as a CDO structure tranches the pool of underlying assets. Liquidity is created in the favored group of buyers, just like the liquidity created for the senior tranche of a CDO. Adverse selection, however, is not eliminated, but transferred and concentrated to other parts of the market.
4 Conclusion

This paper offers a model of opaque over-the-counter markets. Quote seekers search for an attractive price by contacting quote providers in sequence, and possibly repeatedly. I show that a repeat contact to a counterparty reveals a quote seeker’s reduced outside options and worsens the quote from the revisited counterparty.

I also show that the combined effects of market opacity and contact-order uncertainty can create a search-induced adverse selection. As the market grows in size, search-induced adverse selection can become sufficiently severe to cause a market breakdown. Market fragmentation mitigates adverse selection in some parts of the market, but exacerbates it in others. The results here also suggest that information plays a smaller role in forming beliefs and inferences in OTC market than in centralized first-price auctions.
Appendix

A Proofs

Proof of Lemma 2. When \( x \leq \beta_2(v) \),

\[
v_0 \leq x, \beta_2(v') \leq x \Rightarrow R(\beta_1(v'), v_0) > \beta_1(v').
\]

So

\[
G(x) = \frac{1}{\alpha} F_s(x) F_b(\beta_2^{-1}(x)),
\]

\[
\frac{G'(x)}{G(x)} = \frac{f_s(x) F_b(\beta_2^{-1}(x)) + F_s(x) f_b(\beta_2^{-1}(x)) \frac{d\beta_2^{-1}(x)}{dx}}{F_s(x) F_b(\beta_2^{-1}(x))} > \frac{f_s(x)}{F_s(x)},
\]

where \( \alpha \) is given by (7).

When \( x > \beta_2(v) \),

\[
G(x) = \frac{1}{\alpha} \left[ F_s(\beta_2(v)) F_b(v) + \int_{v_0=\beta_2(v)}^{x} \int_{v'=0}^{\beta_1^{-1}(U^{-1}(v_0))} f_s(v_0) f_b(v') dv_0 dv' \right],
\]

where we have used the fact that, for \( \beta_2(v) < v_0 \leq x \),

\[
R(\beta_1(v'), v_0) > \beta_1(v') \Rightarrow v' < \beta_1^{-1}(U^{-1}(v_0)) \leq \beta_2^{-1}(v) \leq \beta_2^{-1}(x),
\]

where the second to last inequality follows from the property of \( U \), defined in (5).

Then

\[
\frac{G'(x)}{G(x)} > \frac{f_s(x)}{F_s(x)}
\]

if and only if

\[
F_b(\beta_1^{-1}(U^{-1}(x))) F_s(x) > F_s(\beta_2(v)) F_b(v) + \int_{v_0=\beta_2(v)}^{x} \int_{v'=0}^{\beta_1^{-1}(U^{-1}(v_0))} f_s(v_0) f_b(v') dv_0 dv',
\]

which holds true because its left hand side less its right hand side is

\[
\int_{v'=\beta_2}^{U(\beta_1(v'))} \int_{v_0=0}^{\beta_1^{-1}(U^{-1}(x))} f_s(v_0) f_b(v') dv_0 dv' > 0.
\]
Proof of Proposition 1.

Step 1: Second contact. Observe that if \( R(b_1, v_0) > b_1 \geq b_1 \), then \( v_0 > \beta_2(\beta^{-1}_1(b_1)) \), as otherwise \( R(b_1, v_0) = \hat{R}(b_1) \leq b_1 \), by the monotonicity of \( \hat{R}(b) - b \).

Thus a seller who rejects a quote of \( b_1 \geq b_1 \) never returns to the same buyer for a second time.

So a repeat contact occurs if and only if all the following conditions hold: the first buyer’s first quote \( \beta_1(v) \leq b_1 \); the second buyer’s first quote \( \beta_1(v') < \beta_2(v) \); and the seller’s value \( v_0 \leq \beta_2(v) \). Because \( \beta_2(v') < \beta_1(v') \) in equilibrium, there exists an \( \epsilon > 0 \) such that \( b_2 > \beta_2(v') \) for all \( b_2 > \beta_2(v) - \epsilon \). That is, \( \mathbb{P}(b_2 > \beta_2(v')|\beta_2(v) > \beta_1(v')) = 1 \) in an open neighborhood of \( \beta_2(v) \).

The first buyer thus quotes as a monopolist. He solves

\[
\max_{b_2}(v - b_2)P(v_0 \leq b_2|v_0 \leq \beta_2(v)) = \max_{b_2}(v - b_2)\frac{F_s(b_2)}{F_s(\beta_2(v))}. \tag{30}
\]

Substituting \( b_2 = \beta_2(v) \) into the first-order condition, we have (11).

Step 2: First-order conditions of first contact. To calculate the first quote of a buyer whose value is \( v \), I separately consider \( v < v \) and \( v \geq v \), since \( \beta_1 \) may not be continuous at \( v \).

Given the seller’s strategy, a first quote of \( b_1 < b_1 \) is accepted only if he is the second buyer visited. The buyer solves

\[
\max_{b_1}(v - b_1)G(b_1) = \max_{b_1} \Pi(b_1), \tag{31}
\]

whose first-order condition is

\[
\Pi'(\beta_1(v)) = (v - \beta_1(v))G'(\beta_1(v)) - G(\beta_1(v)) = 0. \tag{32}
\]

Using Lemma 2 and (11), we have

\[
\Pi'(\beta_2(v)) = G(\beta_2(v)) \left[ (v - \beta_2(v)) \frac{G'(\beta_2(v))}{G(\beta_2(v))} - 1 \right] > G(\beta_2(v)) \left[ (v - \beta_2(v)) \frac{f_s(\beta_2(v))}{F_s(\beta_2(v))} - 1 \right] = 0.
\]

Given the concavity of \( \Pi(\cdot) \), the optimal first quote \( \beta_1(v) \) is given by (12) and satisfies \( \beta_1(v) > \beta_2(v) \). Of course, because the first-order condition is taken in the
region $b < \underline{b}_1$, we still need to separately verify that the buyer does not deviate to quote greater than or equal to $\underline{b}_1$. We also need to verify that deviations of the first quote lead to weakly lower expected profits upon the potential repeat contact. These verifications are provided in Step 3.

We now turn to a buyer with value $v \geq \underline{v}$. As long as his first quote $b_1 \geq \underline{b}_1$, he is not visited for a second time in equilibrium, as shown before. He thus maximizes the expected profit upon the first contact only. Because the probability of being the first is $1/(1 + \alpha)$, the buyer solves

$$\max_{b_1} (v - b_1) \left[ \frac{1}{1 + \alpha} H(b_1) + \frac{\alpha}{1 + \alpha} G(b_1) \right].$$

(33)

Given the concavity of $(v - b)H(b)$ and $(v - b)G(b)$, the optimal first quote $\beta_1(v)$ is given by the first order condition (13). A separate verification that this buyer does not deviate to quote below $\underline{b}_1$ is provided in Step 3.

**Step 3: Verifications.** I verify the equilibrium quotes in an agent-normal form of the game, namely at each information set, a player delegates the action to a “clone.” These clones (or agents) share the same objective function of the original player but they act independently. With this construct, a buyer’s deviation upon the first contact does not affect his second quote.

We now verify that, for a buyer with value $v < \underline{v}$, deviating from $\beta_1(v)$ leads to weakly lower expected profits upon the second contact. With the restriction that $b_1 < \bar{b}$, imitating a buyer with value $\hat{v} > v$ does not affect the expected profit upon the second contact, as the second quote does not change. If the buyer imitate a buyer with value $\hat{v} < v$, then only sellers with value $v_0 \leq \beta_2(\hat{v})$ may return, leading to a lower expected profit.

It remains to verify that buyers do not deviate across the boundary $\underline{b}_1$, which we do now. Let $\beta_1(v^-) = \lim_{v \searrow \underline{v}} \beta_1(v)$, and I will use $v^-$ to denote a number that is right below $v$. If a buyer with value $v \geq \underline{v}$ quotes $b_1 < \beta_1(v^-)$, then his first-order condition at $b_1$ becomes

$$(v - b_1)G'(b_1) - G(b_1) \geq (v - b_1)G'(b_1) - G(b_1)$$

$$> (v - \beta_1(v^-))G'(\beta_1(v^-)) - G(\beta_1(v^-)) = 0,$$

where the last inequality and the last equality follow from the first-order condition.
of the optimization problem of a buyer with value \( v^- \). This implies that quoting \( b_1 < \beta_1(v^-) \) cannot be an equilibrium strategy. The same logic implies that a buyer with value \( v > v^- \) does not quote \( \beta_1(v^-) \). Finally, to prevent a buyer with value \( v \) from imitating a buyer with value \( v^- \), or vice versa, their expected profits must be equal across the boundary quote of \( b_1 \), that is (14):

\[
(v - b_1)(\alpha G(b_1) + H(b_1)) = (v - \beta_1(v^-))\alpha G(\beta_1(v^-)).
\]

Given (14), a buyer with value \( v < v^- \) has

\[
(v - b_1)(\alpha G(b_1) + H(b_1)) > (v - \beta_1(v^-))\alpha G(\beta_1(v^-))
\]

where this inequality follows from \( v < v^- \) and (14). That is, if the buyer imitates a buyer with value \( \hat{v} \geq v^- \), then he can do strictly better by imitating a buyer with value \( v^- \). However, this latter action is ruled out by the buyer’s maximization problem (31). This completes the verification.

**Proof of Proposition 2.** Given that in equilibrium \( \beta_1(v) > \beta_2(v) \) for all \( v > 0 \) and that \( \beta_1(v) \) and \( \beta_2(v) \) are strictly increasing in \( v \), we verify that a buyer’s second quote and first quote are characterized by (15) and (16), respectively. Once a buyer is contacted for a second time, he infers that he is not the last buyer. Suppose that our buyer in question has value \( v \). Because \( \beta_2(v) > \beta_1(v^N) > \beta_2(v^N) \), there exists an \( \epsilon > 0 \) such that \( \beta_2(v^N) < \beta_2(v^-) - \epsilon \). That is, the seller does not return to the \( N \)th buyer if our buyer locally deviates from his equilibrium second quote. As a result, our buyer is concerned only with the quotes of the remaining \( N - 2 \) buyers, whose highest valuation (first order statistic) is denoted \( Z^{(1)} \). A revisited buyer solves

\[
\max_{b_2}(v - b_2)\mathbb{P}(\beta_2(Z^{(1)}) \leq b_2)\mathbb{P}(v_0 \leq b_2) = \max_{b_2}(v - b_2)F_b(\beta_2^{-1}(b_2))^{N-2}F_s(b_2). \tag{34}
\]

First-order condition yields

\[
0 = F_b(\beta_2^{-1}(b_2))^{N-3}[(v - b_2)F_b(\beta_2^{-1}(b_2))f_s(b_2)
\]

\[
+ (v - b_2)(N - 2)f_b(\beta_2^{-1}(b_2))\frac{\beta_2^{-1}(b_2)}{b_2}F_s(b_2) - F_b(\beta_2^{-1}(b_2))F_s(b_2)]. \tag{35}
\]

Substituting in \( b_2 = \beta_2(v) \) and canceling out the non-zero \( F_b(v)^{N-3} \) we get (15).
Let \( Y^{(1)} = \max_{j<N} v^j \) be the highest value of the first \( N-1 \) buyers, whose distribution function is \( F_{Y^{(1)}}(y) = F_b(y)^{N-1} \). A buyer’s first quote is accepted only if three conditions are satisfied: (i) he is the \( N \)th buyer, quoting \( b_N \); (ii) \( b_N^1 > \beta_2(Y^{(1)}) \); and (iii) \( b_N^1 \geq v_0 \). A buyer with value \( v > 0 \) solves

\[
\max_{b_1}(v-b_1)P(\beta_2(Y^{(1)}) \leq b_1)P(v_0 \leq b_1) = \max_{b_1}(v-b_1)F_b(\beta_2^{-1}(b_1))^{N-1}F_s(b_1). \tag{36}
\]

Substituting \( b_1 = \beta_1(v) \) into the first-order condition and dividing by \( F_b(\beta_2^{-1}(\beta_1(v)))^{N-2} \), we have (16). Here I have used the agent-normal form of the game so that the buyer only considers the deviation of his first quote, given that his second quote is \( \beta_2(v) \).

To show that \( \beta_1(v) > \beta_2(v) \), write the right hand side of (16) as a function \( X(\beta_1(v)) \). For all \( v > 0 \), we have

\[
X(v) = -F_s(v)F_b(\beta_2^{-1}(v))\beta_2'(v) < 0,
\]

\[
X(\beta_2(v)) = [(v - \beta_2(v))f_s(\beta_2(v)) - F_s(\beta_2(v))]F_b(v)\beta_2'(v) + (v - \beta_2(v))(N-1)f_b(v)F_s(\beta_2(v)) > 0,
\]

where we have used (15) in the second inequality. So there exists a \( \beta_1(v) \in (\beta_2(v), v) \) that satisfies \( X(\beta_1(v)) = 0 \). Finally, the seller’s strategy is straightforward.

**Proof of Corollary 1.** With \( v_0 = 0 \), the equilibrium of Proposition 2 is still characterized by (15) and (16), but with \( F_s(v) = 1 \) and \( f_s(v) = 0 \) for all \( v > 0 \). Then (15) reduces to

\[
0 = -F_b(v)\beta_2'(v) + (v - \beta_2(v))(N-2)f_b(v).
\]

Multiply both sides by \( F_b(v)^{N-3} \) and rearrange to get

\[
\frac{d}{dv} \left[ \beta_2(v)F_b(v)^{N-2} \right] = v \frac{d}{dv} \left[ vF_b(v)^{N-2} \right].
\]

Integrate both sides from 0 to \( v \) and we have (17). It is straightforward to verify that \( \beta_2(v) \) increases in \( v \). The equation satisfied by \( \beta_1(v) \) simply follows from (16).  

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Proof of Equation 20. Let \( x, y \in (0, \bar{s}) \) and \( x > y \). MLRP implies

\[
f_H(x)f_L(y) > f_H(y)f_L(x). \tag{37}
\]

Integrate \( y \) from 0 to \( x \) in (37) and we have \( f_H(x)F_L(x) > F_H(x)f_L(x) \). Integrate \( x \) from \( y \) to \( \bar{s} \) in (37) and we have \((1 - F_H(y))f_L(y) > (1 - F_L(y))f_H(y)\). Since \( x \) and \( y \) are arbitrary, we combine these two inequalities to get

\[
\frac{F_H(x)}{F_L(x)} < \frac{f_H(x)}{f_L(x)} < \frac{1 - F_H(x)}{1 - F_L(x)},
\]

that is, \( F_H(x) < F_L(x) \) for all \( x \in (0, \bar{s}) \).

Proof of Proposition 3. To verify the equilibrium of Proposition 3, suppose that players adopt the conjectured strategies and that there is a unique cutoff signal that satisfies (23). Given the seller’s acceptance strategy and the random ordering of buyers, a visited buyer assigns probability \( \frac{1}{N} \) that he is the \( k \)th buyer visited, \( k = 1, 2, \ldots, N \), which means the previous \( k - 1 \) buyers all received a signal below \( s^* \). By Bayes’ rule and i.i.d. signals, we have

\[
\frac{\mathbb{P}(v = V_H|s, \text{visit})}{\mathbb{P}(v = V_L|s, \text{visit})} = \frac{p_H}{p_L} \cdot \frac{\mathbb{P}(s, \text{visit}|v = V_H)}{\mathbb{P}(s, \text{visit}|v = V_L)} = \frac{p_H}{p_L} \cdot \frac{f_H(s)}{f_L(s)} \cdot \frac{1}{N} \sum_{k=0}^{N-1} F_H(s^*)^k.
\]

The cutoff signal \( s^* \) must imply an expected asset value of \( DV_H \), or equivalently, a likelihood ratio of \( \frac{DV_H - V_L}{(1 - DV_H)} \). Thus \( s^* \) must satisfy (23).

It remains to show that such \( s^* \) exists. Write the right hand side of (23) as \( J^* \). From MLRP, for some small \( \bar{s} > 0 \), \( f_H(\bar{s}) \leq f_L(\bar{s}) \). By (22), at \( s = \bar{s} \),

\[
J(\bar{s}, N) < \frac{p_H}{p_L} \cdot \frac{f_H(\bar{s})}{f_L(\bar{s})} \leq \frac{p_H}{p_L} < J^*.
\]

and at the upper support \( s = \bar{s} \),

\[
J(\bar{s}, N) = \frac{p_H}{p_L} \cdot \frac{f_H(\bar{s})}{f_L(\bar{s})} > J^*.
\]

So there exists some \( s^* \in (0, \bar{s}) \) such that (23) holds.

When \( s^* \) is unique, then any buyer who receives a signal below (above) \( s^* \) has expected asset value below (above) \( DV_H \). A buyer with a signal \( s \geq s^* \)
has no incentive to quote higher than $DV_H$, as $DV_H$ is acceptable to any seller. If the buyer deviates to quote strictly lower than $DV_H$, then he buys the asset only if the asset is of low value, which implies a nonpositive profit for the buyer. Similarly, a buyer with a signal $s < s^*$ does not quote $DV_H$, as otherwise he makes a negative expected profit. He has no incentive to quote strictly lower than $V_L$, as that gives zero profit. Quoting higher than $V_L$ but lower than $DV_H$ only attracts low-value sellers, for whom a quote of $V_L$ suffices. This completes the verification of the equilibrium.

We now show that $\partial J(s^*, N)/\partial N < 0$. Observe that for all integers $k > j \geq 0$,

$$\frac{F_H(s)^j}{F_L(s)^j} > \frac{F_H(s) + F_H(s)^k}{F_L(s) + F_L(s)^k} > \frac{F_H(s)^k}{F_L(s)^k}.$$  

Iterate it and we get

$$\frac{\sum_{k=0}^{N-1} F_H(s)^k}{\sum_{k=0}^{N-1} F_L(s)^k} > \frac{F_H(s)^N}{F_L(s)^N}.$$  

Then

$$\frac{\sum_{k=0}^{N-1} F_H(s)^k}{\sum_{k=0}^{N-1} F_L(s)^k} > \frac{\sum_{k=0}^{N} F_H(s)^k}{\sum_{k=0}^{N} F_L(s)^k} > \frac{F_H(s)^N}{F_L(s)^N}.$$  

If $\partial J(s^*, N)/\partial s > 0$, then by inverse function theorem,

$$\frac{ds^*}{dN} = -\frac{\partial J(s^*, N)/\partial N}{\partial J(s^*, N)/\partial s^*} > 0.$$  

Finally, (23) and (24) imply that for any fixed $s$,

$$I_{OTC}(s, N) = \frac{f_H(s)/f_L(s)}{f_H(s^*)/f_L(s^*)} \cdot \frac{DV_H - V_L}{(1 - D)V_H}.$$  

By MRLP, $I_{OTC}(s, N)$ is decreasing in $s^*$, and thus decreasing in $N$. \qed

**Proof of Proposition 4.** By Proposition 3, $\partial J(s, N)/\partial s > 0$ implies that $s^*$ is strictly increasing in $N$. Since $s^*$ is bounded above by $\bar{s}$, $\lim_{N \to \infty} s^*$ exists. Suppose for contradiction that the limit is $\bar{s} - \epsilon$ for some $\epsilon > 0$. As $N \to \infty$, $F_H(\bar{s} - \epsilon)^N \to 0$, $F_L(\bar{s} - \epsilon)^N \to 0$, and thus

$$\frac{f_H(s^*)}{f_L(s^*)} \cdot \frac{\sum_{k=0}^{N-1} F_H(s^*)^k}{\sum_{k=0}^{N-1} F_L(s^*)^k} \to \frac{f_H(\bar{s} - \epsilon)}{f_L(\bar{s} - \epsilon)} \cdot \frac{1 - F_L(\bar{s} - \epsilon)}{1 - F_H(\bar{s} - \epsilon)} < 1.$$  

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where the last inequality follows from MLRP (also see the proof of (20)). Given (22), for sufficiently large but finite $N$ (23) cannot hold, a contradiction. Therefore $s^* \to \bar{s}$ as $N \to \infty$.

For any $s < \bar{s}$, there exists some $N$ such that for all $N > N$, $s^* > s$. By MLRP, (23), and (24),

$$I_{OTC}(s, N) = \frac{f_H(s)/f_L(s)}{f_H(s^*)/f_L(s^*)} \cdot J(s^*, N) < J(s^*, N) = \frac{DV_H - V_L}{(1 - D)V_H},$$

or $\mathbb{E}(v|s, \text{visit}) < DV_H$. When $f_H(\bar{s})/f_L(\bar{s}) < \infty$,

$$\lim_{N \to \infty} I_{OTC}(\bar{s}, N) = \lim_{N \to \infty} \frac{f_H(\bar{s})/f_L(\bar{s})}{f_H(s^*)/f_L(s^*)} \cdot J(s^*, N) = \frac{DV_H - V_L}{(1 - D)V_H}.$$ 

\[ \square \]

**Proof of Proposition 5.** Because $s^*$ in the equilibrium of Proposition 3 is strictly increasing in $N$ (by $\partial J(s^*, N)/\partial s^* > 0$), we immediately have $s_k^* < s_N^*$. Now consider the inference of the disfavored group. If (25) holds, then a disfavored buyer who receives the cutoff signal prices the asset exactly at $DV_H$.

That is,

$$f_H(s_{N-K}^*) \cdot \frac{\sum_{k=0}^{N-K-1} F_H(s_{N-K}^*)^k}{\sum_{k=0}^{N-K-1} F_L(s_{N-K}^*)^k} \cdot \frac{F_H(s_k^*)^K}{F_L(s_k^*)^K} = f_H(s_N^*) \cdot \frac{\sum_{k=0}^{N-1} F_H(s_N^*)^k}{\sum_{k=0}^{N-1} F_L(s_N^*)^k},$$

If $s_{N-K}^* \leq s_N^*$, then since $J(s)$ and $F_H(s)/F_L(s)$ both increase in $s$,

$$\frac{f_H(s_{N-K}^*)}{f_L(s_{N-K}^*)} \cdot \frac{\sum_{k=0}^{N-K-1} F_H(s_{N-K}^*)^k}{\sum_{k=0}^{N-K-1} F_L(s_{N-K}^*)^k} \cdot \frac{F_H(s_k^*)^K}{F_L(s_k^*)^K} < \frac{f_H(s_N^*)}{f_L(s_N^*)} \cdot \frac{\sum_{k=0}^{N-1} F_H(s_N^*)^k}{\sum_{k=0}^{N-1} F_L(s_N^*)^k},$$

which implies

$$\frac{\sum_{k=0}^{N-1} F_H(s_N^*)^k}{\sum_{k=0}^{N-1} F_L(s_N^*)^k} \leq \frac{\sum_{k=0}^{N-1} F_H(s_N^*)^k}{\sum_{k=0}^{N-1} F_L(s_N^*)^k},$$

a contradiction. \[ \square \]

**Proof of Proposition 6.** Equation 29 follows because $F_H(s)/F_L(s)$ is increasing in $s$. For the same reason, $I_A(s, N)$ is increasing in $s$. Given $I_A(s^*, N) < I_{OTC}(s^*, N) < I_A(\bar{s}, N)$, there exists some $s^A \in (s^*, \bar{s})$ such that $I_A(s^A, N) = I_{OTC}(s^*, N)$ and $\mathbb{E}(v|s^A, \text{win}) = DV_H$. \[ \square \]
References


