The Demographics of Innovation and Asset Returns*

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Abstract

We study asset-pricing implications of innovation in a general-equilibrium overlapping-generations economy. Innovation increases the competitive pressure on existing firms and workers, reducing the profits of existing firms and eroding the human capital of older workers. Due to the lack of inter-generational risk sharing, innovation creates a systematic risk factor, which we call "displacement risk." This risk helps explain several empirical patterns, including the existence of the growth-value factor in returns, the value premium, and the high equity premium. We assess the magnitude of displacement risk using estimates of inter-cohort consumption differences across households and find support for the model.

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1 Introduction

In this paper we explore asset-pricing implications of innovation. We concentrate on two effects of innovation. First, while innovation expands the productive capacity of the economy, it increases competitive pressure on existing firms and workers, reducing profits of existing firms and eroding the human capital of older workers. Thus, innovation creates a risk factor, which we call the "displacement risk factor." Second, since economic rents from innovation are captured largely by the future cohorts of inventors through the firms they create, existing agents cannot use financial markets to avoid the negative effects of displacement.

We study an overlapping-generations general-equilibrium production economy with multiple intermediate goods, which are used to produce a single consumption good. Innovation creates a stochastically expanding variety of intermediate goods. Intermediate goods are partial substitutes, therefore growth in their variety intensifies competition between their producers and leads to displacement of the established firms by the new entrants. In addition, older workers are not as well adapted to the new technologies as the new cohorts of agents, which implies that innovation diminishes older workers' human capital. Thus, there are two sides to innovation. The bright side is the increased productivity that it brings, which raises aggregate output, consumption, and wages. The dark side is the reduced wage-bill and consumption shares of the older agents.

The displacement risk faced by older agents is a systematic risk factor. Individual Euler equations in our model cannot be aggregated into a pricing model based solely on aggregate consumption because of the wedge between the future consumption of all agents *present currently* and the future *aggregate* consumption: the latter includes the consumption of future cohorts, but the former does not. This wedge is stochastic and driven by innovation shocks. Thus, the standard aggregate consumption-based pricing model must be augmented with the displacement risk factor. This argument helps explain several important empirical patterns in asset returns.

First, the displacement risk factor is connected to cross-sectional differences in stock returns. We assume that existing firms participate in innovation, but some firms are more likely to innovate than others. The more innovative firms derive a larger fraction of their value from future inventions and earn higher valuation ratios, which makes them "growth firms." Because of their relatively high exposure to the innovation shocks, growth firms offer a hedge against displacement risk and, in equilibrium, earn lower average returns than less innovative "value firms." Thus, heterogeneous exposure to displacement risk helps explain the positive average return premium earned by value stocks relative to growth stocks, called the value premium. Moreover, innovation shocks generate co-movement among value stocks and among growth stocks, giving rise to a value-growth factor in stock returns. Hence, our model rationalizes the empirical findings in Fama and French (1993), who document the

empirical success of a multi-factor model featuring a value-growth factor.

Second, the aggregate equity premium in our model is boosted by the stock-market exposure to the displacement risk factor. Large innovation shocks simultaneously lower the value of existing firms through increased competition and reduce consumption of existing agents through the erosion of their human and financial wealths. As a result, agents require a higher premium to hold stocks than could be inferred from the aggregate consumption series using standard pricing models.

Third, the equilibrium interest rate in our model is lower than suggested by the aggregate consumption process and agents' preferences. This is because individual agents' consumption growth is lower on average and riskier than that of aggregate consumption. This property of overlapping-generation economies is noted in the seminal paper of Blanchard (1985) and emphasized recently by Gârleanu and Panageas (2007).[1]

Our model also has implications for the cointegration properties of a) the dividends paid by all corporations that current agents can trade in and b) the dividends paid by all firms at any point in time t, which we refer to as "aggregate" dividends. The latter are cointegrated with (in fact, a constant fraction of) aggregate consumption. However, the former are not, since the future share of aggregate output accruing to the firms existing currently declines towards zero asymptotically due to innovation. The lack of cointegration is empirically realistic and has been recently recognized in the literature as quantitatively important for understanding aggregate market returns.

We test the implications of our model empirically. We identify innovation shocks through their effect on the consumption of individual cohorts and show that inter-generational differences in consumption correlate with the return differences between value and growth stocks. In addition to the empirical tests, we use the empirical moments to calibrate our model and verify that its mechanism can reproduce key asset-pricing patterns quantitatively.

Our paper relates to several strands of the literature. A number of papers use an overlapping-generations framework to study asset-pricing phenomena, e.g., Abel (2003), Constantinides et al. (2002), DeMarzo et al. (2004, 2008), Gârleanu and Panageas (2007), Gomes and Michaelides (2008), or Storesletten et al. (2007). None of these papers, however, considers the displacement risk, which is critical for our results. Our model of innovation is similar to Romer (1990), who studies endogenous sources of growth. We treat growth as exogenous and instead focus on the impact of innovation on financial-asset returns. Consistent with the premise of our model, Hobijn and Jovanovic (2001) document the permanent negative impact of innovation on incumbent firms in the context of the IT revolution. However, they employ a representative-agent framework and hence do not consider the displacement risk

[1] The size of this effect is magnified if we allow for some degree of "catching up with the Joneses," as in Abel (1990).

of innovation across agents.

Our paper also contributes to the theoretical literature on cross-sectional patterns in stock returns, which includes Berk et al. (1999), Gala (2005), Gomes et al. (2003), Carlson et al. (2004, 2006), Papanikolaou (2007), and Zhang (2005) among many. Our contribution is the new approach to the value-premium puzzle. Many of the earlier papers, (e.g., Berk et al. (1999) and Zhang (2005)) use partial or industry equilibrium settings with exogenous pricing of risk. Existing general equilibrium models, (e.g., Gomes et al. (2003) and Papanikolaou (2007)) satisfy the aggregate-consumption CAPM (CCAPM) and thus do not address the challenge of reconciling the value premium with the standard CCAPM empirically. In contrast, we propose a novel source of systematic risk that accounts for return differences between value and growth stocks. Our model implies that the standard CCAPM fails to capture this risk factor.

We also contribute to the vast literature on the equity-premium puzzle, (e.g., Mehra and Prescott (1985), Campbell and Cochrane (1999)). The displacement risk factor helps reconcile a high equity premium with a smooth time series of aggregate consumption.

The rest of the paper is organized as follows. In Section 2, we formulate and in Section 3 we solve our model. Section 4 analyzes qualitative properties of the model, and Section 5 contains a quantitative evaluation, including empirical tests. Section 6 concludes. We collect technical results and proofs in Appendix A. To save space, we have collected a number of model extensions and additional results in an extended appendix, which is available online.

2 Model

2.1 Agents' Preferences and Demographics

We consider a model with discrete and infinite time: $t \in \{\ldots, 0, 1, 2, \ldots\}$. The size of the population is normalized to 1. At each date a mass λ of agents, chosen randomly, die, and a mass λ of agents are born, so that the population remains constant. An agent born at time s has preferences of the form

$$E_s \sum_{t=s}^{s+\tau} \beta^{(t-s)} \frac{\left(c_{t,s}^{\psi} \left(\frac{c_{t,s}}{C_t} \right)^{1-\psi} \right)^{1-\gamma}}{1-\gamma}, \tag{1}$$

where τ is the (geometrically distributed) time of death, $c_{t,s}$ is the agent's consumption at time t, C_t is aggregate consumption at time t, $\beta \in (0,1)$ is a subjective discount factor, $\gamma > 0$ is the agent's relative risk aversion, and $\psi \in [0,1]$ is a constant. Preferences of the

form (1) were originally proposed by Abel (1990), and are commonly referred to as "keeping-up-with-the-Joneses" preferences. When $\psi = 1$, these preferences specialize to the standard constant-relative-risk-aversion preferences. In general, for $\psi \in [0,1]$ agents place a weight ψ on their own consumption (irrespective of what others are consuming) and a weight $1-\psi$ on their consumption relative to average consumption in the population. Our qualitative results hold independently of the keeping-up-with-the-Joneses feature, which only helps at the calibration stage, by reducing the value of the interest rate.

A standard argument allows us to integrate over the distribution of the stochastic times of death and re-write preferences of the form (1) as

$$E_s \sum_{t=s}^{\infty} \left[(1-\lambda)\,\beta\right]^{(t-s)} \frac{\left(c_{t,s}^{\psi}\left(\frac{c_{t,s}}{C_t}\right)^{1-\psi}\right)^{1-\gamma}}{1-\gamma}. \tag{2}$$

2.2 Technology

Final-Good Firms

There is a representative (competitive) final-good producing firm that produces the single final good using two categories of inputs: a) labor and b) a continuum of intermediate goods. Specifically, the production function of a final-good producing firm is

$$Y_t = Z_t \left(L_t^F\right)^{1-\alpha} \left[\int_0^{A_t} \omega_{j,t} \left(x_{j,t}\right)^{\alpha} dj\right]. \tag{3}$$

In equation (3) Z_t denotes a stochastic productivity process, L_t^F captures the efficiency units of labor that enter into the production of the final good, A_t is the number of intermediate goods available at time t, and $x_{j,t}$ captures the quantity of intermediate good j used in the production of the final good. The constant $\alpha \in [0,1]$ controls the relative weight of labor and intermediate goods in the production of the final good, while $\omega_{j,t}$ captures the relative importance placed on the various intermediate goods. We specify $\omega_{j,t}$ as

$$\omega_{j,t} = \left(\frac{j}{A_t}\right)^{\chi(1-\alpha)}, \quad \chi \geq 0. \tag{4}$$

For $\chi = 0$, the production function (3) is identical to the one introduced by the seminal Romer (1990) paper in the context of endogenous growth theory. Our version is slightly more general, since the factor weights $\omega_{j,t}$, which are increasing functions of the intermediate good index j, allow the production function to exhibit a "preference" for more recent intermediate

goods. As we show below, this feature confers an additional degree of control over the individual-firm profit variability, which is helpful for calibration purposes.

Even though our aim here is not to explain the sources of growth in the economy, the production function (3) is useful for our purposes for several reasons: a) Innovation, i.e., an increase in the variety of intermediate goods (A_t) helps increase aggregate output; b) There is rivalry between existing and newly arriving intermediate goods, since increases in A_t strengthen the competition among intermediate-good producers, and c) Heterogeneity in intermediate, rather than final, goods is technically convenient, since we can keep one unit of the final good as numeraire throughout. An exact illustration of the first two properties is provided in the next section, where we solve the model.

The process Z_t follows a random walk (in logs) with drift μ and volatility σ_ε:

$$\log(Z_{t+1}) = \log(Z_t) + \mu + \varepsilon_{t+1}, \quad \varepsilon_{t+1} \sim N(0, \sigma_\varepsilon^2). \tag{5}$$

At each point in time t, the representative final-good firm chooses L_t^F and $x_{j,t}$ (where $j \in [0, A_t]$) so as to maximize its profits

$$\pi_t^F = \max_{L_t^F, x_{j,t}} \left\{ Y_t - \int_0^{A_t} p_{j,t} x_{j,t} dj - w_t L_t^F \right\}, \tag{6}$$

where $p_{j,t}$ is the price of intermediate good j and w_t is the prevailing wage (per efficiency unit of labor).

Intermediate-Good Firms

The intermediate good $x_{j,t}$ is produced by monopolistically competitive firms that own non-perishable blueprints to its production. Each intermediate good is produced by a single firm, while a single firm may produce a measure-zero set of intermediate goods. We assume that the production of the intermediate good $j \in [0, A_t]$ requires one unit of labor (measured in efficiency units) per unit of intermediate good produced, so that the total number of efficiency units of labor used in the intermediate-goods sector is

$$L_t^I = \int_0^{A_t} x_{j,t} dj. \tag{7}$$

The price $p_{j,t}$ of intermediate good j maximizes the profits of the intermediate good producer, taking the demand function of the representative final good firm $x_j\left(p_{j,t}; p_{j' \neq j}, w_t\right) \equiv \arg\max_{x_{j,t}} \pi_t^F$ as given. To simplify notation, we shall write $x_{j,t}\left(p_{j,t}\right)$ instead of $x_{j,t}\left(p_{j,t}; p_{j' \neq j}, w_t\right)$.

Production of the intermediate good j generates profits[2]

$$\pi_t^I(j) = \max_{p_{j,t}} \left\{ (p_{j,t} - w_t)\, x_{j,t}(p_{j,t}) \right\}. \tag{8}$$

2.3 Arrival of New Intermediate Goods and New Agents

New Products

The number of intermediate goods A_t expands over time as a result of innovations. Given our focus on asset pricing, we assume that the innovation process is exogenous for simplicity. The number of intermediate goods in our economy follows a random walk (in logs):[3]

$$\log(A_{t+1}) = \log(A_t) + u_{t+1}. \tag{9}$$

We assume a single aggregate innovation shock for simplicity. As we show in the (online) extended appendix to this paper, our analysis extends to a multi-sector economy with correlated sectoral innovation shocks.

The increment u_{t+1} is i.i.d. across time for simplicity. To ensure its positivity, we assume that u_{t+1} is Gamma distributed[4] with shape parameter k and scale parameter ν.

The intellectual property rights for the production of the $\Delta A_{t+1} \equiv A_{t+1} - A_t$ new intermediate goods belong either to arriving agents or to existing firms. We assume that a fraction $\kappa \in [0,1]$ of the total value of the new blueprints is allocated to arriving entrepreneurs, while the complementary fraction $1-\kappa$ is introduced by established firms and hence belongs indirectly to existing agents, who own these firms.

Workers

New agents are of two types and differ according to their endowments: Entrepreneurs account for a fraction $\phi \in (0,1)$ of new agents and arrive in life with ideas for new firms. We discuss them in the next subsection. In this subsection we focus on workers, who make up a fraction $1-\phi$ of new agents. Workers start life with a constant endowment of $\overline{h}$ hours per period, which they supply inelastically. The ratio of efficiency units of labor to hours is affected by two factors: a) age and experience, and b) skill obsolescence. To capture the first notion,

[2] Any firm produces a zero-measure set of intermediate goods. Hence, it can ignore any feedback effects of the pricing of its intermediate good j on the demand for the other intermediate goods it produces. Therefore, maximization of the overall firm's profits amounts to maximizing the profits from each intermediate good separately.

[3] We choose a random-walk specification in order to ensure that aggregate consumption is a random walk. The assumption of a random walk implies that — for a given u_{t+1} — the increase in production is proportional to the current level of production. This assumption is routinely used in the literature and is sometimes referred to as "standing on the shoulders of giants". See, e.g., Jones (1997).

[4] The gamma probability density function $f(x)$ is proportional to $x^{k-1}\frac{e^{-x/\nu}}{\nu^k}$.

we assume that the ratio of labor efficiency units to hours changes geometrically with age at the rate δ, so that in the absence of skill obsolescence, the ratio of a worker's endowment of efficiency units at time $t > s$ to the respective endowment at the time of birth s is given by $(1+\delta)^{t-s}$.

In the real world younger workers are likely to be more productive in the presence of increased technological complexity than older workers. One potential reason is that their education gives them the appropriate skills for understanding the technological frontier. By contrast, older workers are likely to be challenged by technological advancements. In the extended online appendix we present a simple vintage model of the labor market that introduces imperfect substitution across labor supplied by agents born at different times. To expedite the presentation of the main results, in this section we assume that labor is a homogenous good and that workers' endowment of efficiency units depreciates in a way that replicates the outcome of the more elaborate vintage model.

Specifically, we assume that a worker's total supply of efficiency units of labor is given by $\overline{h}\,(1+\delta)^{t-s}\,q_{t,s}$ with

$$\log(q_{t+1,s}) = \log(q_{t,s}) - \rho u_{t+1} \tag{10}$$

and $\rho \geq 0$. This specification captures the idea that advancements of the technological frontier act as depreciation shocks to the productivity of old workers. Such shocks generate cohort effects in individual consumption and income, which are present in historical data, as we show in Section 5.2. We normalize the initial endowment of efficiency units so that the aggregate number of efficiency units in the economy is constant. In particular, we set

$$q_{s,s} = 1 - (1-\lambda)\,(1+\delta)\,e^{-\rho u_s} \tag{11}$$

and $\overline{h} = \frac{1}{\lambda}$. We assume that $(1-\lambda)\,(1+\delta) \leq 1$. As a result, the number of per-worker efficiency units, $L_t/\,(1-\phi)$ with $L_t = L_t^F + L_t^I$, is always equal to 1, and hence $\bar{h}(1+\delta)^{t-s}q_{t,s}\lambda(1-\lambda)^{t-s}$ can be interpreted as the fraction of total wages that accrues to workers born at time s.[5]

Entrepreneurs and New Value and Growth Firms

Entrepreneurs arrive endowed with ideas for new blueprints. They start a continuum of firms that produce the respective intermediate goods, and introduce them into the stock market.

[5]Note that, with $z = (1-\lambda)(1-\delta)$, our assumptions imply

$$\frac{L_t}{(1-\phi)} = \lambda\bar{h}\sum_{s\leq t} z^{t-s}q_{t,s} = \sum_{s\leq t} z^{t-s}(1-ze^{-\rho u_s})e^{-\rho\sum_{v=s+1}^{t}u_v} = \sum_{s\leq t} z^{t-s}e^{-\rho\sum_{v=s+1}^{t}u_v} - z^{t-s+1}e^{-\rho\sum_{v=s}^{t}u_v} = 1.$$

We assume that these new firms are heterogeneous in their access to new blueprints in future periods. How new blueprints are distributed among firms in future periods is only relevant for the cross-section of stock returns and has no effect on aggregate prices and quantities. For ease of exposition, we make a stylized assumption: some firms can receive blueprints in future periods, while the remainder of the firms cannot. We refer to the first type of firms as "growth" firms, and the latter type as "value" firms. Value firms created at time t are only entitled to a fraction $\eta\kappa$ of the value of blueprints introduced in that period, where $\eta \in (0, 1]$. By contrast, new "growth" firms are entitled to a fraction $(1-\eta)\kappa$ of the value of new blueprints at time t, but they also receive a fraction of the new blueprints in future periods. Specifically, in period t, growth firms born at $s \in (-\infty, t-1]$ obtain a fraction

$$(1-\kappa)\left(\frac{1-\varpi}{\varpi}\right)\varpi^{t-s} \tag{12}$$

of the value of the ΔA_t new blueprints. One can easily relax these stylized assumptions on the distribution of new blueprints to obtain the desired cross-sectional distribution and dynamics of firm characteristics.[6]

To simplify matters, we assume that there are no intra-cohort differences among growth firms and any two growth firms of the same cohort obtain the same value of blueprints in any given period. The geometric decay in the fraction of new blueprints that accrues to a given growth firm as a function of its age ensures that asymptotically the total market capitalization of firms existing at any given, fixed time s goes to zero as a fraction of the aggregate market capitalization as time progresses.

2.4 Asset Markets

There exists a complete set of state-contingent claims. At each point in time existing agents can trade in zero net supply Arrow-Debreu securities contingent on the joint realization of future shocks $\varepsilon_{t+\tau}$ and $u_{t+\tau}$, $\forall\tau > 0$. We denote the corresponding stochastic discount factor by ξ_t, so that the time-s value of a claim paying D_t at time t is given by $E_s\frac{\xi_t}{\xi_s}D_t$.

In addition, agents have access to annuity markets as in Blanchard (1985). (We refer the reader to that paper for details). The joint assumptions of perfect spanning and frictionless annuity markets simplifies the analysis considerably, since feasible consumption choices are

[6] Luttmer (2007) presents an interesting approach to explaining the size distribution of firms in a model that shares some features with Romer (1990). Even though the size distribution does not matter for the insights that we develop in this paper, the methods of Luttmer (2007) could be used to enrich the present model so as to account for the size distribution.

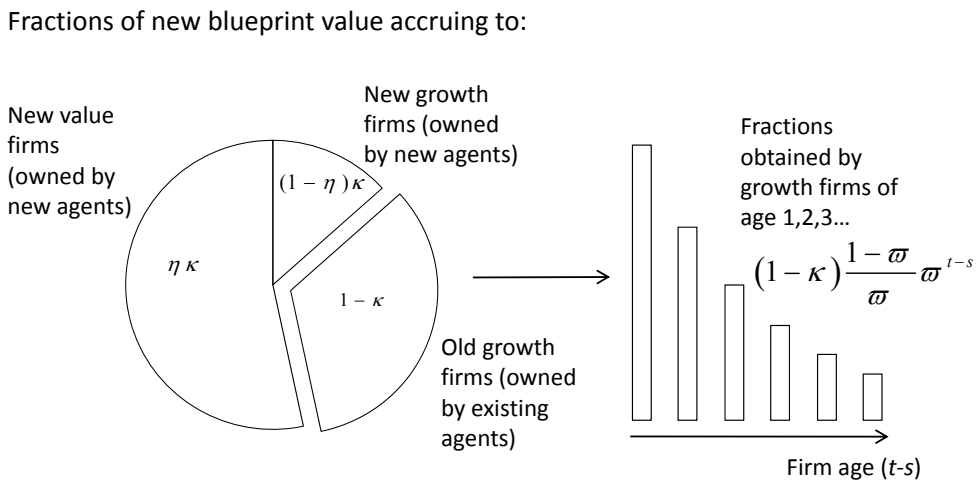

Figure 1: Illustration of the allocation of new blueprint value

constrained by a single intertemporal budget constraint. For a worker, that intertemporal budget constraint is given by

$$E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) c_{t,s}^{w} = E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) w_t q_{t,s} \bar{h} (1+\delta)^{t-s}, \tag{13}$$

where $c_{t,s}^{w}$ denotes the time-t consumption of a representative worker who was born at time s. The left-hand side of (13) represents the present value of a worker's consumption, while the right-hand side represents the present value of his income. Similarly, letting $c_{t,s}^{e}$ denote the time-t consumption of a representative inventor who was born at time s, his intertemporal budget constraint is

$$E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) c_{t,s}^{e} = \frac{1}{\lambda\phi} V_{s,s}, \tag{14}$$

where $V_{s,s}$ is the time-s total market capitalization of new firms created at time s, and therefore the right-hand side represents the wealth at birth of a representative inventor. In order to determine the total market value of firms created at time s, let $\Pi_{j,s}$ be the present value of profits from production of intermediate good j:

$$\Pi_{j,s} = \left[E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s}\right) \pi_{j,t}^{I} \right]. \tag{15}$$

The total market capitalization of all new firms can then be written as

$$V_{s,s} = \kappa \int_{A_{s-1}}^{A_s} \Pi_{j,s} dj + (1-\varpi) E_s \sum_{t=s+1}^{\infty} \left(\frac{\xi_t}{\xi_s}\right) (1-\kappa)\, \varpi^{t-s-1} \int_{A_{t-1}}^{A_t} \Pi_{j,t} dj. \tag{16}$$

The first term in equation (16) is the value of the blueprints for the production of new intermediate goods that are introduced by new firms (both "growth" and "value" firms) at time s. The latter term captures the value of "growth opportunities," that is, the value of blueprints to be received in future periods $t > s$ by growth firms created at time s.

2.5 Equilibrium

The definition of equilibrium is standard. To simplify notation, we let ϕ^e and ϕ^w denote the fraction of entrepreneurs and workers in the population, respectively. An equilibrium is defined as follows.

Definition 1 *An equilibrium is a collection of adapted stochastic processes* $\{x_{j,t},\ L_t^F,\ c_{t,s}^w,\ c_{t,s}^e,\ \xi_t,\ p_{j,t},\ w_t\}$ *with* $j \in [0, A_t]$ *and* $t \geq s$ *such that*

1. *(Consumer optimality): Given* ξ_t*, the process* $c_{t,s}^w$ *(respectively,* $c_{t,s}^e$*) solves the optimization problem (2) subject to the constraint (13) (respectively, constraint (14)).*
2. *(Profit maximization) The prices* $p_{j,t}$ *solve the optimization problem (8) given* L_t^F*,* $x_{j' \neq j,t}$*,* w_t *and* $L_t^F, x_{j,t}$ *solve the optimization problem (6) given* $p_{j,t}, w_t$*.*
3. *(Market clearing). Labor and goods markets clear*

$$L_t^F + L_t^I = (1-\phi), \tag{17}$$

$$\lambda \sum_{s=-\infty}^{t} \sum_{i \in \{w,e\}} (1-\lambda)^{t-s} \phi^i c_{t,s}^i = Y_t. \tag{18}$$

3 Solution

3.1 Equilibrium Output, Profit, and Wages

Consider the intermediate-good markets. We first derive the demand curve of the final-good firm for the intermediate input j at time t. Maximizing (6) with respect to $x_{j,t}$, we obtain

$$x_{j,t} = L_t^F \left[\frac{p_{j,t}}{\omega_{j,t} Z_t \alpha}\right]^{\frac{1}{\alpha-1}}. \tag{19}$$

Substituting this expression into (8) and maximizing over $p_{j,t}$ leads to

$$p_{j,t} = \frac{w_t}{\alpha}, \tag{20}$$

while combining (19) and (20) yields

$$x_{j,t} = L_t^F \left[\frac{w_t}{\omega_{j,t} Z_t \alpha^2} \right]^{\frac{1}{\alpha-1}}. \tag{21}$$

Next, consider the labor markets. Maximizing (6) with respect to L_t^F gives the first-order condition

$$w_t L_t^F = (1-\alpha) Y_t. \tag{22}$$

Substituting (21) into (3) and then into (22) and simplifying yields

$$w_t = \left(\alpha^2\right)^\alpha \left(\frac{1-\alpha}{1+\chi} \right)^{1-\alpha} Z_t A_t^{1-\alpha}. \tag{23}$$

We substitute equation (23) into (21) and then into (17) to obtain

$$x_{j,t} = \frac{1+\chi}{A_t} \left(\frac{j}{A_t} \right)^\chi \frac{(1-\phi)\,\alpha^2}{\alpha^2+1-\alpha}, \tag{24}$$

$$L_t^F = \frac{1-\alpha}{\alpha^2+1-\alpha} (1-\phi). \tag{25}$$

Finally, aggregate output is given by

$$Y_t = \frac{\left(\alpha^2\right)^\alpha \left(\frac{1-\alpha}{1+\chi}\right)^{1-\alpha}}{\alpha^2+1-\alpha} (1-\phi)\, Z_t A_t^{1-\alpha}, \tag{26}$$

which we derive by combining (24) and (25) inside (3). The number of intermediate inputs (A_t) in equation (26) is raised to the power $1-\alpha$. This means that aggregate output is increasing in the number of intermediate inputs. However, the sensitivity of output to the number of inputs depends on the elasticity of substitution between different varieties of intermediate goods. For instance, as α approaches 1, intermediate goods become perfect substitutes, and a larger variety of intermediate goods leads to more competition among firms and lower profits for existing intermediate-good producers without changing the overall productive capacity of the economy.

We now compute the income share of labor and the profits of firms. Total payments to labor $w_t (1-\phi)$ are equal to a fraction $(\alpha^2+1-\alpha)$ of output Y_t, which follows from (23) and

(26). Because of constant returns to scale in the production of final goods, the profits of the representative final-good firm are $\pi_t^F = 0$. The profits from the production of intermediate good j are

$$\pi_{j,t}^I = (1+\chi)\left(\frac{j}{A_t}\right)^{\chi}\frac{Y_t}{A_t}\alpha\left(1-\alpha\right), \tag{27}$$

which is obtained by combining (24) and (20) with (8) and (23). Note that the time series of profits $\pi_{j,t}^I$ is not cointegrated with aggregate output Y_t, since the variety of intermediate goods A_t grows over time and hence asymptotically $\pi_{j,t}^I/Y_t \to 0$. As a result, dividends of an *individual firm* are not co-integrated with aggregate output, which is intuitive because of the constant arrival of competing firms. In comparison, aggregate profits are a constant fraction of total output, as $\int_0^{A_t}\pi_{j,t}^I dj = \alpha\left(1-\alpha\right)Y_t$. This equality follows immediately from the fact that, in a general-equilibrium framework, total income shares must add up to aggregate output: $w_t\left(1-\phi\right)+\pi_t^F+\int_0^{A_t}\pi_{j,t}^I dj = Y_t$.

3.2 The Stochastic Discount Factor

To determine the stochastic discount factor ξ_t, we recall that, since agents have access to a full set of state-contingent securities *after their birth,* a consumer's lifetime consumption profile can be obtained by maximizing (2) subject to a single intertemporal budget constraint (constraint [13] if the agent is a worker and constraint [14] if the agent is an inventor). Attaching a Lagrange multiplier to the intertemporal budget constraint, maximizing with respect to $c_{t,s}^i$, and relating the consumption at time t to the consumption at time s for a consumer whose birth date is s gives

$$c_{t,s}^i = c_{s,s}^i\left(\frac{C_t^{(1-\psi)(1-\gamma)}}{C_s{}^{(1-\psi)(1-\gamma)}}\beta^{-(t-s)}\frac{\xi_t}{\xi_s}\right)^{-\frac{1}{\gamma}} \quad \text{for } i\in\{e,w\}. \tag{28}$$

From this equation, the aggregate consumption at any point in time is

$$C_t = \lambda\sum_{s=-\infty}^{t}\sum_{i\in\{w,e\}}\left(1-\lambda\right)^{t-s}\phi^i c_{s,s}^i\left(\frac{C_t^{(1-\psi)(1-\gamma)}}{C_s{}^{(1-\psi)(1-\gamma)}}\beta^{-(t-s)}\frac{\xi_t}{\xi_s}\right)^{-\frac{1}{\gamma}}. \tag{29}$$

Expressing C_{t+1} in the same way and then using (29) gives

$$C_{t+1} = \left(1-\lambda\right)C_t\left(\beta^{-1}\frac{C_{t+1}^{(1-\psi)(1-\gamma)}}{C_t{}^{(1-\psi)(1-\gamma)}}\frac{\xi_{t+1}}{\xi_t}\right)^{-\frac{1}{\gamma}} + \lambda\sum_{i\in\{w,e\}}\phi^i c_{t+1,t+1}^i. \tag{30}$$

Dividing both sides of (30) by C_t, solving for $\frac{\xi_{t+1}}{\xi_t}$, and noting that $C_t = Y_t$ in equilibrium leads to

$$\frac{\xi_{t+1}}{\xi_t} = \beta \left(\frac{Y_{t+1}}{Y_t}\right)^{-1+\psi(1-\gamma)} \left[\frac{1}{1-\lambda}\left(1-\lambda \sum_{i\in\{w,e\}} \phi^i \frac{c^i_{t+1,t+1}}{Y_{t+1}}\right)\right]^{-\gamma}. \tag{31}$$

To obtain an intuitive understanding of equation (31) it is easiest to focus on the case $\psi = 1$, so that agents have standard CRRA preferences. In this case, the stochastic discount factor consists of two parts. The first term is $\beta \left(\frac{Y_{t+1}}{Y_t}\right)^{-\gamma}$, which is the standard expression for the stochastic discount factor in an (infinitely-lived) representative-agent economy. The second term, namely $1-\lambda \sum_{i\in\{w,e\}} \phi^i \frac{c^i_{t+1,t+1}}{Y_{t+1}}$, gives the proportion of output at time $t+1$ that accrues to agents already alive at time t. Note that only a proportion $1-\lambda$ of existing agents survive between t and $t+1$, and that the arriving generation claims a proportion $\lambda \sum_{i\in\{w,e\}} \phi^i \frac{c^i_{t+1,t+1}}{Y_{t+1}}$ of aggregate output. The combination of these terms yields the consumption growth between t and $t+1$ of the surviving agents.

Equation (31) states an intuitive point: since (ignoring consumption externalities) only agents alive at time t are relevant for asset pricing, it is exclusively their consumption growth that determines the stochastic discount factor, not the aggregate consumption growth, which includes the consumption of agents born at time $t+1$.

The failure of the aggregate-consumption CAPM (CCAPM) in our model is distinct from earlier results obtained in incomplete-market economies (e.g., Constantinides and Duffie (1996), Heaton and Lucas (1996), and Basak and Cuoco (1998)), where various frictions and constraints prevent perfect risk sharing among agents.[7] Our model abstracts from such issues, so that the consumption of *existing* agents is perfectly correlated. Instead, the key economic mechanism is the failure of intergenerational risk sharing. This qualitative distinction is important for empirical work: to test our model, one should look for evidence of imperfect inter-generational risk sharing instead of imperfect risk sharing among existing agents. We undertake this task in Section 5.

To conclude the computation of equilibrium, we need to obtain an expression for the consumption shares $\frac{c^i_{t,t}}{Y_t}$, $i \in \{w, e\}$. This can be done by using the intertemporal budget constraints (14) and (13). Using the fact that the growth rates of consumption and output

[7] Krueger and Lustig (2006) show that non-traded idiosyncratic income shocks by themselves may lead to imperfect risk sharing among existing agents, but are not enough to invalidate the equity premium implications of the CCAPM; they need to be interacted with frictions or portfolio constraints.

in our model are i.i.d. over time, we show (see Proposition 1 in the Appendix) that

$$1-\lambda \sum_{i\in\{w,e\}} \phi^i \frac{c^i_{t+1,t+1}}{Y_{t+1}} = \upsilon(u_{t+1};\theta^e,\theta^w,\theta^g),$$

with

$$\begin{aligned}\upsilon(u_{t+1};\theta^e,\theta^w,\theta^g) \;\equiv\; & 1-\theta^e\alpha\left(1-\alpha\right)\left(\kappa\left(1-e^{-(1+\chi)u_{t+1}}\right)+\left(\frac{1-\varpi}{\varpi}\right)\theta^g\right) \\ & -\theta^w\left(\alpha^2+1-\alpha\right)\left(1-\left(1-\lambda\right)\left(1+\delta\right)e^{-\rho u_{t+1}}\right)\end{aligned} \tag{32}$$

and θ^e, θ^w, and θ^g three appropriate constants solving a system of three nonlinear equations in three unknowns. Given the interpretation of $\upsilon(u_{t+1};\theta^e,\theta^w,\theta^g)$ as the fraction of consumption that accrues to new agents, $(1-\lambda)^{-1}\upsilon(u_{t+1};\theta^e,\theta^w,\theta^g)$ captures the net adjustment of aggregate consumption growth that leads to the consumption growth of existing agents. Accordingly, we refer to $(1-\lambda)^{-1}\upsilon(u_{t+1};\theta^e,\theta^w,\theta^g)$ as the *displacement factor*.

We conclude this subsection, by noting that equation (31) is a robust implication of our analysis. It only relies on the assumption that existing agents can trade in a full set of state-contingent securities, so that their consumption is given by equation (28). Equation (28) would still hold in several realistic, but inessential extensions of the model that would allow for bequests and gifts across generations, government debt, intergenerational transfers mandated by the government, or adjustable and depreciable capital. Such extensions would not change the functional form of equation (31) and would only affect the magnitude of the displacement factor. For instance, in an economy populated by a representative, altruistically-linked dynasty, bequests and gifts between the different generations would ensure that every living member of the dynasty enjoys the same consumption. Accordingly, arriving agents' consumption is equal to per-capita output, and the displacement factor is identically equal to one. Our calibration of the model in Section 5 is robust to such extensions, because it is based directly on the magnitude of the displacement factor in the data.

4 Qualitative Properties of the Equilibrium

The consumption externality aside, the stochastic discount factor in our model depends only on the consumption of existing agents, as we show above using equation (31).

To highlight the departure from the standard paradigm, we consider a limiting case of the model with no aggregate consumption risk. Specifically, suppose that $\sigma = 0$ and $\rho > 0$, and let α approach 1. Equation (26) implies that the volatility of aggregate output, and

of aggregate consumption, approaches zero. Then, according to the standard CCAPM, risk premia must vanish in the limit. This is not the case in our model, as shown in the following lemma.

Lemma 1 *Assume that* $\sigma = 0$, $\rho > 0$, $\kappa = 1$, *and*

$$1 > \beta (1-\lambda)^{\gamma} e^{\mu\psi(1-\gamma)+\frac{1}{2}\sigma^2\psi^2(1-\gamma)^2}, \tag{33}$$

$$1 \leq \beta (1+\delta)^{-\gamma} e^{\mu\psi(1-\gamma)+\frac{1}{2}\sigma^2\psi^2(1-\gamma)^2} E[e^{\rho\gamma u_{t+1}}]. \tag{34}$$

Then an equilibrium exists. Moreover, letting R_t *be the return of any stock,*

$$\lim_{\alpha\to 1} Var\left(\Delta Y_{t+1}\right) = 0, \tag{35}$$

$$\lim_{\alpha\to 1} \frac{\partial\left(\xi_{t+1}/\xi_t\right)}{\partial u_{t+1}} > 0, \tag{36}$$

$$\lim_{\alpha\to 1} \left\{E(R_t) - \left(1+r^f\right)\right\} > 0. \tag{37}$$

The intuition behind Lemma 1 is straightforward. While the volatility of aggregate consumption vanishes as α approaches 1, the volatility of existing agents' consumption does not. As α approaches 1, intermediate inputs behave more and more like perfect substitutes. This implies that innovations have a vanishing effect on aggregate output, their only impact being the redistribution from old to young firms and from old to young agents (since $\rho > 0$). Thus, innovation shocks (u_t) are systematic consumption shocks from the perspective of existing agents and affect the pricing kernel as stated by (36), but they are not aggregate shocks in the conventional sense. Since the profits of existing firms are exposed to the innovation shocks (u_t), stock returns of existing companies are correlated with the consumption growth of existing agents and therefore command a risk premium.

Our model also generates a positive value premium without aggregate uncertainty. To see this, consider a claim on future aggregate dividends,[8] which include the dividends to be paid by the current firms and the dividends to be paid by future firms — i.e., dividends due to the exercise of growth opportunities. On the one hand, since aggregate output (and hence aggregate dividends) are deterministic, the claim on aggregate dividends returns the risk-free rate. On the other hand, this return is a weighted average of the return on current firms — i.e., the market return that Lemma 1 shows to be higher than the risk-free rate — and the return on growth opportunities. Thus, we conclude that there exists a positive spread between average returns on assets in place and growth opportunities, which leads to a positive value premium. Since there are no aggregate shocks, the value premium is driven by innovation risk.

[8] We assume for the purpose of this illustration that such a claim has finite value.

The positive value premium in our model is due to the fact that assets in place and the value of growth opportunities have different exposures to the innovation shocks u_t. Specifically, the value of assets in place is negatively exposed to the innovation shock, while the value of growth opportunities has a positive exposure. According to Lemma 1 (equation [36]), innovation shocks command a positive price of risk, and therefore value stocks must earn a higher average rate of return than growth stocks. From the point of view of the agents, the claim on future growth opportunities embedded in growth stocks acts as a hedge against innovation shocks, driving down the expected return on growth stocks. Thus, the rationale behind the value premium in our model is quite different from the explanations proposed previously.[9] Existing models either take risk factors as exogenous and disconnected from economic fundamentals, or derive equilibrium pricing relationships consistent with the conditional CCAPM, in which case value stocks earn higher average returns because of their higher exposure to the aggregate consumption risk. In our model there exists a fundamental risk factor which affects the return differential between value and growth stocks and is distinct from the aggregate consumption growth. We also note that while the limiting case $\alpha = 1$ is a special case of the model,[10] the qualitative results in Lemma 1, and in particular the existence of a distinct displacement risk factor, hold more generally and illustrate why the CCAPM relationship can understate the risks associated with investing in stocks.

The next Lemma establishes a general relationship between realizations of the displacement risk factor and stock returns.

Lemma 2 *Let* $\Phi \equiv \frac{\Pi^I_{j,t}}{\pi^I_{j,t}}$*, where* j *is the index of any intermediate good in* $[0, A_t]$*, and* $\Pi^I_{j,t}$ *is given by (15).*[11] *Define*

$$R^a_{t+1} = \left(\frac{\Phi}{\Phi - 1}\right)\left(\frac{\pi^I_{j,t+1}}{\pi^I_{j,t}}\right), \quad \forall j \in [0, A_t], \tag{38}$$

$$R^o_{t+1} = \left(\frac{Y_{t+1}}{Y_t}\right)\frac{(1-\kappa)\left(1 - e^{-(1+\chi)u_{t+1}}\right) + \varpi\theta^g}{\theta^g}. \tag{39}$$

[9] A representative sample of papers using structural models to analyze the value premium includes Berk et al. (1999), Gomes et al. (2003), Carlson et al. (2004, 2006), and Zhang (2005).

[10] A caveat behind Lemma 1 is that in the limit $\alpha = 1$ the profits of intermediate goods firms disappear. Hence, even though the rate of return on a stock is well defined in the limit (because rates of return are not affected by the level of dividends and prices), the limiting case $\alpha = 1$ is of limited practical relevance. However, it has theoretical interest, because it illustrates in a simple way the asset pricing implications of the wedge between aggregate consumption and existing agents' consumption.

[11] The ratio Φ does not depend on the index j, and thus all value firms have the same P/E, regardless of which intermediate goods they produce. Since the increments to the log stochastic discount factor and the increments to log profits of value firms are i.i.d. across time, Φ is a constant. We show this formally as part of the proof of Proposition 1.

Then, there exists $w^o_{t,s} \in [0,1]$ such that the gross realized return on any firm can be expressed as a weighted average of R^a_{t+1} and R^o_{t+1}:

$$R_{t+1,s} = \left(1 - w^o_{t,s}\right) R^a_{t+1} + w^o_{t,s} R^o_{t+1}. \tag{40}$$

Specifically, $w^o_{t,s} = 0$ for value firms and $w^o_{t,s} \in (0,1)$ for growth firms. Furthermore,

$$\frac{\partial R^a_{t+1}}{\partial u_{t+1}} < 0, \qquad \frac{\partial R^o_{t+1}}{\partial u_{t+1}} > 0. \tag{41}$$

Lemma 2, which holds for any set of parameters, decomposes the gross return on any stock as a weighted average of two basic returns, namely R^a_{t+1} and R^o_{t+1}. R^a_{t+1} is the gross return on a value stock (assets in place), while R^o_{t+1} can be interpreted as the gross return on a pure "growth opportunity", i.e., as the return on the component of a growth stock's market value that is associated with future rather than existing blueprints. The weight $w^o_{t,s}$ reflects the fraction of a stock's value that is due to growth opportunities. For instance, $w^o_{t,s} = 0$ for a value stock, while $w^o_{t,s} \in (0,1)$ for a growth stock. Equations (38), (39), (26), and (27) imply that the difference $\log\left(R^a_{t+1}\right) - \log\left(R^o_{t+1}\right)$ is not affected by ϵ_{t+1}, while Equation (41) shows that it is a declining function of u_{t+1}. In light of equation (40), the same properties hold true for the log-return differential between a portfolio of value stocks and a portfolio of growth stocks.[12] Thus, the log-return differential between value and growth stocks isolates realizations of the displacement factor, and can act as an empirical proxy, or a "mimicking portfolio," for the unobserved displacement factor. This property of our model is supported by the empirical success of asset pricing models featuring a multi-factor specification, which includes a "value-minus-growth" return as one of the factors. (See Fama and French (1993).) Furthermore, in a multisector version of the baseline model[13] with correlated innovation shocks, we argue that the value-minus-growth return is likely to be larger *within* sectors that experience innovation shocks rather than *across* sectors. This result is consistent with the data, and intuitive. The firms that are most negatively affected by new entrants are the entrants' immediate competitors inside the sector, rather than distant competitors in other sectors. Thus, market value decline in response to a sector-specific innovation shock is strongest for assets in place in the same sector, while market value appreciation is strongest for growth options in the same sector.

We conclude this section by discussing the relationship between the equilibrium stochastic discount factor in our model and the Sharpe-Lintner-Mossin CAPM. Without the consumption externality in preferences ($\psi = 1$), the CAPM relationship holds in our model with

[12] An implication of equation (40) is that the return of a stock is also a weighted average of the two basic returns, R^a_{t+1} and R^o_{t+1}.

[13] This extension is contained in the extended online appendix.

respect to the *total* wealth of existing agents, which includes both their stock holdings and their human capital. The reason is that the ratio of consumption to total wealth is constant in our model, and hence a reasoning similar to the one in Section 3.2 implies that when $\psi = 1$, the stochastic discount factor can be expressed in terms of the growth rate of total wealth of existing agents.[14] Since the two components of total wealth (financial wealth and human capital) are not perfectly correlated, the stock market cannot be used as a proxy for total wealth. This well-known critique of the empirical implementations of the CAPM applies, at a theoretical level, within our model.

5 Quantitative Evaluation

5.1 A First Pass

The main departure of the present model from the literature is the presence of a displacement factor (the term inside the square brackets in equation [31]), which should be positively correlated with the return on a value-minus-growth portfolio. In this section, we employ cross-sectional consumption data to asses whether a displacement factor is present in the data, and whether it exhibits the predicted correlation properties. We also measure some key moments in the data, which we subsequently use as targets in a calibration exercise.

In order to measure the magnitude of the displacement factor, one possible approach is to consider an individual at some time $t > s$, evaluate equation (28) at times t and $t+1$, and use equation (31) to obtain

$$\log\left(\frac{c^i_{t+1,s}}{C_{t+1}}\right) - \log\left(\frac{c^i_{t,s}}{C_t}\right) = \log\left[\frac{1}{1-\lambda}\left(1-\lambda\sum_{i\in\{w,e\}}\phi^i\frac{c^i_{t+1,t+1}}{Y_{t+1}}\right)\right]. \tag{42}$$

Since the left-hand side of equation (42) is observable from cross-sectional consumption data, equation (42) suggests a straightforward way to obtain a measurement of the displacement factor: fix a given set of individuals born at some time s prior to t and compute the time-difference in the log-ratio of their consumption *relative* to aggregate consumption.

To measure the left-hand side of (42) we use CEX data on annual consumption for a sequence of cross-sections of households from 1981-2003. The data are available on the NBER website.[15] We provide further details on the usable sample in the appendix. From this

[14] More generally, when $\psi < 1$, the stochastic discount factor is an exponential affine function of the aggregate consumption growth and the wealth growth of existing agents.

[15] The CEX data on the NBER website were compiled by Ed Harris and John Sabelhaus. See http://www.nber.org/ces_cbo/Cexfam.pdf for a description of the data.

Cohorts prior to	1980	1975	1970	1965
Observations	39678	33878	27844	22482
Wald Test (displacement=0)	11.1	6.86	5.44	5.22
p-value	0.000	0.000	0.000	0.000

Table 1: Wald tests of the joint hypothesis that all estimates of the log displacement factor are equal to zero. (Robust estimates of the variance-covariance matrix.)

dataset we obtain a measurement of the consumption of non-durables and services of each household over four quarters, the year corresponding to the first observation quarter, and the age of the head of household at that time. We adjust for family size by dividing household consumption by the average family-equivalence scales reported in Fernandez-Villaverde and Krueger (2007). Since s should be interpreted as the time when an individual is old enough to join the productive forces of the economy, we identify households whose head of household was twenty years or older at some time s. For each of the cross sections $t = 1981, \ldots, 2003$ we compute the average consumption of these households. (Note that since equation [42] holds for any i and $s < t$, it also holds for averages over households that existed at times $s' \leq s$.[16]). We refer to this cross-sectional average of consumption as "existing agents' consumption." Using data on aggregate consumption of non-durables and services from the St. Louis Fed Economic Research Database (FRED), we construct the log-ratio of existing agents' consumption to aggregate consumption and compute its annual change. In light of equation (42), we refer to this annual change as the log displacement factor.

Figure 2 depicts the log displacement factor for various choices of s: 1980, 1975, 1970, and 1965. Because of issues with the discontinuity in the measurement of consumption in the CEX, we follow Vissing-Jorgensen (2002) and drop cross sections prior to 1984. To ensure that the displacement factor is not merely an artefact of sampling error, we first perform the Wald test of the null hypothesis that the log displacement factor is identically equal to 0. As Table 1 shows, for any value of s, this test rejects the null at any conventional level of significance. We conclude that intergenerational risk is less than perfectly shared. We investigate this issue further in the next section by examining cohort effects.

Besides predicting a non-zero log displacement factor, our model also predicts a positive correlation between the log displacement factor and the return on a value-minus-growth portfolio (HML). Figure 3 gives a visual impression of the comovement between the log displacement factor estimated from cross-sectional consumption data and the HML factor[17]

[16]The practical advantage of using such averages is that we can mitigate measurement error in CEX data.

[17]To be consistent with the model, we recomputed the HML factor from the raw data on the website of Kenneth French, using logarithmic (rather than simple) gross returns for growth and value portfolios and

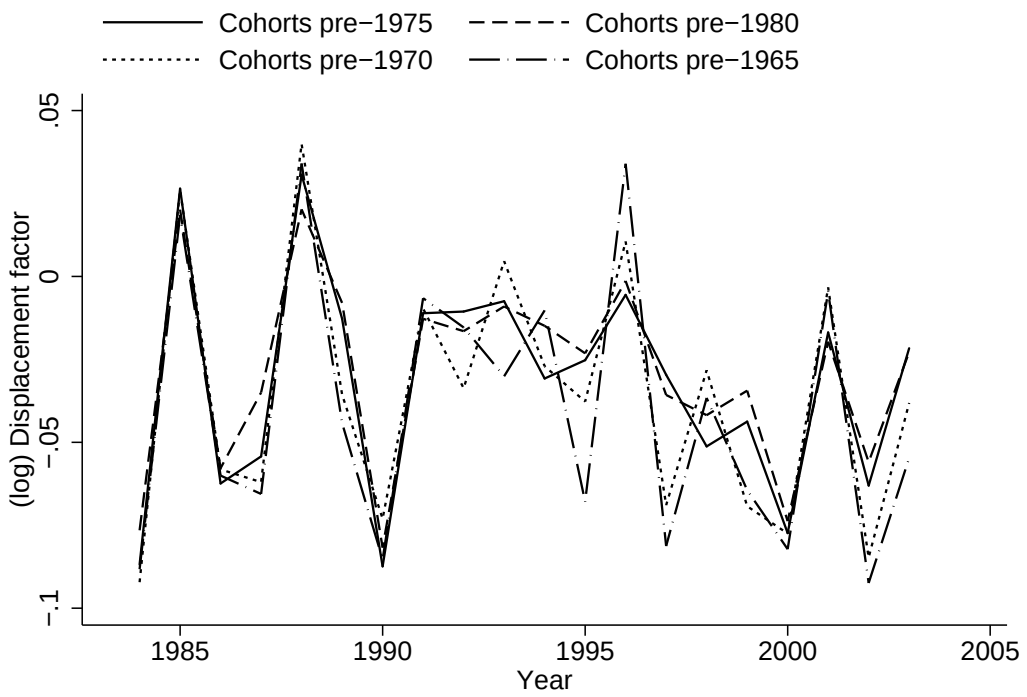

Figure 2: A time-series plot of the estimated log displacement factor (1984-2003). The log displacement factor is computed as the first difference in the log ratio of consumption of various groups of households to aggregate consumption. We group all households whose head of household was 20 years or older in year 1980, 1975, 1970, and 1965 respectively. Data sources: Consumption expenditure survey and St. Louis Fed.

constructed by Fama and French (1993), which is designed to capture the return differential between value and growth stocks. For comparison reasons, we also depict the comovement between the HML factor and aggregate consumption growth. As is well known, aggregate consumption growth has a very weak correlation with HML (-0.01 in the subsample that we consider). By contrast, the correlation between HML and the log displacement factor is 0.27.

To proceed with the quantitative evaluation of the model, one could use the measured variability of the log displacement factor and its correlation with HML to asses whether

taking their difference.

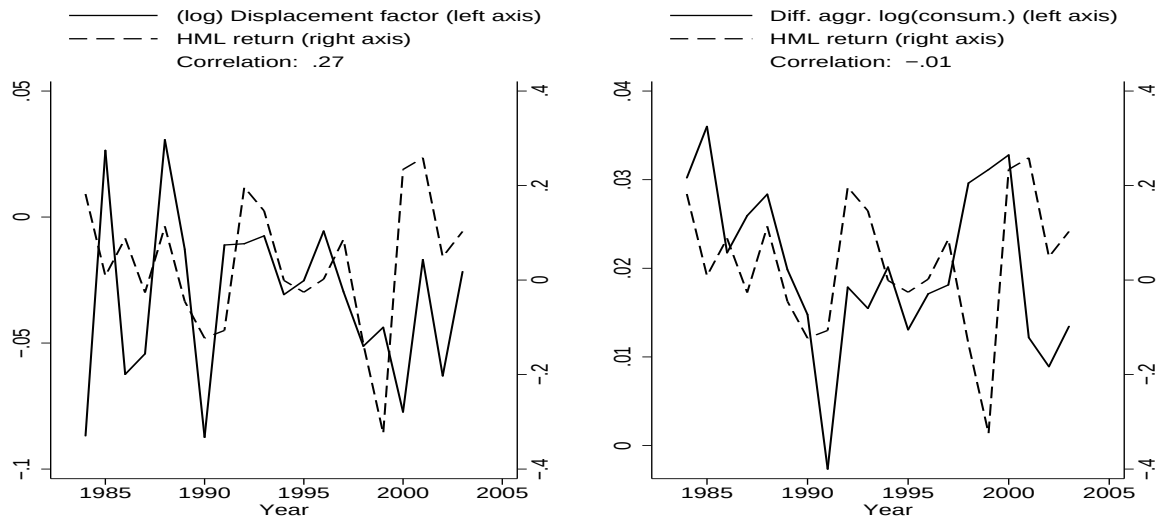

Figure 3: Log displacement factor vs. HML factor (left plot) and Real aggregate consumption growth of non-durables and services vs. HML. (right plot). The computation of the HML factor is identical to Fama and French (1993), except that we use logarithmic rather than simple returns throughout, so as to be consistent with the model. The log displacement factor is computed as the first difference in the log-ratio of consumption of agents that were 20 years or older in $s = 1975$ to aggregate consumption. Results for $s \in \{1965, 1970, 1980\}$ are similar (see Figure 2 for a visual depiction). Data sources: Consumption expenditure survey, St. Louis Fed, and Kenneth French's website.

this extra dimension of systematic risk can account for the key empirical patterns in asset returns. The precision of such an analysis is limited by the length of the historical sample, since consistent cross-sectional consumption data are only available post-1984. As an alternative and complementary approach, we explore an essential implication of our model that individual log consumption exhibits cohort effects that are related to the log displacement factor. Based on this observation, we use the cross-sectional dimension in the CEX to obtain estimates of consumption cohort effects and the displacement factor going back to the early decades of the previous century.

5.2 Cohort Effects and Asset Returns

Econometric specification

According to the model, an individual agent's log consumption can be decomposed into cohort effects (a_s), time effects (b_t), and individual-specific effects $(\varepsilon^i,\ i \in \{e, w\})$:

$$\log c_{t,s}^i = a_s + b_t + \varepsilon_t{}^i, \tag{43}$$

where, according to (28),

$$a_s = \sum_{j\in\{e,w\}} \phi^j \log c_{s,s}^j + \frac{1}{\gamma} \log\left(C_s^{(1-\psi)(1-\gamma)} \beta^{-s} \xi_s\right), \tag{44}$$

$$b_t = -\frac{1}{\gamma} \log\left(\xi_t \beta^{-t} C_t^{(1-\psi)(1-\gamma)}\right), \tag{45}$$

$$\varepsilon_s^i = \log c_{s,s}^i - \sum_{j\in\{e,w\}} \phi^j \log c_{s,s}^j. \tag{46}$$

The next Lemma describes the evolution of the cohort effects according to the model.

Lemma 3 *For any $T \geq 1$, the cohort effect in individual consumption satisfies*

$$a_{s+T} - a_s = -\sum_{i=1}^{T} \log\left(\frac{\upsilon\left(u_{s+i}\right)}{1-\lambda}\right) + z_{s+T} - z_s, \tag{47}$$

where z_s is a series of i.i.d. random variables defined by

$$z_s = (1-\phi) \log\left(\frac{c_{s,s}^w}{Y_s}\right) + \phi \log\left\{\frac{c_{s,s}^e}{Y_s}\right\}.$$

Equation (47) shows why cohort analysis is useful for our purposes. First, it allows us to test for the presence of perfect intergenerational risk sharing. Under perfect risk sharing across generations, all cohort effects are zero.[18] In contrast, cohort effects are non-zero in our model and, moreover, they are non-stationary, since the first term on the right-hand side of (47) is a random walk with drift. Second, and more important, the permanent shocks to the consumption cohort effects (increments to the random-walk component in

[18] The discussion at the end of Section 3.2 implies that, under perfect intergenerational risk sharing, individual consumption equals aggregate consumption per capita irrespective of cohort. Accordingly, individual consumption should exhibit time, but not cohort effects.

(47)) are equal to the unobserved log displacement factor $\log\left(\frac{v(u_{s+i})}{1-\lambda}\right)$.[19] Hence, by using standard econometric techniques to isolate the variance of these permanent increments, we can obtain a measurement of the variance of the displacement factor. Third, cohort effects allow us to exploit the cross-sectional dimension of the CEX, rather than rely on its short time dimension. By using the cross-sectional dimension of the CEX, we can estimate cohort effects for individuals that were born as early as the first decades of the last century. This allows us to obtain a long time path of cohort effects (about 80 years). In light of equation (47), this long path contains information about the behavior of the displacement factor over the same period.

We estimate cohort effects in CEX data by running a regression of household log consumption on time and cohort dummies. To make our results robust to the presence of age effects in the data (for instance due to borrowing constraints early in life, or changes in consumption patterns due to the aging of children), we also allow for age effects, which we model either by including flexible parametric spline functions of age or simply age dummies. We also include a control for log household size.[20]

It is well understood in the empirical literature that linear trends in age, cohort, and time effects cannot be identified separately if age effects are included in equation (43).[21] However, it is possible to uniquely identify differences in differences of cohort effects $(a_{s+1} - a_s - (a_s - a_{s-1}))$ without any normalizing assumptions and even after including a full set of age dummies.[22] Under the null hypothesis that all cohort effects are zero ($a_s = 0$), so should be their differences in differences ($a_{s+1} - a_s - (a_s - a_{s-1}) = 0$). Hence, the first hypothesis we test is that $a_{s+1} - a_s - (a_s - a_{s-1}) = 0$. The three columns of Table 2 report

[19] As we illustrate in the online appendix, the focus on the permanent component of consumption cohorts adds robustness to our analysis with respect to frictions affecting agents' life-cycle of earnings. While such frictions may affect the transitory dynamics of cohort effects and asset returns, they do not change the qualitative relationship between the permanent component of consumption cohorts and the stochastic discount factor.

[20] As a robustness check, we also adjust for family size by dividing by the average family-equivalence scales reported in Fernandez-Villaverde and Krueger (2007). The two approaches produce very similar results, since our estimate for the coefficient of log family size implies an adjusted household consumption that is very similar to the average family-equivalence scales reported in Fernandez-Villaverde and Krueger (2007).

[21] Some of the literature addresses this problem by following Deaton and Paxson (1994) and making the normalizing assumption that the time effects add up to zero and are orthogonal to the time trend. In our model, the time effects b_t follow a random walk and hence such an assumption is not appropriate.

[22] See McKenzie (2006) for a proof. The easiest way to see why identification is possible is to allow for age effects in equation (43) and note that equation $\log c^i_{t,s} = a_s + b_t + \gamma_{t-s} + \varepsilon^i$ implies

$$E\log c^i_{t+1,s+1} - E\log c^i_{t+1,s} - \left(E\log c^i_{t,s} - E\log c^i_{t,s-1}\right) = a_{s+1} - a_s - (a_s - a_{s-1})\,.$$

	No age effects	Parametric Age Effects	Age Dummies
Wald Test $\Delta a_{s+1} - \Delta a_s = 0$	31.3	4.21	4.25
p-value	0.000	0.000	0.000
Observations	52245	52245	52245
R-squared	0.373	0.382	0.384

Table 2: Results from regression of log consumption on time dummies (one dummy for each quarter), annual cohort dummies, and various specifications of age effects. In the first specification the regression does not contain age effects, while the second specification allows age effects parameterized via a cubic spline. The third specification allows for a full set of age dummies. The Wald test refers to the test that $a_{s+1} - a_s - (a_s - a_{s-1}) = 0$ for all s. Standard errors are computed using a robust covariance matrix clustered by cohort and quarter. The CEX data are from 1980-2003 and include observations on cohorts as far back as 1911.

the results of estimating equation (43) including a) no age effects, b) parametric age effects, and c) a full set of age dummies. The model with parametric age effects is fitted by assuming that age effects are given by a function $h(age)$ which we parameterize with a cubic spline having knots at ages 33, 45, and 61. The first row reports the results from a Wald test of $a_{s+1} - a_s - (a_s - a_{s-1}) = 0$ for all s. The second row reports the associated $p-$values. The Wald test rejects the hypothesis that cohort effects are identically zero.

We document the magnitude of variation of cohort effects in the first four rows of Table 3. The first row contains estimates of the standard deviation[23] of the first differences in cohort effects, i.e., $\Delta a_{s+1} \equiv a_{s+1} - a_s$.[24] As we discussed above, *permanent shocks* to consumption cohort effects are equal to the log displacement factor in our model. The second row of the table reports estimates of the standard deviation of permanent shocks to consumption cohort effects, obtained using the methods of Beveridge and Nelson (1981) after fitting an ARIMA (1,1,1) model to the estimated cohort effects. We report two additional estimates of the standard deviation of permanent shocks. The third row contains Newey-West estimates of the long-run variance of Δa_s using 10 lags. In the fourth row, we report the standard deviation

[23]We use only cohorts from 1927-1995 for the calculations in Table 3, because cohorts prior to 1927 and after 1995 are not sufficiently populated. With this choice of sample, the minimal cohort has 199 observations, the first quartile of cohorts has 521 observations and the median cohort has 657 observations. Accordingly, cohorts are sufficiently well populated so that our variance estimate of the first differences of cohorts is not materially affected by sampling error. We would also like to point out that our estimates of the variance of *permanent* components of cohort effects are less affected by (i.i.d) measurement error than the variance of first differences, because heteroscedasticity- and autocovariance-consistent variance estimators control for the moving-average error structure introduced by the noisy measurement of first differences.

[24]Note that Δa_{s+1} is identified up to an additive constant, and hence its standard deviation is identified.

	Param. Age Effects	Age Dummies
Std. Dev. (Cohort−Lagged Cohort)	0.030	0.030
Std. Dev. (Perm. Component (Beveridge Nelson))	0.021	0.023
Std. Dev. (Perm. Component (Newey West))	0.020	0.020
Std. Dev. (Perm. Component $\sqrt{10}$(10-year Aveg))	0.019	0.018
$\frac{\text{Cov(coh.diffs; FF)}}{\text{Var(coh.diffs)}}$ (Newey West)	3.39	3.92
$\frac{\text{Cov(10-year coh.diffs; 10-year FF)}}{\text{Var(10-year coh.diffs)}}$	2.93	3.43
Observations	68	68

Table 3: Various moments of the permanent components of the estimated cohort effects.

of rolling ten-year averages of Δa_s, normalized by $\sqrt{10}$. All three alternative estimates of the volatility of the permanent component of the cohort effects are similar. These estimates form target moments for the calibration exercise of the next subsection.

The last two rows in Table 3 relate increments in consumption cohort effects to cross-sectional differences in stock returns. According to our model increments of the permanent component (the random-walk component) of cohort effects should co-vary positively with the growth-value return differential. In the fifth row of Table 3 we use the Newey-West variance-covariance matrix to estimate this covariance, normalized by the long-run variance of the consumption cohort effects as obtained in the third row of the table. As a robustness check, we also report in the sixth row of the table the results from computing the covariance of 10-year consumption cohort differences and 10-year cumulative returns on the growth-value returns, normalized by the variance of 10-year consumption-cohort differences. These numbers form targets for our calibration exercise.

Figure 4 depicts the estimated cohort effects against the cumulative sum of the (negative of) the HML factor,[25] after removing a deterministic trend from both series. According to equation (47), the random-walk component of the cohort effects is identical to the (negative of) the cumulative sum of the log displacement factor, which in turn should be correlated with the cumulative sum of the HML factor. The left panel of the figure suggests such covariation. (The right panel of Figure 4 is identical to the left panel, except that we split the sample into pre-1945 and post-1945 subsamples and remove two separate deterministic trends in the two subsamples.)

Table 4 reports further results on the relationship between consumption cohort effects, innovation activity, and stock returns. Specifically, the first row in Table 4 reports the average difference between the log gross return on the first nine book-to-market decile portfolios

[25] We report the cohort effects from 1927 onward, since data on the Fama-French HML factor are available from 1927 onwards. We also report results up to 1995 because of the sparsity of data on cohorts post 1995.

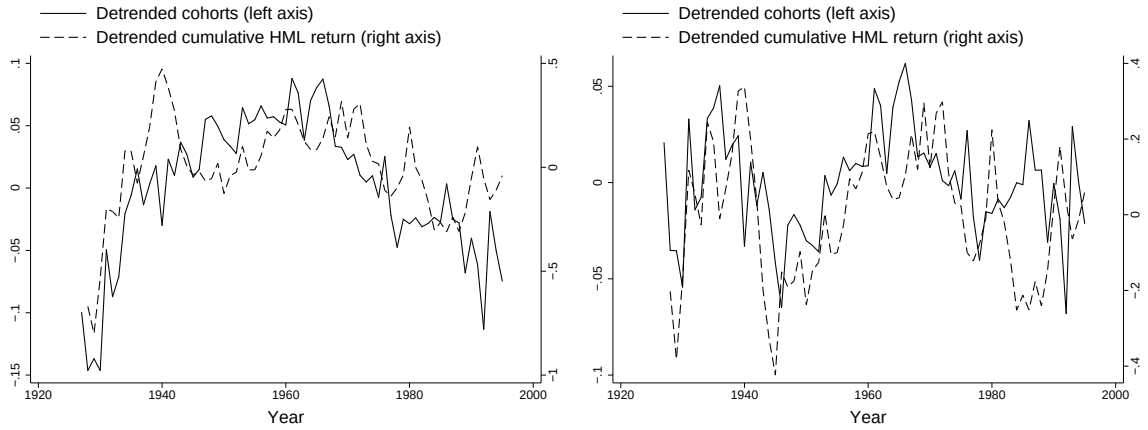

Figure 4: Left panel: Consumption cohort effects and cumulative returns on a growth-value portfolio (negative of the HML factor) after removing a constant time trend from both series. Right panel: Same as left panel, except that we split the sample into pre-1945 and post-1945 subsamples and remove two separate deterministic trends in the two subsamples. A full set of age dummies was used in estimating the consumption cohort effects.

and the respective return on the 10th-decile portfolio. The second row of Table 4 reports the estimates of the betas of these return differences with respect to the increments of the permanent component of log consumption cohort effects. Stocks in low book-to-market deciles (growth stocks) have lower average returns than stocks in high book-to-market deciles (value stocks), which is the well-known value premium. The second row of the table shows that displacement risk exposure decreases across the book-to-market deciles, and therefore growth stocks offer a hedge against the displacement risk. The third row explores an additional implication of the model, namely that the return differential between value and growth stocks is related to innovation activity. Using increases in the aggregate stock of trademarks as a proxy for innovation activity, we report the betas of portfolio differentials on the (percentage) increments in trademarks.[26] The declining pattern of these betas is consistent with the model.[27]

Since the displacement risk in the model is generated partly by shocks to agents' human

[26]We are grateful to F. Belo for providing us the data on trademarks and suggesting this additional test of the model's mechanism.

[27]These betas remain practically unchanged when we include aggregate consumption growth as an additional regressor and compute "multi-factor betas".

Portfolio	1-10	2-10	3-10	4-10	5-10	6-10	7-10	8-10	9-10
Mean log return	-0.033	-0.020	-0.023	-0.028	-0.015	-0.011	-0.016	0.005	0.009
β_1	6.293	7.162	5.609	4.995	5.125	4.184	3.090	3.474	2.636
β_2	1.744	1.822	1.980	1.258	1.163	1.379	1.138	0.153	-0.129

Table 4: Average of returns on long-short book-to-market decile portfolios and their betas with respect to the permanent component of consumption-cohort innovations (β_1) and percentage increases in trademarks (β_2). The first row reports average annual returns. The label n-10 denotes the average log return on a portfolio long the stocks in the nth decile and short those in the 10th decile. The second row (β_1) reports the covariances between these returns and innovations in the permanent component of log consumption cohort effects, normalized by the long-run variance of the latter. To isolate permanent components, covariances and variances are computed using the Newey-West approach with 10 lags. The third row (β_2) reports coefficients from regressing the respective portfolio differentials on the percentage increase in trademarks. The data on returns are from K. French's website (annual 1927-2007). The data on consumption cohorts are from the Consumption expenditure survey. The compilation of trademark data follows Greenwood and Uysal (2005) and covers the time-frame 1930-2000.

capital, as an additional test of the model's mechanism we estimated cohort effects in individual earned income.[28] (Indeed, equations [10] and [11] imply the presence of cohort effects in income data that should be correlated with the cumulative return of a growth-minus-value portfolio.) Consistent with the model, the results using earned-income cohort effects were very similar to the results using consumption cohort effects and we omit them to save space.

5.3 Calibration

Our empirical results suggest that the key predictions of the model are qualitatively consistent with the data. In this section we assess whether the model can account quantitatively for the empirical relationships between asset returns, aggregate consumption growth, and consumption cohort effects.

Our parameter choices are summarized in Table 5. The values of μ and σ are chosen to match the moments of aggregate consumption growth. The parameter α controls the share of profits in aggregate income in the model, according to equation (20). We set $\alpha = 0.8$, which implies a profit share of 16%. In yearly NIPA data for the U.S. since 1929, the average share of (after-depreciation) profits and interest payments is about 15% of national income,

[28] We define earned income as disposable income net of related them to the stock market returns, in the same manner as we did with consumption cohort effects.

or 18% if one imputes that 1/3 of proprietor's income is due to profits.[29] The parameter λ is chosen to capture the arrival of new agents. In post-war data, the average birth rate is about 0.016. Immigration rates are estimated to be between $0.002 - 0.004$, which implies an overall arrival rate of new agents between 0.018 and 0.02. We take the time-discount factor to be close to 1, since in an overlapping generations model the presence of death makes the "effective" discount factor of agents equal to $\beta(1-\lambda)$. Given a choice of $\lambda = 0.02$, the effective discount rate is 0.98, which is a standard choice in the literature. The constant ψ influences the growth rate of agents' marginal utilities, and hence is important for the determination of interest rates. We choose $\psi = 0.5$ in order to approximately match observed interest rates. On behavioral grounds, this assumption implies that an individual places equal weights on his own consumption and on his consumption relative to the aggregate. In the online extended appendix, we investigate the model's performance when agents have standard CRRA preferences. With the exception of the interest rate, which becomes 5.7%, the model's performance is slightly improved along all dimensions.

In the real world, income is hump-shaped as a function of age, whereas in the model age effects are assumed to follow a geometric trend. Since an agent's initial consumption — and hence the stochastic discount factor — is affected by the present value of earnings over the life cycle, we calibrate δ so that, inside the model, the present value of income computed using the empirical age-earnings profile coincides with the one computed using a simple geometric trend with parameter δ. Specifically, we use the estimated age-log earnings profiles of Hubbard et al. (1994) and determine δ so that

$$E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{w_t}{w_s}\right) (1+\delta)^{t-s} \left(\frac{A_t}{A_s}\right)^{-\rho} \quad = \quad E_s \sum_{t=s}^{\infty} \Lambda_{t-s} G_{t-s} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{w_t}{w_s}\right) \left(\frac{A_t}{A_s}\right)^{-\rho},$$

where Λ_{t-s} is an agents' survival probability at age $t-s+20$ (conditional on surviving until age 20) obtained from the National Center for Health Statistics and G_{t-s} is the age-earnings profile, as estimated by Hubbard et al. (1994).

The innovation shocks u_t are drawn from a Gamma distribution with parameters k and ν. Parameters ρ and χ control the exposures of labor and dividend income to the shock u_t. We

[29] Since in our model there is no financial leverage, it seems appropriate to combine dividend and interest payments. Moreover, it also seems appropriate to deduct depreciation from profits, because otherwise the relative wealth of agents e and w would be unduly affected by a quantity that should not be counted as income of either. We note here that our choice of a profit share of 16 percent is consistent with the real business cycle literature, which assumes a capital share (i.e. profits prior to depreciation) of 1/3 and deducts investment from gross profits to obtain dividends. Since in stochastic steady state, investment and depreciation are typically close to each other, the share of net output that accrues to equity holders is approximately equal to the number we assume here.

β	0.999	k	0.25
ψ	0.5	ν	0.05
μ	0.015	ρ	0.9
σ	0.032	κ	0.9
α	0.8	χ	4
λ	0.018	ϖ	0.87
δ	0.012	η	0.9

Table 5: Baseline parameters used in the calibration.

choose k, ν, ρ, and χ jointly to match a) the volatility of the permanent component of consumption cohort effects as reported in Table 3, b) the volatility of the permanent component of income cohort effects,[30] c) the volatility of dividend growth of the market portfolio, and d) the correlation between dividend growth of the market portfolio and aggregate consumption.

The parameter κ controls the proportion of growth opportunities owned by existing firms, which are therefore tradeable, while ϖ controls the decay of existing firms' growth opportunities (high ϖ means that growth opportunities are front-loaded). As a consequence, these two parameters jointly determine the aggregate price-to-earnings ratio, as well as the return properties of growth firms. We therefore calibrate them to the aggregate price-to-earnings ratio and the covariance between HML returns and the log displacement factor, as estimated in the previous subsection. We chose this covariance as a target in calibration because it is directly linked (through the consumer's Euler equation) to the average value premium. In the online extended appendix we consider alternative calibrations that show, in particular, that our results are robust to lower values of κ.

The parameter η affects only the relative weight of assets in place and growth opportunities in firms' values, but it does not affect any aggregate quantity. Therefore we use it to calibrate the cross-sectional spread in price-to-earnings ratios between the top and the bottom price-to-earnings deciles of firms.

We treat the risk-aversion coefficient γ as a free parameter and examine the model's ability to match a number of moments of asset returns and fundamentals for a range of values of γ. As can be seen in Table 6, with $\gamma = 10$ the model can match about 66% of the equity premium and about 80% of the value premium. As γ increases to 12, the model does well in almost all dimensions. In interpreting these results, it should be noted that the model has no financial leverage, which, as Barro (2006) argues, implies that the unlevered equity

[30] We obtain the permanent component of income cohort effects by using earned log income on the left-hand side of Equation (43), estimating the resulting cohort effects and isolating their permanent component, as we did for consumption.

	Data	$\gamma = 10$	$\gamma = 12$	$\gamma = 15$
Aggregate log consumption growth rate	0.017	0.017	0.017	0.017
Aggregate log consumption volatility	0.033	0.032	0.032	0.032
Riskless rate	0.010	0.022	0.015	0.014
Equity premium	0.061	0.040	0.051	0.062
Aggregate earnings / price	0.075	0.103	0.108	0.119
Dividend volatility	0.112	0.10	0.101	0.101
Correl. (divid. growth, cons.growth)	0.2	0.189	0.189	0.189
Std $(\Delta\alpha_s^{\text{perm}})$	0.023	0.024	0.024	0.023
$\frac{cov\left(\Delta\alpha_s^{\text{perm}}, \log R^g - \log R^a\right)}{var\left(\Delta\alpha_s^{\text{perm}}\right)}$	3.92	4.226	4.378	4.598
Std $(\Delta w_s^{\text{perm}})$	0.022	0.023	0.023	0.023
Earnings / price 90th Perc.	0.120	0.11	0.118	0.132
Earnings / price 10th Perc.	0.04	0.041	0.039	0.041
Average value premium	0.081	0.064	0.081	0.097
Std. dev. of value premium	0.120	0.104	0.105	0.105
$\mathrm{E}\left(\log R^o - \log R^a\right)$		0.102	0.121	0.141

Table 6: Data and model moments for different values of risk aversion γ. Data on consumption, the riskless rate, the equity premium, and dividends per share are from Campbell and Cochrane (1999). Data on the aggregate E/P ratio are from the long sample (1871-2005) on R. Shiller's website. The E/P for value and growth firms are the respective E/P ratios of firms in the bottom and top book-to-market deciles from Fama and French (1992). The value premium is computed as the difference in value weighted returns of stocks in the top and bottom book-to-market deciles, available from the website of Kenneth French. Std $(\Delta\alpha_s^{\text{perm}})$ denotes the standard deviation of the permanent component of consumption cohort effects as estimated in Table 3. Std $(\Delta w_s^{\text{perm}})$ refers to the cohort effects of earned income. $\mathrm{E}\left(\log R^o - \log R^a\right)$ is the expected return difference between assets in place and growth opportunities.

premium produced by the model should account for two thirds of the levered equity premium estimated empirically. Moreover, in the model, there is no time variation in interest rates, stock return volatility, and the conditional equity premium. Therefore, it is not surprising that the model needs relatively high levels of risk aversion to match the data. However, even in the absence of time-varying conditional moments of returns, levels of risk aversion around 10 explain a substantial fraction of return moments. Therefore, the evidence in Table 6 suggests that the model's mechanisms are quantitatively powerful enough to match the salient moments of asset returns and macroeconomic fundamentals.

5.4 Inspecting the mechanism

In this section, we attribute the quantitative performance of our model to its key ingredients.

Our model produces large equity and value premia for three reasons. First, current agents' consumption growth is more volatile than aggregate consumption growth because of the displacement risk. Second, current firms' dividends are more sensitive to the displacement risk factor than current agents' consumption. Third, there is co-skewness between current firms' dividend growth and current agents' consumption growth.

A simple back-of-the-envelope calculation helps illustrate the magnitude of each factor. Taking logarithms of the pricing kernel in equation (31), using (5), (26), and the definition of $\upsilon(u_{t+1})$ in equation (32) leads to

$$\Delta \log \xi_{t+1} + \text{const} = \underbrace{(-1+\psi\,(1-\gamma))\varepsilon_{t+1}}_{\text{std. dev.: 0.18}} - \underbrace{\gamma\upsilon(u_{t+1}) + (-1+\psi\,(1-\gamma))\,(1-\alpha)\,u_{t+1}}_{\text{std. dev.: 0.2}}. \quad (48)$$

The numbers under the two components in equation (48) are the annual standard deviation of each term. The stochastic component of returns on the aggregate stock market portfolio equals $\varepsilon_{t+1} + f(u_{t+1})$, where $f(u_{t+1})$ is an appropriate function of the displacement shock. The volatility of ε_t is 0.032 and the volatility of the term $f(u_{t+1})$ is approximately 0.95. Thus, if all the components of aggregate market returns and the stochastic discount factor were jointly normally distributed, then the equity premium would equal approximately $0.18 \times 0.032 + 0.2 \times 0.095 = 0.025$. The difference between this number and the equity premium of 0.04 in our base-case calibration owes to the fact that the shock u_{t+1} is not Gaussian, making the consumption growth of existing agents and the stock market returns co-skewed.

In our model, volatility of stock market returns is largely induced by dividend growth volatility, which is equal to $\sqrt{0.032^2 + 0.095^2} = 0.1$. The model can generate a higher volatility of stock market returns than of consumption growth because future dividends of existing firms are not co-integrated with future aggregate consumption, or even with existing agents' consumption. This lack of co-integration allows dividend growth to be much more volatile than consumption growth, with both driven by *permanent* shocks. Such long-run dynamics of dividends and consumption are mutually consistent. Dividends of existing firms become a negligible fraction of aggregate consumption over time, while the aggregate dividends paid by all firms at any point in time are a constant fraction of aggregate consumption.

6 Conclusion

Innovation activity raises productivity and aggregate output. The benefits of innovative activity, however, are unequally shared. In an overlapping-generations model where the

young benefit more from innovative activity than the old, and existing agents cannot trade with unborn generations, we show that the process of innovation can give rise to a new risk factor, the displacement risk factor. At a qualitative level, this factor can help raise the equity premium in aggregate time-series data and explain the return differential between value and growth stocks in cross-sectional data.

Cross-sectional consumption data allow us to test quantitatively for the presence, and measure the magnitude, of the displacement risk factor. Our empirical results suggest that displacement risk is non-trivial and is related to the HML factor introduced by Fama and French (1993), while our calibration exercises suggest that this new source of risk is quantitatively important enough to explain significant fractions of the equity and the value premia.

Our model abstracts from many elements of asset-price behavior, intergenerational transfers, life-cycle effects, cross-sectional characteristics of firms, etc., to highlight the economic intuition behind the displacement risk factor. Our framework can be enriched and extended in a number of important directions, which we leave for future work.

Appendix

A Auxiliary Results and Proofs

Proposition 1 *Let ζ be defined as*

$$\zeta \equiv \beta (1-\lambda)^{\gamma} e^{\mu\psi(1-\gamma)+\frac{\sigma^2}{2}\psi^2(1-\gamma)^2}$$

and consider the solution to the following system of three equations in three unknowns θ^e, θ^w, and θ^g:

$$\theta^e = \frac{1-\zeta E_t\left[e^{\psi(1-\alpha)(1-\gamma)u_{t+1}}\upsilon(u_{t+1};\theta^e,\theta^w,\theta^g)^{1-\gamma}\right]}{1-\zeta E_t\left[e^{((1-\alpha)\psi(1-\gamma)-(1+\chi))u_{t+1}}\upsilon(u_{t+1};\theta^e,\theta^w,\theta^g)^{-\gamma}\right]} \tag{49}$$

$$\theta^w = \frac{1-\zeta E_t\left[e^{\psi(1-\alpha)(1-\gamma)u_{t+1}}\upsilon(u_{t+1};\theta^e,\theta^w,\theta^g)^{1-\gamma}\right]}{1-\zeta(1-\lambda)(1+\delta)E_t\left[e^{((1-\alpha)\psi(1-\gamma)-\rho)u_{t+1}}\upsilon(u_{t+1};\theta^e,\theta^w,\theta^g)^{-\gamma}\right]} \tag{50}$$

$$\theta^g = \frac{\zeta E_t\left[e^{(1-\alpha)\psi(1-\gamma)u_{t+1}}\upsilon(u_{t+1};\theta^e,\theta^w,\theta^g)^{-\gamma}\left(1-e^{-(1+\chi)u_{t+1}}\right)(1-\kappa)\right]}{1-\varpi\zeta E_s\left[e^{(1-\alpha)\psi(1-\gamma)u_{t+1}}\upsilon(u_{t+1};\theta^e,\theta^w,\theta^g)^{-\gamma}\right]}. \tag{51}$$

Here,

$$\begin{aligned}\upsilon(x;\theta^e,\theta^w,\theta^g) \equiv\ & 1-\theta^e\alpha(1-\alpha)\left(\kappa\left(1-e^{-(1+\chi)x}\right)+(1-\varpi)\theta^g\right) \\ & -\theta^w\left(\alpha^2+1-\alpha\right)\left(1-(1-\lambda)(1+\delta)e^{-\rho x}\right).\end{aligned} \tag{52}$$

Assuming positivity of the numerators and denominators in (49) and (50) and positivity of the denominator in (51), there exists an equilibrium with stochastic discount factor

$$\frac{\xi_{t+1}}{\xi_t} = \beta\left(\frac{Y_{t+1}}{Y_t}\right)^{-1+\psi(1-\gamma)}\left[\frac{1}{1-\lambda}\upsilon(u_{t+1};\theta^e,\theta^w,\theta^g)\right]^{-\gamma}. \tag{53}$$

Proof of Proposition 1. To prove Proposition 1 we conjecture that the expression $\frac{c^i_{t+1,t+1}}{Y_{t+1}}$ is exclusively a function of u_{t+1}, and then confirm our conjecture based on the resulting expression for $\frac{\xi_{t+1}}{\xi_t}$. To start, we note that if $\frac{c^i_{t+1,t+1}}{Y_{t+1}}$ is exclusively a function of u_{t+1}, then an appropriate function $f(u_{t+1})$ exists such that the stochastic discount factor is given by $\frac{\xi_{t+1}}{\xi_t}$ $= \beta\left(\frac{Y_{t+1}}{Y_t}\right)^{-1+\psi(1-\gamma)} \times f(u_{t+1})$.

To determine $c^i_{t+1,t+1}$ for a worker ($i = w$) under this conjecture for $\frac{\xi_{t+1}}{\xi_t}$ we use (28), (10), and the fact that $h_{t,s} = \overline{h}\,(1+\delta)^{t-s}$ inside the intertemporal budget constraint (13) to obtain

$$c^w_{s,s} = \overline{h} q_{s,s} w_s \left[\frac{E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{w_t}{w_s}\right) (1+\delta)^{t-s} \left(\frac{A_t}{A_s}\right)^{-\rho}}{E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{Y_t^{(1-\psi)(1-\gamma)}}{Y_s^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s}\right)^{-\frac{1}{\gamma}}} \right]. \tag{54}$$

Under our conjecture the expression ξ_{t+1}/ξ_t is a deterministic function of ε_{t+1} and u_{t+1}, and it follows that the distribution of $\frac{\xi_t}{\xi_s}$ for $t \geq s$ depends *only* on $t - s$. The same is true for A_t/A_s and for w_t/w_s (by equation [23]). Therefore, the expectations in both the numerator and the denominator inside the square brackets of equation (54) are time-invariant constants. Hence, using (10) we can express (54) as

$$c^w_{s,s} = \overline{h} \left(1 - (1-\lambda)(1+\delta) e^{-\rho u_s}\right) w_s \theta^w, \tag{55}$$

where θ^w is defined as

$$\theta^w \equiv \frac{E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{w_t}{w_s}\right) (1+\delta)^{t-s} \left(\frac{A_t}{A_s}\right)^{-\rho}}{E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{Y_t^{(1-\psi)(1-\gamma)}}{Y_s^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s}\right)^{-\frac{1}{\gamma}}}, \tag{56}$$

governing the ratio between the value of earned wages and consumption.

The initial consumption of new business owners born at time s can be computed in a similar fashion. To start, we observe that

$${\Pi^I}_{j,s} = \pi^I_{j,s} \left[E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{\pi^I_{j,t}}{\pi^I_{j,s}}\right) \right]. \tag{57}$$

Our conjecture on $\frac{\xi_{t+1}}{\xi_t}$ and (27) imply that the expression inside square brackets in (57) is a constant. Observing that $A_s - A_{s-1} = A_s\left(1 - e^{-u_s}\right)$ (from [9]) and that $\int_{A_{s-1}}^{A_s} \pi^I_{j,s} dj = \left(1 - e^{-(1+\chi)u_s}\right) \alpha (1-\alpha) Y_s$ (from [27]), and using (57) inside (16) gives

$$\begin{aligned} V_{s,s} &= \alpha(1-\alpha) Y_s \times \left\{ E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{\pi^I_t}{\pi^I_s}\right) \right\} \\ &\quad \times \left\{ \kappa \left(1 - e^{-(1+\chi)u_s}\right) + (1-\varpi) E_s \sum_{t=s+1}^{\infty} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{Y_t}{Y_s}\right) (1-\kappa) \left(1 - e^{-(1+\chi)u_t}\right) \varpi^{t-s-1} \right\}. \end{aligned}$$

It will be useful to define

$$\theta^e \equiv \frac{E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{\pi_t^I}{\pi_s^I}\right)}{E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{Y_t^{(1-\psi)(1-\gamma)}}{Y_s^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s}\right)^{-\frac{1}{\gamma}}}, \tag{58}$$

which, in analogy to θ^w, governs the ratio between the values of future dividends (therefore cashflows to an entrepreneur owning a value firm, and consumption) and

$$\theta^g \equiv E_s \sum_{t=s+1}^{\infty} \left(\frac{\xi_t}{\xi_s}\right) \left(\frac{Y_t}{Y_s}\right) (1-\kappa) \left(1 - e^{-(1+\chi)u_t}\right) \varpi^{t-s-1}, \tag{59}$$

which encodes the (normalized) value of growth options.

The maintained conjecture that ξ_{t+1}/ξ_t is a deterministic function of ε_{t+1} and u_{t+1} and equation (27) imply that θ^e and θ^g are both constants. Using (28) inside (14),

$$c_{s,s}^e = \frac{\theta^e}{\phi} \frac{\alpha (1-\alpha) Y_s}{\lambda} \left\{\kappa \left(1 - e^{-(1+\chi)u_s}\right) + (1-\varpi)\theta^g\right\}. \tag{60}$$

Combining (55) and (60) and noting that s in equations (60) and (55) is arbitrary, we obtain

$$\begin{aligned} \sum_{i \in \{w,e\}} \phi^i \frac{c_{t+1,t+1}^i}{Y_{t+1}} &= \theta^e \frac{1}{\lambda} \left\{\kappa \left(1 - e^{-(1+\chi)u_{t+1}}\right) + (1-\varpi)\theta^g\right\} \alpha (1-\alpha) \\ &\quad + \overline{h}\theta^w \left(1 - (1-\lambda)(1+\delta) e^{-\rho u_{t+1}}\right) \left(\alpha^2 + 1 - \alpha\right), \end{aligned} \tag{61}$$

which is a deterministic function of u_{t+1}. Using (61) and $\overline{h} = 1/\lambda$ inside (31) verifies the conjecture that there exists an equilibrium with $\frac{\xi_{t+1}}{\xi_t} = \beta \left(\frac{Y_{t+1}}{Y_t}\right)^{-1+\psi(1-\gamma)} \times f(u_{t+1})$ where $f(u_{t+1})$ is given by $f(u_{t+1}) = \left[\frac{1}{1-\lambda} \upsilon(u_{t+1}; \theta^e, \theta^w, \theta^g)\right]^{-\gamma}$. This proves equation (53).

To obtain equations (49), (50), and (51) we use the formula (53) for the growth of the stochastic discount factor in the definitions (56), (58), and (59). We start by computing the term inside square brackets in equation (57). Since $\left(\frac{\xi_{i+1}}{\xi_i}\right) \left(\frac{\pi_{j,i+1}^I}{\pi_{j,i}^I}\right)$ is an i.i.d random variable

for any i, it follows that

$$\begin{aligned}
E_s \sum_{t=s}^{\infty} \left(\frac{\xi_t}{\xi_s}\right)\left(\frac{\pi_{j,t}^I}{\pi_{j,s}^I}\right) &= \sum_{t=s}^{\infty} E_s \left(\frac{\xi_t}{\xi_s}\right)\left(\frac{\pi_{j,t}^I}{\pi_{j,s}^I}\right) = \sum_{t=s}^{\infty} E_s \prod_{i=s}^{t-1} \left(\frac{\xi_{i+1}}{\xi_i}\right)\left(\frac{\pi_{j,i+1}^I}{\pi_{j,i}^I}\right) \\
&= \sum_{t=s}^{\infty} \prod_{i=s}^{t-1} E_s \left(\frac{\xi_{i+1}}{\xi_i}\right)\left(\frac{\pi_{j,i+1}^I}{\pi_{j,i}^I}\right) = \sum_{t=s}^{\infty} \left[E_s \left(\frac{\xi_{s+1}}{\xi_s}\right)\left(\frac{\pi_{j,s+1}^I}{\pi_{j,s}^I}\right)\right]^{t-s} \\
&= \frac{1}{1 - E_s \left(\frac{\xi_{s+1}}{\xi_s}\right)\left(\frac{\pi_{j,s+1}^I}{\pi_{j,s}^I}\right)} \qquad (62) \\
&= \frac{1}{1 - \zeta E_s \left[e^{((1-\alpha)\psi(1-\gamma)-(1+\chi))u_{s+1}} \upsilon(u_{s+1}; \theta^e, \theta^w, \theta^g)^{-\gamma}\right]},
\end{aligned}$$

where the last equality follows from (53):

$$\begin{aligned}
\mathrm{E}_s \left[\frac{\xi_{s+1}}{\xi_{s+1}} \frac{\pi_{j,s}^I}{\pi_{j,s}^I}\right] &= \beta \mathrm{E}_s \left[\left(Z_{s+1} e^{(1-\alpha)u_{s+1}}\right)^{\psi(1-\gamma)-1} \left(Z_{s+1} e^{-(\alpha+\chi)u_{s+1}}\right)\right] \\
&= \beta \mathrm{E}_s \left[Z_{s+1}^{\psi(1-\gamma)} e^{((1-\alpha)\psi(1-\gamma)-(1+\chi))u_{s+1}}\right].
\end{aligned}$$

Following a similar reasoning,

$$\begin{aligned}
& E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right)\left(\frac{w_t}{w_s}\right)(1+\delta)^{t-s}\left(\frac{A_t}{A_s}\right)^{-\rho} \\
&= \frac{1}{1 - (1-\lambda)(1+\delta)\zeta E_s \left[e^{((1-\alpha)\psi(1-\gamma)-\rho)u_{s+1}} \upsilon(u_{s+1}; \theta^e, \theta^w, \theta^g)^{-\gamma}\right]} \qquad (63)
\end{aligned}$$

and

$$\begin{aligned}
& E_s \sum_{t=s}^{\infty} (1-\lambda)^{t-s} \left(\frac{\xi_t}{\xi_s}\right)\left(\frac{Y_t^{(1-\psi)(1-\gamma)}}{Y_s^{(1-\psi)(1-\gamma)}} \beta^{-(t-s)} \frac{\xi_t}{\xi_s}\right)^{-\frac{1}{\gamma}} \\
&= \frac{1}{1 - \zeta E_s \left[e^{\psi(1-\alpha)(1-\gamma)u_{s+1}} \upsilon(u_{s+1}; \theta^e, \theta^w, \theta^g)^{1-\gamma}\right]}. \qquad (64)
\end{aligned}$$

Finally,

$$
\begin{aligned}
& E_s \sum_{t=s+1}^{\infty} \left(\frac{\xi_t}{\xi_s}\right)\left(\frac{Y_t}{Y_s}\right)\left(1-e^{-(1+\chi)u_t}\right)(1-\kappa)\,\varpi^{t-s-1} \\
&= E_s \sum_{t=s+1}^{\infty} \left[\prod_{i=s+1}^{t}\left(\frac{\xi_i}{\xi_{i-1}}\right)\left(\frac{Y_i}{Y_{i-1}}\right)\varpi\right]\varpi^{-1}\left(1-e^{-(1+\chi)u_t}\right)(1-\kappa) \\
&= \sum_{t=s+1}^{\infty} E_s\left\{\left[\prod_{i=s+1}^{t-1}\left(\frac{\xi_i}{\xi_{i-1}}\right)\left(\frac{Y_i}{Y_{i-1}}\right)\varpi\right]\left(\frac{\xi_t}{\xi_{t-1}}\right)\left(\frac{Y_t}{Y_{t-1}}\right)\left(1-e^{-(1+\chi)u_t}\right)(1-\kappa)\right\} \qquad (65) \\
&= \left\{\sum_{t=s+1}^{\infty}\left\{\prod_{i=s+1}^{t-1} E_s\left(\frac{\xi_i}{\xi_{i-1}}\right)\left(\frac{Y_i}{Y_{i-1}}\right)\varpi\right\}\right\} \times E_t\left[\left(\frac{\xi_{t+1}}{\xi_t}\right)\left(\frac{Y_{t+1}}{Y_t}\right)\left(1-e^{-(1+\chi)u_{t+1}}\right)(1-\kappa)\right] \\
&= \frac{\zeta E_s\left[e^{(1-\alpha)\psi(1-\gamma)u_{s+1}}\upsilon(u_{s+1};\theta^e,\theta^w,\theta^g)^{-\gamma}\left(1-e^{-(1+\chi)u_{s+1}}\right)(1-\kappa)\right]}{1-\varpi\zeta E_s\left\{e^{(1-\alpha)\psi(1-\gamma)u_{s+1}}\upsilon(u_{s+1};\theta^e,\theta^w,\theta^g)^{-\gamma}\right\}}.
\end{aligned}
$$

Combining (65) with (59) leads to (51). Similarly, combining (58) with (64) and (62) leads to (49), while combining (56), (63), and (64) implies (50). ■

Proof of Lemma 1. To establish that the equity premium is non-zero in the limit, it suffices to show that

$$\lim_{\alpha\to 1}\operatorname{cov}\left\{R_t,(\xi_{t+1}/\xi_t)\right\}\neq 0. \qquad (66)$$

Since $\kappa=1$, all stocks have rate of return

$$R_t=\frac{\left(\pi^I_{t+1}/\pi^I_t\right)+\left(\Pi_{t+1}/\pi_{t+1}\right)}{\Pi_t/\pi_t}.$$

Equation (57) implies that (Π_t/π_t) is a constant. Therefore, in order to establish (66) it suffices to show that $\lim_{\alpha\to 1}\operatorname{cov}\left(\left(\pi^I_{t+1}/\pi^I_t\right),(\xi_{t+1}/\xi_t)\right)\neq 0$. To see that this is the case note that $\lim_{\alpha\to 1}\left(\pi^I_{t+1}/\pi^I_t\right)=e^{\mu-(1+\chi)u_{t+1}}$. Hence, in order to establish (66), we need to show that ξ_{t+1}/ξ_t is a non-degenerate function of u_{t+1} as $\alpha\to 1$. Given that

$$\lim_{\alpha\to 1}\left(\frac{\xi_{t+1}}{\xi_t}\right) = \beta(1-\lambda)^{\gamma}e^{\mu(-1+\psi(1-\gamma))}\left[1-\theta^w\left(1-(1-\lambda)(1+\delta)e^{-\rho u_{t+1}}\right)\right]^{-\gamma}, \qquad (67)$$

the lemma holds as long as a solution $\theta^w>0$ exists to Equation (50), an equation that simplifies to

$$
\begin{aligned}
& \theta^w\left(1-\zeta(1-\lambda)(1+\delta)E\left[e^{-\rho u}\left(1-\theta^w\left(1-(1-\lambda)(1+\delta)e^{-\rho u_t}\right)\right)^{-\gamma}\right]\right) \\
&= 1-\zeta E\left[\left(1-\theta^w\left(1-(1-\lambda)(1+\delta)e^{-\rho u_t}\right)\right)^{1-\gamma}\right]. \qquad (68)
\end{aligned}
$$

By expanding the right-hand side of (68), the equation further simplifies to

$$1 \;=\; \zeta E\left[\left(1-\theta^{w}\left(1-(1-\lambda)(1+\delta)\,e^{-\rho u_t}\right)\right)^{-\gamma}\right]. \tag{69}$$

As the right-hand side increases in θ^w, and the probability of the event $\{u_t \in (0,\epsilon)\}$ is strictly positive for all $\epsilon > 0$, conditions (33) and (34) are necessary and sufficient for the existence of a solution $\theta^w > 0$. (Note that $\theta^w \leq 1$.) ■

Proof of Lemma 2. Without loss of generality, we focus on the representative growth firm. (A value firm is a special case of a growth firm with no growth opportunities). We start by showing the following result.

Lemma 4 *The (end of) period-t value of the representative growth firm created at time s is given by*

$$P_{t,s} = \alpha\,(1-\alpha)\,Y_t\left[(\Phi-1)\,N_{t,s} + (1-\varpi)\,\varpi^{t-s}\theta^{g}\Phi\right], \tag{70}$$

where

$$\begin{aligned} N_{t,s} \;=\; & (1-\eta)\kappa\left(\frac{A_s}{A_t}\right)^{1+\chi}\left(1-e^{-(1+\chi)u_s}\right) + \\ & \sum_{n=s+1}^{t}(1-\varpi)\,(1-\kappa)\,\varpi^{n-(s+1)}\left(\frac{A_n}{A_t}\right)^{1+\chi}\left(1-e^{-(1+\chi)u_n}\right). \end{aligned}$$

Proof of Lemma 4. The value of a growth firm is given by the value of all its assets in place and all its growth options.

$$\begin{aligned} P_{t,s} \;=\; & (1-\eta)\kappa\left(\int_{A_{s-1}}^{A_s}\Pi^{I}_{j,t}dj\right) + \sum_{n=s+1}^{t}(1-\varpi)\,(1-\kappa)\,\varpi^{n-(s+1)}\left(\int_{A_{n-1}}^{A_n}\Pi^{I}_{j,t}dj\right) \\ & + (1-\varpi)\left[E_t\sum_{n=t+1}^{\infty}\left(\frac{\xi_n}{\xi_t}\right)\varpi^{n-(s+1)}\left(\int_{A_{n-1}}^{A_n}\Pi^{I}_{j,n}dj\right)(1-\kappa)\right] \end{aligned}$$

Using the definition of Φ, and noting that $\int_{A_{s-1}}^{A_s}\pi^{I}_{j,t}dj = \left(\frac{A_s}{A_t}\right)^{1+\chi}\left(1-e^{-(1+\chi)u_s}\right)$ along with the definition of θ^g in equation (59) leads to (70). ■

The gross return on a growth firm R^{g}_{t+1} at time $t+1$ is given by sum of the time-$t+1$ dividends from all the blueprints collected by the firm up to and including period $t+1$, $\alpha\,(1-\alpha)\,Y_{t+1}N_{t+1,s}$, and the end-of-period price $P_{t+1,s}$, divided by the beginning-of-period price $P_{t,s}$:

$$R^{g}_{t+1,s} \equiv \frac{\alpha\,(1-\alpha)\,Y_{t+1}N_{t+1,s} + P_{t+1,s}}{P_{t,s}}.$$

Using the definitions of R^a_{t+1} and R^o_{t+1}, as given in the statement of the Lemma, the gross rate of return on any growth firm can be expressed as in equation (40), where $w^o_{t,s}$ is the relative weight of growth opportunities in the value of the firm, and is obtained from Lemma 4 as

$$w^o_{t,s} = \frac{(1-\varpi)\,\varpi^{t-s}\theta^g\Phi}{(\Phi-1)\,N_{t,s} + (1-\varpi)\,\varpi^{t-s}\theta^g\Phi}.$$

Combining (27) and (26) with (38) and (39), we obtain that the return on assets in place has a negative loading on the innovation shock, $\frac{\partial R^a_{t+1}}{\partial u_{t+1}} < 0$, while the return on the growth-opportunity component of the firm value has a positive loading, $\frac{\partial R^o_{t+1}}{\partial u_{t+1}} > 0$. ■

Proof of Lemma 3. Equation (44) implies that

$$a^i_{s+1} - a^i_s = \log c^i_{s+1,s+1} - \log c^i_{s,s} + \frac{1}{\gamma}\log\left(C^{(1-\psi)(1-\gamma)}_{s+1}\beta^{-(s+1)}\xi_{s+1}\right) - \frac{1}{\gamma}\log\left(C^{(1-\psi)(1-\gamma)}_{s}\beta^{-s}\xi_{s}\right). \quad (71)$$

Using (53) inside (71) along with $C_s = Y_s$ and simplifying gives

$$a^i_{s+1} - a^i_s = \log c^i_{s+1,s+1} - \log c^i_{s,s} - \log\left(\frac{C_{s+1}}{C_s}\right) - \log\left[\frac{1}{1-\lambda}\upsilon(u_{s+1};\theta^e,\theta^w,\theta^g)\right].$$

Using the definitions of a_s and z_s, (55), and (60), noting that $C_s = Y_s$, and simplifying gives

$$a_{s+1} - a_s = -\log\left[\frac{1}{1-\lambda}\upsilon(u_{s+1};\theta^e,\theta^w,\theta^g)\right] + z_{s+1} - z_s. \quad (72)$$

Equation (47) follows immediately. ■

B Data Description

The CEX data are from the NBER website as compiled by Ed Harris and John Sabelhaus. See http://www.nber.org/ces_cbo/Cexfam.pdf for a detailed description of the data. In short, the data set compiles the results from the four consecutive quarterly interviews in the CEX into one observation for each household. We follow a large literature (see e.g. Vissing-Jorgensen (2002)) and drop from the sample households with incomplete income responses, households who haven't completed one of the quarterly interviews, and households that reside in student housing. To ensure that data selection does not unduly affect the results, we also ran all the regressions on the raw data including dummies for reporting status and the number of completed interview quarters. The results were not affected in any essential way.

A more delicate issue concerns the definition of consumption. In section 5.1 we used consumption of non-durables and services throughout (i.e. total consumption expenditure excluding medical and educational expenses, housing, furniture and automotive related expenses), because in this section we are interested in the Euler equation of existing agents. In section 5.2, however, we followed Fernandez-Villaverde and Krueger (2007) and used a comprehensive measure of consumption expenditure. Specifically, we used exactly the same definition as Harris and Sabelhaus. Our choice is motivated by our model; according to the model, cohort effects are determined by the intertemporal budget constraint of agents born at different times, so that total consumption expenditure seems to be the appropriate concept for the estimation of cohort effects. To test if this choice materially affects our conclusions, we also ran the results using consumption of non-durables and services instead of total consumption expenditure. Using consumption of non-durables and services, the volatility of first differences of cohort effects was larger; however, the there was not a big difference between the variance of the permanent components of the cohort effects, no matter which concept of consumption we used, consistent with the view that the two concepts of consumption share the same stochastic trend.

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