# Trading Frenzy and Its Impact on Real Investment

Itay Goldstein, Emre Ozdenoren, and Kathy Yuan*

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ABSTRACT

We study a model where a capital provider learns from the price of a firm's stock when making a decision how much capital to provide for new investment. The efficiency of the investment decision and the volatility of prices and investments depend crucially on the strategic interaction among speculators in the market. We show that speculators almost always coordinate either too much or too little, reducing the effectiveness of the financial market in guiding the investment decision. Our model gives rise to phenomena similar to what has happened in the recent crisis where speculators coordinate to 'run' on a stock, pushing down its price in a way that deprives the firm of capital and exacerbates the underlying problem. We study policy measures that alter speculators' behavior and improve the efficiency of investment.

*Goldstein is from the University of Pennsylvania; e-mail: itayg@wharton.upenn.edu. Ozdenoren is from the London Business School and the University of Michigan; e-mail: eozdenoren@london.edu and emreo@umich.edu. Yuan is from the London School of Economics; e-mail: K.Yuan@lse.ac.uk.

## I. Introduction

Recent events have focused attention again on the role of financial markets in the economy. Traditionally, financial markets are perceived to be crucial for a well-functioning economy. One channel via which this happens goes back to Hayek (1945), according to which market prices provide important information for various decision makers, and this increases real efficiency. On the other hand, given wide swings in market prices and the resulting disruptions in real economic activity, commentators are increasingly calling regulatory authorities to put restrictions on financial market trading. For example, after the fall of Bear Stearns and Lehman Brothers, some had the view that damaging short sales led to the collapse in the price, which later amplified itself in the collapse of these firms themselves after they were deprived of further capital due to the deteriorating stock price. This led regulatory authorities in the US and the UK to put restrictions on short selling activities.[1]

At the heart of this debate lies the interaction among speculators in a financial market and the effect that it has on asset prices and real economic activities. Providing a theoretical model that analyzes these issues is important for enriching the debate and guiding future policy. Our goal in this paper is to develop such a model.

In our model, a capital provider has to decide how much capital to provide to a firm for the purpose of making new real investment. The decision of the capital provider depends on his assessment of the productivity of the proposed investment, as it determines his expected profit. In his decision, the capital provider uses some private information and also the information conveyed by the price of the firm's traded asset as determined in the financial market. The reliance of capital provision on financial-market prices establishes the effect that the stock market has on the real economy, an effect that was referred to in the literature as the *feedback effect*.

The effectiveness by which the financial market guides capital provision and real investment depends on the process of price formation and hence on the strategic interaction among speculators in the financial market. This strategic interaction is the center of our theory. Our model has many small speculators, each one having access to two signals about the productivity of the firm's investment. The first signal is independent across speculators

[1]In executing a naked-shortsale-ban order on July 15, 2008, the SEC concluded that "there now exists a substantial threat of sudden and excessive fluctuations of securities prices generally and disruption in the functioning of the securities markets that could threaten fair and orderly markets." It found that "short sale, in particular naked short sale, has exacerbated a loss of investor confidence and caused further panic selling and lead to counterparties to Bear Stearns unwilling to make secured funding (`http://www.sec.gov/rules/other/2008/34-58166.pdf`)."

(conditional on the realization of the productivity), while the second one is correlated among them.[2] Speculators choose whether to buy or sell shares of the firm's asset in the market.

In this environment, speculators would make their trading decisions based on the two signals. We identify two strategic effects that determine the relative importance of each signal. First, due to the price mechanism, the sale (purchase) of shares by other speculators reduces (increases) the price and then the profit from the selling (buying). This generates strategic substitutes – speculators wish to act differently from one another – and reduces the weight speculators put on the correlated signal. Second, due to the feedback effect from the price to the capital provision decision, a coordinated sale (purchase) by many speculators transmits negative (positive) information to the capital provider and leads to a reduction (an increase) in the amount of capital provided and in the amount of investment undertaken. This reduces (increases) the underlying value of the security and increases the profit from selling (buying). The result is strategic complementarities that make speculators put a larger weight on the correlated signal. Absent strategic interaction, speculators would make the trading decision giving weights to the two signals based on their relative precision. Due to the two strategic effects, however, the weights diverge from this rule. The sum of these two strategic effects shapes the coordination in speculators' trading, which ends up affecting the information conveyed by the price, the efficiency of the investment decision, and the volatility of price and real investment.

In general, our model demonstrates that the financial market indeed has an important role in the economy. The information conveyed to decision makers (here, the capital provider) as a result of speculative trading is indeed important for the efficiency of the decisions. Yet, the presence of strategic interactions among speculators implies that almost always there is either too little or too much coordination in their actions, reducing the efficiency of real investment and increasing excess (i.e., non-fundamental) volatility. This is the damaging aspect of financial-market trading, which potentially could be alleviated with appropriate policy.

For example, we find that when there is small variance in noise/liquidity trading in the financial market, speculators tend to coordinate their trading too much by putting excessive weights on their correlated signals. As a result, the common noise in their correlated signals has a too big effect on prices and investments, reducing efficiency and increasing volatility. This echoes very clearly some of the events mentioned in the opening paragraph of our introduction. Essentially, our model gives rise to a 'run' on a stock by many speculators,

[2] In our model, the correlation is perfect, but this is for expositional clarity and is not essential.

who are driven by common noise in their correlated signals (e.g. rumor), leading to a price decline, lack of provision of new capital, and collapse of real value.

On the other hand, there are circumstances in which our model predicts there will be too little coordination in speculators' trades. This happens when there is large variance in noise/liquidity trading. Speculators then realize that the price becomes very volatile due to factors that are beyond speculative trading and hence the ability to affect real decisions is hampered. As a result, they end up coordinating too little and the price is controlled by variation in noise trading (e.g. "irrational exuberance" among general market participants), failing to provide decision makers proper guidance in their real decisions.

These observations suggest that some policy steps may be needed to alter patterns of coordination and improve the usefulness of the financial market in the economy. One type of policy we consider targets the cost of capital for the capital provider. A counter-cyclical policy reduces the cost of capital when fundamentals are weak and increases it when fundamentals are strong. Such a policy mitigates the incentive of speculators to coordinate, and thus is desirable when there is excessive coordination. Similarly, pro-cyclical policy, which reduces the cost of capital in good times and increases it in bad times would be desirable when there is too little coordination in the financial market.

Other policy measures target directly the trading environment in the financial market. In general, the effect of noise trading on the incentives of speculators to coordinate is opposite to its effect on the socially desirable level of coordination. Speculators wish to coordinate more when there is little noise in the market because this is when they can affect real investments. Yet, this is exactly when coordination is not needed to reduce noise and improve the informational content of the price. The government can potentially fix this by affecting the amount of liquidity/noise in the market. Increasing liquidity when it is at a low level will reduce coordination and improve efficiency. The government can also attempt to achieve more efficient levels of coordination by affecting the informational environment. For example, restricting communication among speculators can reduce the correlation in their signal and reduce their ability to coordinate. Transparency measures such as releasing a public signal to all market participants increases the information precision of commonly shared information, reduces the dependent on informed revealed in the financial trading for real decision-making, and hence lowers speculators' incentive to coordinate.

Our paper builds on a small, but growing, branch of models in financial economics that consider the feedback effect from trading in financial markets to corporate investments. Empirical evidence for this link is provided by Baker, Stein, and Wurgler (2003), Luo (2005), and Chen, Goldstein, and Jiang (2007). On the theoretical side, Earlier contributions to this literature include Fishman and Hagerty (1992); Leland (1992); Khanna, Slezak, and Bradley (1994); Boot and Thakor (1997); Dow and Gorton (1997); Subrahmanyam and Titman (1999); and Fulghieri and Lukin (2001).

Several recent papers in this literature are more closely related to the mechanism in our paper. Ozdenoren and Yuan (2008) show that the feedback effect from asset prices to the real value of a firm generates strategic complementarities. In their paper, however, the feedback effect is modeled exogenously and is not based on learning. Goldstein and Guembel (2008) do analyze learning by a decision maker, and show that this might lead to manipulation of the price by a single potentially informed trader. Hence, the manipulation equilibrium in their paper is not a result of strategic complementarities among heterogeneously informed traders.[3] Dow, Goldstein, and Guembel (2007) show that the feedback effect generates complementarities in the decision to produce information, but not in the trading decision.[4] Goldstein, Ozdenoren, and Yuan (2009) analyze strategic complementarities in currency trading due to learning by a decision maker (central bank). Their paper, however, lacks the price mechanism and thus the inherent strategic substitutability in financial markets due to this mechanism.

The remainder of this paper is organized as follows. In Section II, we present the model setup and characterize the equilibrium of the model. In Section III, we solve the model. In Section IV, we analyze the determinants of coordination among speculators in our model. In Section V, we discuss the implications for the efficiency of investments and the volatility of prices and investments. In Section VI, we discuss policy implications. Section VII concludes.

## II. Model

The model has one firm and a traded asset. There is a capital provider who has to decide how much capital to provide to the firm for the purpose of making an investment. There are three dates, $t = 0, 1, 2$. At date 0, speculators trade in the asset market based on their information about the fundamentals of the firm. At date 1, after observing the asset price and receiving private information, the capital provider of the firm decides how much capital the firm can have and the firm undertakes investment accordingly. Finally, at date 2, the cash flow is realized and agents get paid.

[3] See also Khanna and Sonti (2004), where manipulation happens as a result of the feedback effect. In their paper, feedback is exogenous and not based on learning.

[4] Complementarities in the decision to produce information also arise due to other reasons in several other papers. For example see, Froot, Scharfstein, and Stein (1992); Hirshleifer, Subrahmanyam, and Titman (1994); Bru and Vives (2002); and Veldkamp (2006a and 2006b).

*A. Investment*

The firm in this economy has access to a production technology, which at time $t = 2$ generates cash flow $\tilde{F}I$. Here $I$ is the amount of investment financed by the capital provider, and $\tilde{F} \geq 0$ is the level of productivity. Let $\tilde{f}$ denote the natural log of productivity, $\tilde{f} = \ln \tilde{F}$. We assume that the productivity, $\tilde{f}$, is unobservable with an ex ante normal distribution with mean $\bar{f}$ and variance $\sigma_f^2$. We use $\tau_f$ to denote $1/\sigma_f^2$. Focusing on the natural log of the productivity parameter is important for the tractability of our model and is the key behind the methodological contribution of our paper.

At time $t = 1$ the capital provider chooses the level of capital $I$. Providing capital is costly and the capital provider must incur a private cost of: $C(I) = \frac{1}{2}cI^2$, where $c > 0$. This cost can be thought of as the cost of raising the capital, which is increasing in the amount of capital provided, or as effort incurred in monitoring the investment (which is also increasing in the size of the investment). The capital provider's benefit increases in the size of the investment and in the firm's productivity. We assume that his benefit is the same as the firm's output, i.e., he gets $\tilde{F}I$. Then, he chooses $I$ to maximize the value of the cash flow from investing in the firm's production technology minus his cost of raising capital $C(I)$, conditional on his information set, $\mathcal{F}_l$, at $t = 1$:

$$I = \arg\max_I E[\tilde{F}I - C(I)|\mathcal{F}_l]. \tag{1}$$

The solution to this maximization problem is:

$$I = \frac{E[\tilde{F}|\mathcal{F}_l]}{c}. \tag{2}$$

The capital provider's information set, denoted by $\mathcal{F}_l$, consists of a private signal $\tilde{s}_l$ received at date 0 and the asset price observed at the date 0, $P$ (we will elaborate on this next). That is, $\mathcal{F}_l = \{\tilde{s}_l, P\}$. The private signal $\tilde{s}_l$ is a noisy signal about $\tilde{f}$ with precision $\tau_l$: $\tilde{s}_l = \tilde{f} + \sigma_l \tilde{\epsilon}_l$, where $\tilde{\epsilon}_l$ is distributed normally with mean zero and standard deviation one and $\tau_l = 1/\sigma_l^2$.

*B. Speculative Trading*

There are two assets in the economy available for speculators to trade in. The first is a riskless asset, which generates a zero fixed net return at the final date. This asset has a perfectly elastic supply. The payoff of the second asset replicates the payoff of the firm.

That is, the payoff is $\tilde{F}I$, which is realized at the final date $t = 2$. The price of this risky asset at time $t = 0$ is denoted by $P$.[5]

There is a measure-one continuum of heterogeneously informed risk-neutral speculators indexed by $i \in [0, 1]$. Each speculator is endowed with two signals at time 0 about $\tilde{f}$, the productivity of the firm. The first signal, $\tilde{s}_i = \tilde{f} + \sigma_s \tilde{\epsilon}_i$, is privately observed where $\tilde{\epsilon}_i$ is independently normally distributed across speculators with mean zero and unit variance. The precision of this signal is denoted as $\tau_s = 1/\sigma_s^2$. The second signal is $\tilde{s}_c = \tilde{f} + \sigma_c \tilde{\epsilon}_c$. This signal is observed by all speculators and $\tilde{\epsilon}_c$ is independently and normally distributed with mean zero and unit variance and $\tau_c = 1/\sigma_c^2$.[6]

We assume that each speculator can buy or sell up to a unit of the risky asset. The size of speculator $i$'s position is denoted by $x(i) \in [-1, 1]$. This position limit can be justified by limited capital and/or borrowing constraints faced by speculators.[7]

Due to risk neutrality, speculators choose their positions to maximize expected profits. For example, a speculator's profit from shorting one unit of the asset is given by $P - \tilde{F}I$, where $\tilde{F}I$ is the asset payoff and $P$ is the price of the asset. Formally speculator $i$ chooses $x(i)$ to solve:

$$\max_{x(i) \in [-1,1]} x(i)\, E\left[\tilde{F}I - P | \mathcal{F}_i\right] \tag{3}$$

where $\mathcal{F}_i$ denotes the information set of speculator $i$ and consists of $\tilde{s}_i$ and $\tilde{s}_c$. Since each speculator has measure zero and is risk neutral, an informed speculator optimally chooses to either short up to the position limit, or buy up to the position limit.

[5] Note that this risky asset can be considered as the stock of the firm except that the dividend payments to the buyers of the asset are from short-sellers rather than from the firm directly. That is, this asset can be interpreted as a derivative whose payoff mimics that of the stock of the firm. In reality, stock prices not only reflect the fundamental value of the firm, but also are directly related to the amount of fund available to the firm (for example, firm can raise more money through secondary equity offerings, which is not modeled here in the paper). If the second effect is also present, the feedback result in our paper would be much stronger.

[6] Our results remain the same but with expositional complexity in an alternative setup where the second signal is specified as a heterogenous private signal with a common noise component $\tilde{\epsilon}_c$ and an agent-specific noise component $\tilde{\epsilon}_{2i}$. That is, $\tilde{s}_{ci} = \tilde{f} + \sigma_c \tilde{\epsilon}_c + \sigma_{\epsilon 2} \tilde{\epsilon}_{2i}$, where $\tilde{\epsilon}_c$ and $\tilde{\epsilon}_{2i}$ are independently normally distributed variables with mean zero and variance one. In this setup, the second signal can be observed by the capital provider as well. Essentially, for our results to go through, speculators need to share some correlated information to facilitate coordination and this information cannot be entirely filtered out by the capital provider.

[7] The specific size of this position limit on asset holdings is not crucial for our results. What is crucial is that informed speculators cannot take unlimited positions; if they do, strategic interaction among informed speculators will become immaterial.

We denote the aggregate demand by speculators by $X = \int_0^1 x(i)\, di$, which is given by the fraction of speculators who buy the asset minus who short the asset multiplied by the position limit.

### C. *Equilibrium*

At date 0, conditional on private information, each speculator decides whether to submit a market order to buy or sell a unit of the asset to a Walrasian auctioneer. The Walrasian auctioneer sets a price to clear the market. The noise supply of the risky asset is exogenously given by $Q(\tilde{\xi}, P)$, a continuous function of the exogenous demand shock $\tilde{\xi}$ and price $P$. We assume that $Q(\tilde{\xi}, P)$ is strictly decreasing in $\tilde{\xi}$, and increasing in $P$, that is, the supply curve is upward slopping in price. The supply shock $\tilde{\xi} \in \mathbb{R}$ is independent of other shocks in the economy, and $\tilde{\xi} \sim N(0, \sigma_\xi^2)$. The shock $\tilde{\xi}$, can be interpreted as asset holdings of noise (liquidity) traders. To solve the model in closed form, we assume that $Q(\tilde{\xi}, P)$ takes the following functional form:

$$Q(\tilde{\xi}, P) = 1 - 2\Phi\left(\frac{\tilde{\xi} - \ln(\delta P)}{\sigma_s}\right), \tag{4}$$

where $\Phi(\cdot)$ denotes the cumulative standard normal distribution function.

As we will show later, price does not not fully reveal the firm's productivity. We now turn to definition of equilibrium.

Definition 1: [Equilibrium with Market Orders ] An equilibrium consists of a price function, $P(\tilde{f}, \tilde{\epsilon}_c, \tilde{\xi}) : \mathbb{R}^3 \to \mathbb{R}$, an investment policy for the capital provider $I(\tilde{s}_l, P) : \mathbb{R}^2 \to \mathbb{R}$, strategies for speculators, $x(\tilde{s}_i, \tilde{s}_c) : \mathbb{R}^2 \to [-1, 1]$, and the corresponding aggregate demand $X(\tilde{f}, \tilde{\epsilon}_c)$, such that:

- For speculator $i$, $x(\tilde{s}_i, \tilde{s}_c) \in \arg\max_{x(i)\in[-1,1]} x(i)E\left[\tilde{F}I - P|\tilde{s}_i, \tilde{s}_c\right]$;
- The capital provider's investment is $I(\tilde{s}_l, P) = E\left[\tilde{F}|\tilde{s}_l, P\right]/c$.
- The market clearing condition for the risky asset is satisfied:

$$Q(\tilde{\xi}, P) = X(\tilde{f}, \tilde{\epsilon}_c) = \int x(\tilde{f} + \sigma_s\tilde{\epsilon}_i, \tilde{f} + \sigma_c\tilde{\epsilon}_c) d\Phi(\tilde{\epsilon}_i). \tag{5}$$

A *linear monotone equilibrium* is an equilibrium where $x(\tilde{s}_i, \tilde{s}_c) = 1$ if $\tilde{s}_i + k\tilde{s}_c \geq g$ for constants $k$ and $g$, and $x(\tilde{s}_i, \tilde{s}_c) = -1$ otherwise. In words, in a monotone linear equilibrium a speculator buys the asset if and only if a linear combination of her signals is above a cutoff $g$, and sells it otherwise.

## III. Solving the Model

In this section, we explain the main steps that are required to solve our model. Restricting to a linear monotone equilibrium, we first use the market clearing condition to determine the asset price. We then characterize the information content of the asset price to derive the capital provider's belief on $\tilde{f}$ based on $\{P, \tilde{s}_l\}$ and solve for the optimal investment problem. Finally, given the capital provider's investment decision and the asset price, we solve for individual speculators' optimal trading decision to complete the equilibrium characterization.

In a linear monotone equilibrium, speculators short the asset whenever $\tilde{s}_i + k\tilde{s}_c \leq g$ or, equivalently, $\sigma_s\tilde{\epsilon}_i \leq g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c$. Hence, their aggregate selling can be characterized by: $\Phi\left(\left(g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c\right)/\sigma_s\right)$. Conversely, they purchase the asset whenever $\tilde{s}_i + k\tilde{s}_c \geq g$ or, equivalently, $\sigma_s\tilde{\epsilon}_i \geq g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c$. Hence, their aggregate purchase can be characterized by $1 - \Phi\left(\left(g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c\right)/\sigma_s\right)$. The net holding from speculators is then:

$$X\left(\tilde{f}, \tilde{\epsilon}_c\right) = 1 - 2\Phi\left(\frac{g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c}{\sigma_s}\right). \tag{6}$$

The market clearing condition together with Equation (4) indicate that

$$1 - 2\Phi\left(\frac{g - (1+k)\tilde{f} - k\sigma_c\tilde{\epsilon}_c}{\sigma_s}\right) = 1 - 2\Phi\left(\frac{\tilde{\xi} - \ln(\delta P)}{\sigma_s}\right). \tag{7}$$

Therefore the equilibrium price is given by

$$\delta P = \exp\left((1+k)\tilde{f} + k\sigma_c\tilde{\epsilon}_c - g + \tilde{\xi}\right) = \exp\left(\tilde{f} + k\tilde{s}_c - g + \tilde{\xi}\right), \tag{8}$$

which can be rewritten as

$$z(P) \equiv \frac{g + \ln(\delta P)}{1+k} = \tilde{f} + \frac{k}{1+k}\sigma_c\tilde{\epsilon}_c + \frac{1}{1+k}\tilde{\xi} = \left(\frac{1}{1+k}\right)\tilde{f} + \frac{k}{1+k}\tilde{s}_c + \frac{1}{1+k}\tilde{\xi}. \tag{9}$$

From the above equation, we can see that $z(P)$, which is a sufficient statistic for the information in $P$, provides some information about the realization of the productivity shock $\tilde{f}$. Yet, the signal $z(P)$ is not fully revealing of $\tilde{f}$, as it is also affected by the noise in the common signal $\tilde{\epsilon}_c$ and by the noisy demand $\tilde{\xi}$. Since the capital provider observes $z(P)$, he will use it to update his belief about the productivity. Note that $z(P)$ is distributed normally with a mean of $\bar{f}$ and a variance of $\sigma_p^2 = (k/(1+k))^2\sigma_c^2 + (1/(1+k))^2\sigma_\xi^2$. We denote the precision of $z(P)$ as a signal for $\tilde{f}$ as:

$$\tau_p = \frac{1}{\sigma_p^2} = \frac{(1+k)^2\tau_c\tau_\xi}{k^2\tau_\xi + \tau_c}. \tag{10}$$

After characterizing the information content of the price, we can derive the capital provider's belief on $\tilde{f}$. That is, conditional on observing $\tilde{s}_l$ and $z(P)$, the capital provider believes that $\tilde{f}$ is distributed normally with mean

$$\frac{\tau_f}{\tau_f+\tau_l+\tau_p}\bar{f}+\frac{\tau_l}{\tau_f+\tau_l+\tau_p}s_l+\frac{\tau_p}{\tau_f+\tau_l+\tau_p}z(P) \tag{11}$$

and variance $1/(\tau_f+\tau_l+\tau_p)$. Then, using the capital provider's investment rule in Equation (1) and taking expectations, we can express the level of investment as:

$$\begin{aligned} I &= \frac{1}{c}E[\tilde{F}|\tilde{s}_l=s_l,P]=\frac{1}{c}E[\exp\left(\tilde{f}\right)|\tilde{s}_l=s_l,P] \\ &= \frac{1}{c}\exp\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p z(P)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right). \end{aligned} \tag{12}$$

Given the capital provider's investment policy in (12) and the price in (8), we can now write speculator $i$'s expected profit from buying the asset given the information that is available to her (shorting the asset would give the negative of this):

$$\begin{aligned} & E\left[\tilde{F}I-P|\tilde{s}_i,\tilde{s}_c\right] \\ = & \frac{1}{c}E\left[\exp\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p z(P)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}+\tilde{f}\right)|\tilde{s}_i,\tilde{s}_c\right] \\ & -E\left[\frac{1}{\delta}\exp\left(\tilde{f}+k\tilde{s}_c-g+\tilde{\xi}\right)|\tilde{s}_i,\tilde{s}_c\right]. \end{aligned} \tag{13}$$

Note that we made use here of the fact that $\tilde{F}=\exp\left(\tilde{f}\right)$. Conditional on observing $\tilde{s}_i$ and $\tilde{s}_c$ speculator $i$ believes that $\tilde{f}$ is distributed normally with mean

$$\frac{\tau_f}{\tau_f+\tau_s+\tau_c}\bar{f}+\frac{\tau_s}{\tau_f+\tau_s+\tau_c}\tilde{s}_i+\frac{\tau_c}{\tau_f+\tau_s+\tau_c}\tilde{s}_c \tag{14}$$

and variance $1/(\tau_f+\tau_s+\tau_c)$. Hence, substituting for $z(P)$ (from (9)) and taking expectations, Equation (13) can be rewritten as:

$$\begin{aligned} E\left[\tilde{F}I-P|\tilde{s}_i,\tilde{s}_c\right] = \frac{1}{c}\exp & \left(\begin{array}{c} \frac{\tau_f\bar{f}+\tau_p\frac{k}{1+k}\tilde{s}_c+\frac{1}{2}}{\tau_f+\tau_l+\tau_p}+\frac{\left(\tau_f+2\tau_l+\tau_p\left(1+\frac{1}{1+k}\right)\right)}{\tau_f+\tau_l+\tau_p}\left(\frac{\tau_f\bar{f}+\tau_s\tilde{s}_i+\tau_c\tilde{s}_c}{\tau_f+\tau_s+\tau_c}\right) \\ +\left(\frac{\tau_f+2\tau_l+\left(1+\frac{1}{1+k}\right)\tau_p}{\tau_f+\tau_l+\tau_p}\right)^2\frac{1}{2(\tau_f+\tau_s+\tau_c)}+\frac{1}{2}\left(\frac{\tau_l}{\tau_f+\tau_l+\tau_p}\right)^2\sigma_l^2 \\ +\frac{1}{2}\left(\frac{\tau_p}{\tau_f+\tau_l+\tau_p}\right)^2\left(\frac{1}{1+k}\right)^2\sigma_\xi^2 \end{array}\right) \\ -\frac{1}{\delta}\exp & \left(\frac{\tau_f\bar{f}+\tau_s\tilde{s}_i+\tau_c\tilde{s}_c+\frac{1}{2}}{\tau_f+\tau_s+\tau_c}+k\tilde{s}_c-g+\frac{1}{2}\sigma_\xi^2\right). \end{aligned} \tag{15}$$

In equilibrium, a speculator who receives a private signal $\tilde{s}_i=g-k\tilde{s}_c$ must be indifferent between buying the asset or shorting it. That is,

$$E\left[P-\tilde{F}I|\tilde{s}_i=g-k\tilde{s}_c,\tilde{s}_c\right]=0. \tag{16}$$

Substituting $\tilde{s}_i = g - k\tilde{s}_c$ into (15), and taking logs, the indifference condition for the marginal investor of (16) becomes:

$$
\begin{aligned}
& \ln\frac{1}{c} + \left( \frac{\tau_f \bar{f} + \tau_p \frac{k}{1+k}\tilde{s}_c + \frac{1}{2}}{\tau_f + \tau_l + \tau_p} + \frac{\left(\tau_f + 2\tau_l + \left(1 + \frac{1}{1+k}\right)\tau_p\right)}{\tau_f + \tau_l + \tau_p} \left( \frac{\tau_f \bar{f} + \tau_s (g - k\tilde{s}_c) + \tau_c \tilde{s}_c}{\tau_f + \tau_s + \tau_c} \right) \right) \\
& + \left( \frac{\tau_f + 2\tau_l + \left(1 + \frac{1}{1+k}\right)\tau_p}{\tau_f + \tau_l + \tau_p} \right)^2 \frac{1}{2(\tau_f + \tau_s + \tau_c)} + \frac{1}{2}\left( \frac{\tau_l}{\tau_f + \tau_l + \tau_p} \right)^2 \sigma_l^2 \\
& + \frac{1}{2}\left( \frac{\tau_p}{\tau_f + \tau_l + \tau_p} \right)^2 \left( \frac{1}{1+k} \right)^2 \sigma_\xi^2 \\
= & \ln\frac{1}{\delta} + \frac{\tau_f \bar{f} + \tau_s (g - k\tilde{s}_c) + \tau_c \tilde{s}_c + \frac{1}{2}}{\tau_f + \tau_s + \tau_c} + k\tilde{s}_c - g + \frac{1}{2}\sigma_\xi^2. \qquad (17)
\end{aligned}
$$

In a linear monotone equilibrium this indifference condition must hold for all $\tilde{s}_c$. So the coefficient $\tilde{s}_c$ in the above expression must be zero. Using this we solve for the speculator's cutoff strategy and characterize the equilibrium. The result is provided in the following proposition. The proof of this proposition as well as all other proofs that are omitted from text are in the Appendix.

PROPOSITION 1: There is a unique linear monotone equilibrium. In the equilibrium, the (strictly positive) weight $k^*$ on the common signal is the unique real root of:

$$
\begin{aligned}
0 = & -\left(\tau_s\tau_l + (\tau_f + \tau_s + \tau_c)(\tau_f + \tau_l + \tau_c)\right)k^3 + \tau_c(\tau_l - 2\tau_s - \tau_f - \tau_c)k^2 \qquad (18) \\
& + \tau_c(\tau_c - \tau_s)k + \tau_c^2 - \frac{\tau_c}{\tau_\xi}\left(\tau_s\tau_l + (\tau_f + \tau_l)(\tau_f + \tau_s + \tau_c)\right)k + \frac{\tau_c^2\tau_l}{\tau_\xi}.
\end{aligned}
$$

The weight speculators put on the common signal in equilibrium, $k^*$ captures the degree of coordination among speculators in their trading decisions. When $k^*$ is high, speculators put a large weight on the common information when deciding whether to sell or buy the stock, and this leads to large coordination among them. In the upcoming sections we develop a series of results on the determinants of coordination and its implications for the efficiency of the investment decision and for the volatility of prices.

## IV. The Determinants of Speculators' Coordination

The weight that speculators put on the common signal in this model is affected by the degree to which there are strategic complementarities or strategic substitutes among them. Strategic substitutes are generated in our model by the price mechanism. Since the aggregation of speculators' orders affects the price, when many of them decide to short sell (purchase) the asset, the price is low (high), and the profit from short selling (purchasing) is low. This creates an incentive for speculators to act differently from others – their incentive to short sell (or buy) the asset decreases if many others are expected to do so – and thus leads speculators to put less weight on the common signal in their trading decision. Strategic complementarities, on the other hand, are generated here by the feedback effect that prices have on the investment decision and thus on the real value of the firm. When many speculators decide to short sell (buy) the asset, the price declines (rises), and this transmits a negative (positive) signal to the capital provider that leads to a reduction (an increase) in the level of investment. Then, the value of the firm decreases (increases) and this increases the profit from short selling (the purchase). This creates an incentive for speculators to coordinate and act like each other, and thus to put more weight on the common signal. The resulting level of $k^*$ reflects the sum of these two effects in addition to the raw effect that the precision of the signals (private and common) has on the weights they should receive. In the rest of this section, we isolate the various determinants of coordination to understand the impact of each factor on the equilibrium level of coordination.

### A. *Impact of Learning by the Capital Provider*

To get a clearer understanding of the two effects, let us start by shutting down one of them. In particular, suppose that there is no feedback effect from prices to real values, because the capital provider does not learn from the price. In this case, the capital provider's decision on how much capital to provide becomes (this equation is analogous to Equation (12) in the full model):

$$\begin{aligned} I &= \frac{1}{c} E[\tilde{F} | \tilde{s}_l = s_l] \\ &= \frac{1}{c} \exp\left( \frac{\tau_f}{\tau_f + \tau_l} \bar{f} + \frac{\tau_l}{\tau_f + \tau_l} s_l + \frac{1}{2(\tau_f + \tau_l)} \right). \end{aligned} \tag{19}$$

We again solve for the linear monotone equilibrium where speculators short sell the asset if and only if $\tilde{s}_i + k_{BM}\tilde{s}_c \leq g_{BM}$ (the subscript $BM$ stands for 'benchmark'), and purchase the asset otherwise. Given the investment rule in (19), the expected profit for speculator $i$ from buying the asset, given the information available to her, becomes (this equation is analogous to Equation (13) in the full model):

$$\begin{aligned} & E[\tilde{F}I - P | \tilde{s}_i, \tilde{s}_c] \\ = \; & E\left[ \frac{1}{c} \exp\left( \frac{\tau_f \bar{f} + \tau_l s_l}{\tau_f + \tau_l} + \frac{1}{2(\tau_f + \tau_l)} \right) \tilde{F} | \tilde{s}_i, \tilde{s}_c \right] - E\left[ \frac{1}{\delta} \exp\left( \tilde{f} + k_{BM}\tilde{s}_c - g_{BM} + \tilde{\xi} \right) | \tilde{s}_i, \tilde{s}_c \right]. \end{aligned} \tag{20}$$

We know that a speculator observing $\tilde{s}_i = g_{BM} - k_{BM}\tilde{s}_c$ is indifferent between buying and shorting the asset. Following similar steps to those in the full model, we obtain:

$$\begin{aligned}
& \ln\left(\frac{1}{c}\right) + \frac{\tau_f \bar{f} + \frac{1}{2}}{\tau_f + \tau_l} + \left(\frac{\tau_f + 2\tau_l}{\tau_f + \tau_l}\right)\left(\frac{\tau_f \bar{f} + \tau_s\left(g_{BM} - k_{BM}\tilde{s}_c\right) + \tau_c \tilde{s}_c}{\tau_f + \tau_s + \tau_c}\right) \\
& + \left(\frac{\tau_f + 2\tau_l}{\tau_f + \tau_l}\right)^2 \frac{1}{2(\tau_f + \tau_s + \tau_c)} + \frac{1}{2}\left(\frac{\tau_l}{\tau_f + \tau_l}\right)^2 \sigma_l^2 \\
= \ & \ln\frac{1}{\delta} + \frac{\tau_f \bar{f} + \tau_s\left(g_{BM} - k_{BM}\tilde{s}_c\right) + \tau_c \tilde{s}_c + \frac{1}{2}}{\tau_f + \tau_s + \tau_c} + k_{BM}\tilde{s}_c - g_{BM} + \frac{1}{2}\sigma_\xi^2.
\end{aligned} \tag{21}$$

Finally, since the above equality must be satisfied for all $\tilde{s}_c$ we set the coefficient of $\tilde{s}_c$ to zero:

$$\left(\frac{\tau_f + 2\tau_l}{\tau_f + \tau_l}\right)\left(\frac{1}{\tau_f + \tau_s + \tau_c}\right)\left(-k_{BM}\tau_s + \tau_c\right) = \left(\frac{1}{\tau_f + \tau_s + \tau_c}\right)\left(-k_{BM}\tau_s + \tau_c\right) + k_{BM}. \tag{22}$$

Then, we obtain the weight that speculators put on the common signal in the case of no feedback effect from price to real investment:

$$k_{BM} = \frac{\tau_l \tau_c}{\left(\tau_f + \tau_s + \tau_c\right)\left(\tau_f + \tau_l\right) + \tau_l \tau_s}. \tag{23}$$

The following proposition states the properties of $k_{BM}$.

Proposition 2: If the capital provider does not learn from the price when making lending decisions, the weight speculators put on the common signal $k_{BM}$ is strictly below the equilibrium weight $k^*$ they put in the full model (with a feedback effect). Moreover, in the equilibrium of the full model, $k^*$ is strictly below $\tau_c/\tau_s$, the precision ratio of the two signals held by the speculators.

We can see that when we shut down the feedback effect from the price to real investment, the weight that speculators put on the common signal decreases. This is in line with our discussion above, according to which the feedback effect from prices to real investment is a source of complementarity in speculators' strategies, making them want to put more weight on the common signal. Moreover, as one would expect, without a feedback effect the substitutability among speculators' strategies reduces the weight that speculators put on the common signal below the ratio between the precision of the two signals held by the speculators, $\tau_c/\tau_s$, which is the weight that speculators would be expected to put on the common signal absent any strategic effects. Interestingly, even with the feedback effect, $k^*$ is less than $\tau_c/\tau_s$ highlighting the strength of the substitution effect from the price.

*B. Impact of Noise Trading*

The comparison with the case of no feedback clarifies that the feedback effect from the price to the real investment has a crucial impact. It increases the speculators incentive to coordinate to influence the decision of the capital provider. Clearly, in a model with feedback, the ability of speculators to transmit a message to the capital provider via the price depends on the amount of noise trading. As the following proposition states, this affects the weight speculators end up putting on the common signal.

PROPOSITION 3: The equilibrium weight $k^*$ that speculators put on the common signal in the presence of feedback effects (i.e., the full model) is decreasing in the variance of noise demand $1/\tau_\xi$.

The intuition here goes as follows: With high variance in the noise demand, there is high variance in the market price for reasons that are not related to speculators' trades. As a result, the ability of speculators to impact the capital provider's decision via coordination diminishes. This reduces the incentive of speculators to act like each other and thus reduces the equilibrium level of $k^*$.

*C. Impact of Information Structure*

Since the linear monotone equilibrium is unique, we are able to establish comparative statics with respect to the informativeness of various signals regarding the equilibrium level of coordination. We summarize these results in the next proposition.

PROPOSITION 4: Equilibrium level of coordination decreases in the precision of the prior and the private signals: $\partial k^*/\partial \tau_f < 0$ and $\partial k^*/\partial \tau_s < 0$. If the prior is not too precise equilibrium level of coordination decreases in the precision of the lender's signal and increases in the precision of the common signal: $\partial k^*/\partial \tau_l < 0$ and $\partial k^*/\partial \tau_c > 0$ for small enough $\tau_f$.

We find that if the prior is more precise then the ability of speculators to coordinate is reduced. This is not surprising since the capital provider relies more on the prior when it becomes more precise. The scope for speculators to affect the capital provider's belief is much limited. Therefore, speculators reduce the weight they put on the common signal and shift toward the prior which is now more informative about the capital provider's action and the final payoff of the asset.

We also find that if the the speculators' private information is less precise, speculators coordinate better in equilibrium. If each speculator holds a very sharp private signal about the fundamental, each bases the buying or selling decision mostly on the private signal rather than the noisy common signal, and hence there is less incentive to coordinate.

Finally, for volatile enough underlying fundamentals, if the capital provider's private information is less precise, or the common signal is more precise, speculators coordinate better in equilibrium. Intuitively, if the capital provider holds a precise signal, he relies less on the information revealed in the market price. In equilibrium, this gives speculators little incentive to coordinate since their ability to affect the capital provider's beliefs is limited. Hence, the equilibrium weight on the common signal is lower in this case. Finally, the incentive to coordinate is largest when the common signal is very precise. In this case, speculators put a larger weight on the common signal and the capital provider cannot ignore the information revealed in the market price.

We also note that simulations indicate that $\partial k^*/\partial \tau_c > 0$ no matter what the value of $\tau_f$. Simulations show, however, that the restriction on small $\tau_f$ is necessary for $\partial k^*/\partial \tau_l > 0$. For large $\tau_f$, an increase in $\tau_l$ leads to a larger $k^*$, that is, a higher coordination among speculators. Intuitively, this indicates that when the prior is very precise (higher $\tau_f$), there is little room for speculators to coordinate their buying or selling in order to affect the capital provider's beliefs. However, in this case, if the capital provider's private signal becomes more precise, she will shift the weight towards her private signal rather than the prior in making decisions. This will allow the speculators to move away from the prior, giving speculators a higher incentive to affect the capital provider's beliefs.

## V. Coordination, Efficiency, and Volatility

While the last section established that learning from the price by the capital provider enhances speculators' coordination, and that this effect becomes stronger when there is less variance in the noise trading that affects the price, we are ultimately interested in the effect that coordination has on the efficiency of investment decisions and on market volatility.

As our efficiency criteria we use the ex ante expected net benefit of investment (i.e. expected net benefit before any of the signals are realized given the prior belief that $\tilde{f}$ is normally distributed with mean $\bar{f}$ and precision $\tau_f$.) We keep the information structure the same as before, and in particular, in the interim stage we allow the capital provider to obtain information only from her private signal and the price. So our efficiency criteria is given by:

$$\max_I E_0 \left[ E \left[ \tilde{F} I - \frac{1}{2} c I^2 \, | \tilde{s}_l = s_l, P \right] \right] \tag{24}$$

where a speculator purchases the asset if $\tilde{s}_i + k\tilde{s}_c \geq g$ and shorts it otherwise for constants $k$

and $g$ and $P$ is the market clearing price. We denote the optimal level of coordination $k_{OP}$ to be the one that maximizes (24).

The following lemma establishes a strong link between the optimal level of coordination, $k_{OP}$, and the accuracy of the information that can be inferred from the market price, $\tau_p$:

LEMMA 1: The efficient level of coordination that maximizes (24) is $k_{OP} = \tau_c/\tau_\xi$ which also maximizes $\tau_p$, the precision of the information that the capital provider infers from the market price. Ex ante efficiency increases in $k$ for $k < k_{OP}$ and decreases for $k > k_{OP}$.

Essentially, the capital provider cares about the events in the security market only to the extent that it affects the quality of the information she has when making the investment decision. Hence, the level of coordination that is optimal is the one that maximizes the accuracy of the information in the market price.

Examining the expression for $\tau_p$ in (10), we can see that there is a tradeoff in setting the level of coordination optimally. The tradeoff arises because there are two sources of noise in the price, one coming from the noise demand and the other one from the noise in the common signal. A high level of coordination reduces the effect of the first source of noise – as speculative trading becomes more prominent than noise trading – and increases the effect of the second source of noise – as the weight on the common signal is higher. Therefore optimal level of coordination will be high when the potential damage from noise demand is high ($\tau_\xi$ is low) or when the potential damage from noise in the common signal is low ($\tau_c$ is high). In the opposite cases optimal level of coordination will be low. One can easily verify from (10) that, on balance, optimal coordination is given by $k_{OP} = \tau_c/\tau_\xi$.

From Proposition 3 we know that when speculators maximize their profits they coordinate more when the variance in the noise demand is low ($\tau_\xi$ is high). From this we can see that there is a conflict here between the profit incentives of speculators and the efficiency of the investment. The next proposition establishes that in fact the equilibrium level of coordination is higher than the optimal level when $\tau_\xi$ is high and lower when $\tau_\xi$ is low.

PROPOSITION 5: There exists $\bar{\tau}_\xi$ such that $k_{OP} > k^*$ for $\tau_\xi < \bar{\tau}_\xi$ and $k_{OP} < k^*$ for $\tau_\xi > \bar{\tau}_\xi$.

Deviations from the optimal level of coordination $k_{OP}$ are also manifested in the market by higher levels of volatility. The following proposition establishes the link between the level of coordination and excess volatility – volatility that does not come from the variability in fundamental – of price and investment.

PROPOSITION 6: (a) Excess volatility of asset price is minimized at $k = k_{OP}$ (where its value is $1/(\tau_c + \tau_\xi)$), decreases in $k$ when $k < k_{OP}$ and increases in $k$ when $k > k_{OP}$. In particular, when $k > k_{OP}$, excess volatility of asset prices is higher because prices are more sensitive to the noise component in speculators' common signal $\tilde{\epsilon}_c$. When $k < k_{OP}$, excess volatility of asset prices is higher because prices are more sensitive to the noise demand $\tilde{\xi}$.

(b) Excess volatility of investment is minimized at $k = k_{OP}$ (where its value is $1/(\tau_l + \tau_c + \tau_\xi))$, decreases in $k$ when $k < k_{OP}$ and increases in $k$ when $k > k_{OP}$. When $k > k_{OP}$ excess volatility of investment is higher because investment is more sensitive to the noise component in speculators' common signal $\tilde{\epsilon}_c$. When $k^* < k_{OP}$, excess volatility of investment is higher because investment is more sensitive to the noise demand $\tilde{\xi}$.

This proposition indicates that the strategic interactions among speculators in the financial markets often lead to the excess (non-fundamental) volatility in prices as well as real activities. The source of this excess volatility could come from either too low coordination (that is, when the market is characterized by a high amount of trading by noise investors) or too high coordination (that is, when the market is illiquid and the noise in the correlated signals among speculators is high).

## VI. Policy Implications

As demonstrated earlier, the equilibrium level of coordination is either too much or too little, both of which lead to excess volatility in the asset market and real economy. To curb extreme and disruptive swings in these markets, government may consider stepping in under certain conditions. Next we describe two broad types of policy interventions: those that directly regulate the funding of the firms and those that oversee the trading of the security markets.

### *A. Funding Policies Contingent on Economic Conditions*

We consider policies that affect the capital provider's cost of capital in order to influence the speculators' behavior. Policies that affect *ex ante* cost of capital will change the level of investment but not speculators' incentive to coordinate. To see this suppose speculators influence the belief of the capital provider to convince her that productivity is low. If policy is not contingent on the realization of the fundamental, the capital provider will cut back her investment exactly as before (now from a different level). Consequently, coordination incentives of speculators are unchanged since they can still profit from affecting the capital provider's beliefs. However, if the cost of capital is contingent on the realization of fundamental, the capital provider will further decrease or increase her investment (depending on policy) which in turn makes coordination more or less attractive for speculators. Therefore, for policies that affect the cost of capital to be effective the government needs to condition the capital provider's cost of investment on fundamental related information that will be revealed *ex-post* (such as various economic indicators).

To illustrate this point we assume that the government can affect the cost of investment based on the ex post realization of the fundamental itself. This policy can be implemented, for example, as an ex post subsidy (tax) to investment. The level of $\tilde{F}$ could be tied to the GDP or a general industry performance index which is less subject to manipulation by individual investors. More specifically we assume that the government sets the capital provider's cost so that the net cost of investment is $C(I, \tilde{F}) = \frac{1}{2}c\tilde{F}^{\beta}I^2$ when the realized fundamental is $\tilde{F}$. In this case, a counter-cyclical funding policy can be implemented with a positive $\beta$, which leads to a higher cost of investment when productivity is high (large $\tilde{F}$.) Similarly, a pro-cyclical funding policy can be implemented with a negative $\beta$, which reduces the cost of investment for larger $\tilde{F}$.

Next we analyze the new game with government policy in place. That is, the government chooses $\beta$ to affect the capital provider's cost. After the policy choice the capital provider and speculators play the same game as before. Equilibrium of the game with policy intervention is also defined just like before (except that the capital provider now maximizes $E[\tilde{F}I - C(I, \tilde{F})|\mathcal{F}_l] = E[\tilde{F}I - \frac{1}{2}c\tilde{F}^{\beta}I^2|\mathcal{F}_l]$.) Once again we look for linear monotone equilibria where $x(\tilde{s}_i, \tilde{s}_c) = 1$ if $\tilde{s}_i + k\tilde{s}_c \geq g$ for constants $k$ and $g$, and $x(\tilde{s}_i, \tilde{s}_c) = -1$ otherwise. Equilibrium price is still given by Equation (8). Following steps that are similar to the ones that we used in solving the standard model we solve for the speculator's cutoff strategy and characterize the equilibrium of the game with policy intervention.

Proposition 7: There is a unique linear monotone equilibrium of the game with policy intervention for $\beta$ close to zero. In the equilibrium, the (strictly positive) weight $k(\beta)$ on the common signal is the unique real root of:

$$\frac{\tau_p}{\tau_f + \tau_l + \tau_p}\frac{k(\beta)}{1 + k(\beta)} + \frac{\tau_f + (2 - \beta)\,\tau_l + \left(\left(\frac{1-\beta}{1+k(\beta)} + 1\right)\right)\tau_p}{\tau_f + \tau_l + \tau_p}\left(\frac{-\tau_s k(\beta) + \tau_c}{\tau_f + \tau_s + \tau_c}\right) - \frac{-\tau_s k(\beta) + \tau_c}{\tau_f + \tau_s + \tau_c} - k(\beta) = 0.$$

Utilizing the equilibrium condition in Proposition 7, we can characterize the comparative statistics of $k(\beta)$ with respect to $\beta$, a policy instrument controlled by the central planner. The following proposition presents the result.

Proposition 8: For $\beta$ close to zero, a counter-cyclical policy ($\beta > 0$) leads to less coordination among speculators and a pro-cyclical policy ($\beta < 0$) leads to more coordination among speculators.

Recall from Proposition 5 that there exists $\bar{\tau}_\xi$ such that $k^*$ is smaller than $k_{OP}$ for $\tau_\xi$ less than $\bar{\tau}_\xi$ and the reverse is true for $\tau_\xi$ larger than $\bar{\tau}_\xi$. In the former case there is too little coordination and in the latter case there is too much coordination. In either case moving coordination closer to the optimal level increases informational efficiency and lowers excess volatility.[8] Combining this with Proposition 8 we obtain the next corollary:

Corollary 1: There exists $\bar{\tau}_\xi$ such that following a counter-cyclical policy ($\beta > 0$) when $\tau_\xi$ is larger than $\bar{\tau}_\xi$ and a pro-cyclical policy ($\beta < 0$) when $\tau_\xi$ is less than $\bar{\tau}_\xi$ improves efficiency and reduces excess volatility.

The above corollary implies that when $\tilde{\xi}$ has low variance (which can be interpreted as low market liquidity) government should adopt counter-cyclical policies. To understand this recall that when market liquidity is low, speculators coordinate too much, because it is easier for them to impact the capital provider's beliefs through their impact on the price. By adopting a counter-cyclical policy, government diminishes the ability of the speculators to influence the capital provider. In the case that speculators attempt to convince the capital provider that fundamentals are bad (by short selling the asset), the counter-cyclical funding policy proposed in the above corollary gives capital provider an additional incentive to invest which makes short selling and coordination less desirable. Conversely for similar reasons when market liquidity is high, government should adopt pro-cyclical policies. Adopting such state-contingent funding policies would increase the information efficiency of asset prices, decrease the excess volatility in the financial market as well as in the real economy.

### B. *Intervention in Security Trading*

Another type of intervention is to increase the informational efficiency of market prices by changing the information environment in the security markets. One of key determinants of strategic trading in the security market is the level of noise trading. To increase efficiency and curb excess volatility, the government may directly control noise trading. The following corollary states this relationship which follows from Proposition 5.

Corollary 2: The equilibrium $k^*$ is closer to $k_{OP}$ if government increases $\tau_\xi$ when $\tau_\xi < \bar{\tau}_\xi$ and decreases $\tau_\xi$ when $\tau_\xi > \bar{\tau}_\xi$.

[8] See Lemma 1 and Proposition 6.

That is, when $\tilde{\xi}$ has high variance (or when the market has high liquidity), intervention should focus on absorbing this liquidity. Conversely, when $\tilde{\xi}$ has low variance, government should step in and provide market liquidity. This market liquidity intervention can be in the form of buying and selling market indices – an asset management program. This policy encourages coordination among speculators when there is too little coordination and discourages coordination when there is too much coordination. By doing so it increases the informational efficiency of prices.

Interventions can also target directly the information available to the speculators or capital provider as described by the following corollary (which follows immediately from Proposition 4):

COROLLARY 3: When there is too much (little) coordination the government can move the equilibrium level of coordination towards $k_{OP}$ by increasing (decreasing) $\tau_l$ and/or $\tau_s$, or by decreasing (increasing) $\tau_c$.

The government may intervene by making the balance-sheet information available to capital providers (which increases $\tau_l$), or by restricting information sharing among the speculators (which reduces $\tau_c$) with the purpose of reducing strategic collusion. In an environment where quality of the information held by speculators is heterogenous, government may prevent a coordinated 'run' on a firm and improve the general signal precision of the market participants ($\tau_s$) by imposing a transaction cost on trading which makes market participation less attractive to those holding less precise signals. It is unclear, however, whether a short-sale ban will achieve the objectives stated in Corollary 4. Banning short-sales may achieve a temporarily high price. However, the resulting higher price is a more noisy signal to the capital providers (since it does not reveal information held by some albeit pessimistic speculators), and hence, has smaller impact on promoting real economic activities.

COROLLARY 4: By releasing public news, $\tilde{s}_n = \tilde{f} + \tilde{\epsilon}_n \sigma_n$ where $\tau_n = 1/\sigma_n^2$, to all market participants, the government can reduce the equilibrium level of coordination.

This result follows immediately from Proposition 4 since by releasing a public signal to all market participants, the precision of the prior becomes $(\tau_f + \tau_n)$, which is more precise. The implication for transparency policy contrasts the common intuition that more public information might have a perverse effect on information efficiency of the market.[9] This might

[9]There is a large literature on transparency. Some recent works include Morris and Shin (2002, 2005), Heinemann and Cornand (2004), Woodford (2005), Svensson (2005), Hellwig (2005), Angeletos and Pavan (2007), and Amador and Weill (2007).

be due to the fact that more precise public information increases the ability of speculators to coordinate or lower the incentive for speculators to act on their private information causing the aggregate variables such as price or trading volume less informative. In our setting, transparency policy unambiguously lowers speculators's ability to coordinate and increases their incentive to act on their private information. This is because that the commonly shared information (i.e., their prior) is very informative of everyone's final payoff and there is little need to guess other people's actions to infer about the final outcome.

## VII. Conclusion

We study strategic interactions among speculators in financial markets and their real effects. Two opposite strategic effects exist. On the one hand, speculators wish to act differently from each other as a certain action by other speculators changes the price in a way that reduces the profit for other speculators from this action. On the other hand, due to the feedback effect from price to real investment, a certain action by speculators changes the real value of the firm in a way that increases the incentive of other speculators to take this action. We characterize which effect dominates when and analyze the resulting level of coordination in speculators' actions.

The interaction among speculators affects the informational content of the price. Since prices affect real investment in our model, we can ask what level of coordination is most efficient for real investment. In general, speculators' incentives to coordinate go in opposite direction to the optimal level of coordination. Speculators want to coordinate more when there is little variation in noise trading, but this is when coordination is less desirable from an efficiency point of view. Hence, our model shows that there is always either too much or too little coordination, and this reduces the efficiency of investment and creates excess volatility in the price.

By analyzing the feedback mechanism between financial market trading and real investment activities, our model has implications for policy measures that can alter the level of coordination and improve efficiency. We consider changes to the cost of capital for the firm in correlation with firm fundamentals, and also measures that directly affect the trading environment by changing liquidity, transparency, and the precision of various sources of information.

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## Appendix

**Proof of Proposition 1:** In the proposed equilibrium, (17) must hold for all $\tilde{s}_c$. Therefore, the coefficient of $\tilde{s}_c$ must be zero. That is:

$$\frac{\tau_p}{\tau_f+\tau_l+\tau_p}\frac{k}{1+k}+\left(\frac{\tau_f+2\tau_l+\left(1+\frac{1}{1+k}\right)\tau_p}{\tau_f+\tau_l+\tau_p}\right)\left(\frac{-\tau_s k+\tau_c}{\tau_f+\tau_s+\tau_c}\right)-\frac{-\tau_s k+\tau_c}{\tau_f+\tau_s+\tau_c}-k=0.$$

Substituting for $\tau_p$ and rearranging, this equation can be rewritten as:

$$\begin{aligned}&-\left(\tau_s\tau_l+\left(\tau_f+\tau_s+\tau_c\right)\left(\tau_f+\tau_l+\tau_c\right)\right)k^3+\tau_c\left(\tau_l-2\tau_s-\tau_f-\tau_c\right)k^2\\&+\tau_c\left(\tau_c-\tau_s\right)k+\tau_c^2-\frac{\tau_c}{\tau_\xi}\left(\tau_s\tau_l+\left(\tau_f+\tau_l\right)\left(\tau_f+\tau_s+\tau_c\right)\right)k+\frac{\tau_c^2\tau_l}{\tau_\xi}=0.\end{aligned}\tag{25}$$

Next we show that the above cubic equation can be solved for $k$ and has a unique strictly positive root for $k$. To see this first consider the following function:

$$\begin{aligned}H(k)&=-\left(\tau_s\tau_l+\left(\tau_f+\tau_s+\tau_c\right)\left(\tau_f+\tau_l+\tau_c\right)\right)k^3+\left(\tau_c\tau_l-2\tau_s\tau_c-\left(\tau_f+\tau_c\right)\tau_c\right)k^2\\&\quad+\tau_c\left(\tau_c-\tau_s\right)k+\tau_c^2.\end{aligned}\tag{26}$$

The discriminant for $H(k)=0$ is:

$$\begin{aligned}&4\left(\tau_c\tau_l-2\tau_s\tau_c-\left(\tau_f+\tau_c\right)\tau_c\right)^3\tau_c^2-\left(\tau_c\tau_l-2\tau_s\tau_c-\left(\tau_f+\tau_c\right)\tau_c\right)^2\tau_c^2\left(\tau_c-\tau_s\right)^2\\&-4\left(\tau_s\tau_l+\left(\tau_f+\tau_s+\tau_c\right)\left(\tau_f+\tau_l+\tau_c\right)\right)\tau_c^3\left(\tau_c-\tau_s\right)^3\\&+18\left(\tau_s\tau_l+\left(\tau_f+\tau_s+\tau_c\right)\left(\tau_f+\tau_l+\tau_c\right)\right)\left(\tau_c\tau_l-2\tau_s\tau_c-\left(\tau_f+\tau_c\right)\tau_c\right)\left(\tau_c-\tau_s\right)\tau_c^3\\&+27\left(\tau_s\tau_l+\left(\tau_f+\tau_s+\tau_c\right)\left(\tau_f+\tau_l+\tau_c\right)\right)^2\tau_c^4\end{aligned}$$

which can be rewritten as:

$$\tau_c^3\left(\begin{array}{c}32\tau_c^4\tau_f+64\tau_c^4\tau_l+91\tau_c^3\tau_f^2+184\tau_c^3\tau_f\tau_l+80\tau_c^3\tau_f\tau_s+32\tau_c^3\tau_l^2\\+160\tau_c^3\tau_l\tau_s+86\tau_c^2\tau_f^3+174\tau_c^2\tau_f^2\tau_l+152\tau_c^2\tau_f^2\tau_s+60\tau_c^2\tau_f\tau_l^2\\+326\tau_c^2\tau_f\tau_l\tau_s+72\tau_c^2\tau_f\tau_s^2+4\tau_c^2\tau_l^3+104\tau_c^2\tau_l^2\tau_s+144\tau_c^2\tau_l\tau_s^2\\+27\tau_c\tau_f^4+54\tau_c\tau_f^3\tau_l+72\tau_c\tau_f^3\tau_s+27\tau_c\tau_f^2\tau_l^2+162\tau_c\tau_f^2\tau_l\tau_s\\+68\tau_c\tau_f^2\tau_s^2+90\tau_c\tau_f\tau_l^2\tau_s+152\tau_c\tau_f\tau_l\tau_s^2+28\tau_c\tau_f\tau_s^3\\+71\tau_c\tau_l^2\tau_s^2+56\tau_c\tau_l\tau_s^3+4\tau_f^2\tau_s^3+4\tau_f\tau_l\tau_s^3+4\tau_f\tau_s^4+8\tau_l\tau_s^4\end{array}\right)>0.$$

Therefore, the equation $H(k)=0$ has a unique real root. Since $H(-\infty)=\infty$, $H(\infty)=-\infty$ and $H(0)=\tau_c^2>0$ the only real root occurs for $k>0$. Next consider the last two terms of Equation (25):

$$-\frac{\tau_c}{\tau_\xi}\left(\tau_s\tau_l+\left(\tau_f+\tau_l\right)\left(\tau_f+\tau_s+\tau_c\right)\right)k+\frac{\tau_c^2\tau_l}{\tau_\xi}\tag{27}$$

Note that when $k < 0$ Equation (27) is strictly positive, so there can not be a real root of Equation (25) for $k < 0$. Moreover, Equation (27) has a strictly negative derivative when $k > 0$ so the left side of Equation (25) decreases faster than $H(k)$ and thus crosses zero only once.

After characterizing $k$, we note that in the constructed linear equilibrium, the value of $g$ is given by the following equation:

$$g = -\left[\ln\frac{\delta}{c} + \frac{\tau_f\bar{f} + \frac{1}{2}}{\tau_f + \tau_l + \tau_p} - \frac{\tau_f\bar{f} + \frac{1}{2}}{\tau_f + \tau_s + \tau_c} + \frac{\left(\tau_f + 2\tau_l + \left(1 + \frac{1}{1+k}\right)\tau_p\right)}{\tau_f + \tau_l + \tau_p}\left(\frac{\tau_f\bar{f}}{\tau_f + \tau_s + \tau_c}\right)\right.$$
$$+\left(\frac{\tau_f + 2\tau_l + \left(1 + \frac{1}{1+k}\right)\tau_p}{\tau_f + \tau_l + \tau_p}\right)^2 \frac{1}{2(\tau_f + \tau_s + \tau_c)} + \frac{1}{2}\left(\frac{\tau_l}{\tau_f + \tau_l + \tau_p}\right)^2 \sigma_l^2$$
$$\left. + \frac{1}{2}\left(\frac{\tau_p}{\tau_f + \tau_l + \tau_p}\right)^2 \left(\frac{1}{1+k}\right)^2 \sigma_\xi^2 - \frac{1}{2}\sigma_\xi^2\right] \Big/ \left(1 + \frac{\tau_s}{\tau_f + \tau_s + \tau_c}\frac{\left(\tau_l + \left(\frac{1}{1+k}\right)\tau_p\right)}{\tau_f + \tau_l + \tau_p}\right) \quad (28)$$

Finally, we need to establish that a speculator observing a private signal below $g - k\tilde{s}_c$ prefers to short sell and a speculator observing a signal above $g - k\tilde{s}_c$ prefers not to short sell. Note that the derivative of a speculator's payoff from short selling in (15) with respect to $\tilde{s}_i$ is $\tau_s/(\tau_f + \tau_s + \tau_c)$ times

$$\frac{1}{\delta}\exp\left(\frac{\tau_f\bar{f} + \tau_s\tilde{s}_i + \tau_c\tilde{s}_c + \frac{1}{2}}{\tau_f + \tau_s + \tau_c} + k\tilde{s}_c - g + \frac{1}{2}\sigma_\xi^2\right)$$
$$-\frac{1}{c}\left(\frac{\left(\tau_f + 2\tau_l + \tau_p\left(1 + \frac{1}{1+k}\right)\right)}{\tau_f + \tau_l + \tau_p}\right)\exp\left(\begin{array}{c} \frac{\tau_f\bar{f} + \tau_p\frac{k}{1+k}\tilde{s}_c + \frac{1}{2}}{\tau_f + \tau_l + \tau_p} + \frac{\left(\tau_f + 2\tau_l + \tau_p\left(1 + \frac{1}{1+k}\right)\right)}{\tau_f + \tau_l + \tau_p}\left(\frac{\tau_f\bar{f} + \tau_s\tilde{s}_i + \tau_c\tilde{s}_c}{\tau_f + \tau_s + \tau_c}\right) \\ + \left(\frac{\tau_f + 2\tau_l + \left(1 + \frac{1}{1+k}\right)\tau_p}{\tau_f + \tau_l + \tau_p}\right)^2 \frac{1}{2(\tau_f + \tau_s + \tau_c)} + \frac{1}{2}\left(\frac{\tau_l}{\tau_f + \tau_l + \tau_p}\right)^2\sigma_l^2 \\ + \frac{1}{2}\left(\frac{\tau_p}{\tau_f + \tau_l + \tau_p}\right)^2\left(\frac{1}{1+k}\right)^2\sigma_\xi^2 \end{array}\right)$$

Note that the above is strictly negative whenever the speculator's payoff is zero for a given $\tilde{s}_i$ and $\tilde{s}_c$. This implies that for a given $\tilde{s}_c$ there is a unique $\tilde{s}_i$ at which the speculator is indifferent between buying the asset or shorting it and that the speculator wants to buy for $\tilde{s}_i$ above this level and short below it. QED.

**Proof of Proposition 2:** We plug $k_{BM}$ in the right side of Equation (18) to obtain:

$$-\left(\frac{\tau_s\tau_l}{\tau_f + \tau_s + \tau_c} + (\tau_f + \tau_l + \tau_c)\right)\left(\frac{\tau_l\tau_c}{(\tau_f + \tau_s + \tau_c)(\tau_f + \tau_l) + \tau_l\tau_s}\right)^3$$
$$+\left(\frac{\tau_c\tau_l - \tau_s\tau_c}{\tau_f + \tau_s + \tau_c} - \tau_c\right)\left(\frac{\tau_l\tau_c}{(\tau_f + \tau_s + \tau_c)(\tau_f + \tau_l) + \tau_l\tau_s}\right)^2$$
$$+\tau_c\left(\frac{\tau_c - \tau_s}{\tau_f + \tau_s + \tau_c}\right)\left(\frac{\tau_l\tau_c}{(\tau_f + \tau_s + \tau_c)(\tau_f + \tau_l) + \tau_l\tau_s}\right) + \frac{\tau_c^2}{\tau_f + \tau_s + \tau_c}$$
$$\frac{\tau_l\tau_c^2}{\tau_\xi(\tau_f + \tau_s + \tau_c)}\left(\frac{-(\tau_s\tau_l + (\tau_f + \tau_l)(\tau_f + \tau_s + \tau_c))}{(\tau_f + \tau_s + \tau_c)(\tau_f + \tau_l) + \tau_l\tau_s} + 1\right)$$

which (after some tedious algebra) can be shown to be strictly positive. Therefore, $k_{BM}$ is strictly less than the equilibrium weight that the speculators put on the common signal when the capital provider learns from price. The statement $k_{BM} < \tau_c/\tau_s$ is immediate from Equation (23). Similarly, to show that $k^* < \tau_c/\tau_s$, we plug $\tau_c/\tau_s$ in the right side of Equation (18) and find that it is strictly negative. QED.

**Proof of Proposition 3:** Consider the last two terms in Equation (18):

$$-\frac{\tau_c}{\tau_\xi}\left(\tau_s\tau_l + \left(\tau_f + \tau_l\right)\left(\tau_f + \tau_s + \tau_c\right)\right)k + \frac{\tau_c^2\tau_l}{\tau_\xi}.$$

Denote by $k^*\left(\tau_\xi\right)$ the equilibrium $k$ for a given $\tau_\xi$. We want to show that $k^*\left(\tau_\xi\right)$ is decreasing in $1/\tau_\xi$. Take a fixed $\hat{\tau}_\xi > 0$. Note that the above sum is negative at $k = k^*\left(\hat{\tau}_\xi\right)$ (since $k^*\left(\hat{\tau}_\xi\right) > \tau_l\tau_c/\left(\left(\tau_f + \tau_s + \tau_c\right)\left(\tau_f + \tau_l\right) + \tau_l\tau_s\right)$ by Proposition 2.) As $1/\tau_\xi$ increases this sum becomes more negative at $k = k^*\left(\hat{\tau}_\xi\right)$. This means that for $\tau_\xi > \hat{\tau}_\xi$ the value of Equation (18) is strictly negative at $k = k^*\left(\hat{\tau}_\xi\right)$ and thus $k^*\left(\tau_\xi\right) < k^*\left(\hat{\tau}_\xi\right)$.QED.

**Proof of Proposition 4**

We start with a lemma:

LEMMA 2: The expression

$$\begin{aligned}
D\left(k\right) = {} & -3k^2\left(\left(\tau_l + \tau_c + \tau_f\right)\left(\tau_c + \tau_f + \tau_s\right) + \tau_l\tau_s\right) \\
& +2\tau_c k\left(\tau_l - 2\tau_s - \tau_f - \tau_c\right) + \tau_c\left(\tau_c - \tau_s\right) - \frac{\tau_c}{\tau_\xi}\left(\tau_l\tau_s + \left(\tau_l + \tau_f\right)\left(\tau_c + \tau_f + \tau_s\right)\right)
\end{aligned}$$

is negative at $k = k^*$.

**Proof of Lemma 2** We know that Equation (18) crosses zero once and from above so its derivative with respect to $k$ is negative at $k^*$. QED

Now we proceed with the proof of Proposition 4. To see $\partial k^*/\partial\tau_f < 0$ we take the total derivative of Equation (18) with respect to $\tau_f$ to obtain:

$$\frac{\partial k^*}{\partial \tau_f} = \frac{k^3\left(2\tau_f + 2\tau_c + \tau_l + \tau_s\right) + \tau_c k^2 + \tau_c\frac{k}{\tau_\xi}\left(2\tau_f + \tau_c + \tau_s + \tau_l\right)}{D\left(k\right)} < 0.$$

Taking total derivative of Equation (18) with respect to $\tau_s$ and using Lemma 2 establishes that $\partial k^*/\partial\tau_s < 0$.

Next we show $\partial k^*/\partial\tau_l < 0$ for small enough $\tau_f$. Taking total derivative of Equation (18) with respect to $\tau_l$ we see that the derivative is given by:

$$\left(\tau_s + \left(\tau_c + \tau_f + \tau_s\right)\right)k^3 - \tau_c k^2 + \frac{\tau_c}{\tau_\xi}\left(\tau_c + \tau_f + 2\tau_s\right)k - \frac{\tau_c^2}{\tau_\xi}$$

divided by $D(k)$. The numerator is negative if and only if

$$k > \frac{\tau_c}{2\tau_s + \tau_f + \tau_c}.$$

We directly verify that the value of Equation (18) at $\tau_c / (2\tau_s + \tau_f + \tau_c)$ is positive if $\tau_f$ is small enough. The last result then again follows from Lemma 2.

Finally, taking total derivative of Equation (18) with respect to $\tau_c$ we obtain $\partial k^* / \partial \tau_c$ equals

$$\left[ \begin{array}{c} k^3 (2\tau_c + 2\tau_f + \tau_s + \tau_l) + k^2 (2\tau_c + \tau_f - \tau_l + 2\tau_s) - k (2\tau_c - \tau_s) - 2\tau_c \\ + \frac{k}{\tau_\xi} (\tau_l \tau_s + (\tau_f + \tau_l)(2\tau_c + \tau_f + \tau_s)) - 2\tau_c \frac{\tau_l}{\tau_\xi} \end{array} \right] / D(k).$$

Using Equation (18) we can write the numerator as:

$$\frac{1}{\tau_c} \left[ - \left(2\tau_s \tau_l + \tau_l \tau_f + \tau_s \tau_f + \tau_f^2\right) k^3 + \tau_c^2 \left( k^3 + k^2 - k - 1 + \frac{k}{\tau_\xi} (\tau_f + \tau_l) - \frac{\tau_l}{\tau_\xi} \right) \right].$$

Equation (18) evaluated at $k = 1$ is strictly negative thus in equilibrium $k^* < 1$. Moreover the expression

$$k^3 + k^2 - k - 1 + \frac{\tau_l}{\tau_\xi} (k - 1) < 0$$

for $k \in (0, 1)$ . Therefore the numerator of $\partial k^* / \partial \tau_c$ is negative for small enough $\tau_f$. Using Lemma 2 establishes that $\partial k^* / \partial \tau_c > 0$ for small enough $\tau_f$. QED

**Proof of Lemma 1:**

We substitute optimal $I$ into Equation (24) and compute the expectations:

$$\begin{aligned}
& \frac{1}{c}E\left[\exp\left(\tilde{f}\right)\exp\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p z(P)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
& -\frac{1}{2}\frac{1}{c}E\left[\exp 2\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p z(P)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
= & \frac{1}{c}E\left[\exp\left(2\tilde{f}+\frac{\tau_f\left(\bar{f}-\tilde{f}\right)+\tau_l\sigma_l\tilde{\epsilon}_l+\tau_p\left(z(P)-\tilde{f}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
& -\frac{1}{2}\frac{1}{c}E\left[\exp 2\left(\tilde{f}+\frac{\tau_f\left(\bar{f}-\tilde{f}\right)+\tau_l\sigma_l\tilde{\epsilon}_l+\tau_p\left(z(P)-\tilde{f}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
= & E\frac{1}{c}\left[\exp\left(\frac{\tau_f+2\tau_l+2\tau_p}{\tau_f+\tau_l+\tau_p}\tilde{f}+\frac{\tau_f\bar{f}}{\tau_f+\tau_l+\tau_p}+\frac{\tau_l\sigma_l\tilde{\epsilon}_l+\tau_p\left(z(P)-\tilde{f}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
& -\frac{1}{2}\frac{1}{c}E\left[\exp 2\left(\tilde{f}+\frac{\tau_f\left(\bar{f}-\tilde{f}\right)+\tau_l\sigma_l\tilde{\epsilon}_l+\tau_p\left(z(P)-\tilde{f}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right)\right] \\
= & \frac{1}{c}\exp\left(2\bar{f}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}+\frac{1}{2}\frac{(\tau_f+2\tau_l+2\tau_p)^2\frac{1}{\tau_f}}{(\tau_f+\tau_l+\tau_p)^2}+\frac{1}{2}\frac{\tau_l+\tau_p}{(\tau_f+\tau_l+\tau_p)^2}\right) \\
& -\frac{1}{c}\frac{1}{2}\exp\left(2\bar{f}+\frac{1}{(\tau_f+\tau_l+\tau_p)}+2\left(\frac{(\tau_l+\tau_p)^2\frac{1}{\tau_f}}{(\tau_f+\tau_l+\tau_p)^2}+\frac{\tau_l+\tau_p}{(\tau_f+\tau_l+\tau_p)^2}\right)\right) \\
= & \frac{1}{c}\exp\left(2\bar{f}+\frac{1}{\tau_f}\frac{\tau_f+2\tau_l+2\tau_p}{\tau_f+\tau_l+\tau_p}\right)-\frac{1}{c}\frac{1}{2}\exp\left(2\bar{f}+\frac{1}{\tau_f}\frac{\tau_f+2\tau_l+2\tau_p}{\tau_f+\tau_l+\tau_p}\right) \\
= & \frac{1}{c}\frac{1}{2}\exp\left(2\bar{f}+\frac{1}{\tau_f}\frac{\tau_f+2\tau_l+2\tau_p}{\tau_f+\tau_l+\tau_p}\right).
\end{aligned}$$

Therefore the maximization problem can be viewed as maximizing the following expression in $k$ :

$$\exp\left(\frac{\tau_f+2\tau_l+2\tau_p}{\tau_f+\tau_l+\tau_p}\right),$$

and this is equivalent to maximizing $\tau_p$. Moreover $\tau_p$ is increasing for $k<\tau_c/\tau_\xi$ and decreasing for $k<\tau_c/\tau_\xi$ which proves the last statement. QED.

**Proof of Proposition 5:** Since $\tau_p = \left((1+k)^2\tau_c\tau_\xi\right)/\left(k^2\tau_\xi+\tau_c\right)$, its maximum is achieved when $k=\tau_c/\tau_\xi$. We plug $k=\tau_c/\tau_\xi$ in the right side of Equation (18) to obtain:

$$\begin{aligned} & -\left(\tau_s\tau_l+\left(\tau_f+\tau_s+\tau_c\right)\left(\tau_f+\tau_l+\tau_c\right)\right)\left(\frac{\tau_c}{\tau_\xi}\right)^3+\tau_c\left(\tau_l-2\tau_s-\tau_f-\tau_c\right)\left(\frac{\tau_c}{\tau_\xi}\right)^2 \\ & +\tau_c\left(\tau_c-\tau_s\right)\left(\frac{\tau_c}{\tau_\xi}\right)+\tau_c^2-\frac{\tau_c}{\tau_\xi}\left(\tau_s\tau_l+\left(\tau_f+\tau_l\right)\left(\tau_f+\tau_s+\tau_c\right)\right)\left(\frac{\tau_c}{\tau_\xi}\right)+\frac{\tau_c^2\tau_l}{\tau_\xi} \\ = \; & -\frac{\tau_c^2}{\tau_\xi^3}\left(\tau_c+\tau_\xi\right)\left(\tau_c^2+\tau_f^2-\tau_\xi^2+2\tau_c\tau_f+\tau_c\tau_l+\tau_f\tau_l+\tau_c\tau_s+\tau_f\tau_s+2\tau_l\tau_s-\tau_l\tau_\xi+\tau_s\tau_\xi\right). \end{aligned}$$

There exists $\bar{\tau}_\xi$ such that the above expression is negative for $\tau_\xi<\bar{\tau}_\xi$ and positive for $\tau_\xi>\bar{\tau}_\xi$. To see this note that the sign of the above expression is the negative of the sign of the last part in brackets. It is easy to see that the last part is positive at $\tau_\xi=0$, may increase as $\tau_\xi$ increases at first but will eventually decrease in $\tau_\xi$ and cross once and for all to the negative region. Using the logic in the proof of Proposition 1, this establishes the statement in the proposition. QED.

**Proof of Proposition 6:** (a) The market clearing price is

$$P=\frac{1}{\delta}\exp\left(\left(1+k\right)\tilde{f}+k\sigma_c\tilde{\epsilon}_c-g+\tilde{\xi}\right),$$

and its excess volatility is defined as non-fundamental volatility which can be written as the volatility of the following:

$$z(P)-\tilde{f}=\frac{g+\ln\left(\delta P\right)}{1+k}-\tilde{f}=\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{1}{1+k}\tilde{\xi}.$$

It is straightforward to show that when $k=k_{OP}=\tau_c/\tau_\xi$, its excess volatility is the lowest and is

$$\text{Excess Volatility (Asset Price)}=\frac{1}{\tau_c+\tau_\xi}.$$

The rest of the statement follows immediately. QED.

(b) When $k=k_{OP}=\tau_c/\tau_\xi$, $\tau_p=\tau_c+\tau_\xi$. We know that:

$$I=\frac{1}{c}\exp\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p\left(\tilde{f}+\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{1}{1+k}\tilde{\xi}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right).$$

Take logs on both sides, we obtain:

$$\ln I=\ln\left(\frac{1}{c}\right)+\left(\frac{\tau_f\bar{f}+\tau_l s_l+\tau_p\left(\tilde{f}+\frac{k}{1+k}\sigma_c\tilde{\epsilon}_c+\frac{1}{1+k}\tilde{\xi}\right)}{\tau_f+\tau_l+\tau_p}+\frac{1}{2(\tau_f+\tau_l+\tau_p)}\right).$$

We can define the excess volatility of the real investment as the volatility of the following:

$$\frac{(\tau_f + \tau_l + \tau_p)\left(\ln I - \ln\left(\frac{1}{c}\right)\right) - \frac{1}{2} - \tau_f \bar{f}}{\tau_l + \tau_p} - \tilde{f} = \frac{\tau_l \sigma_l \epsilon_l + \tau_p \left(\frac{k}{1+k}\sigma_c \tilde{\epsilon}_c + \frac{1}{1+k}\tilde{\xi}\right)}{\tau_l + \tau_p}$$

It is straightforward to show that when $k = k_{OP} = \tau_c/\tau_\xi$, $\tau_p = \tau_c + \tau_\xi$,and the excess volatility of the real investment is the lowest which is

$$\text{Excess Volatility (Real Investment)} = \frac{1}{\tau_l + \tau_c + \tau_\xi}.$$

The rest of the statement follows immediately. QED.

**Proof of Proposition 7:**

Given the adjusted cost of investment solution to capital provider's problem is given by:

$$I = \frac{E[\tilde{F}|\mathcal{F}_l]}{cE[\tilde{F}^\beta|\mathcal{F}_l]} = \frac{1}{c}\exp\left((1-\beta)\left(\frac{\tau_f \bar{f} + \tau_l s_l + \tau_p z(P)}{\tau_f + \tau_l + \tau_p}\right) + \frac{(1-\beta^2)}{2(\tau_f + \tau_l + \tau_p)}\right).$$

Given the investment policy and the price in (8), we can now write speculator $i$'s expected profit from buying the asset given the information that is available to her:

$$\begin{aligned} & E\left[\tilde{F}I - P|\tilde{s}_i, \tilde{s}_c\right] \qquad (29) \\ = \; & \frac{1}{c}E\left[\exp\left((1-\beta)\left(\frac{\tau_f \bar{f} + \tau_l s_l + \tau_p z(P)}{\tau_f + \tau_l + \tau_p}\right) + \frac{(1-\beta^2)}{2}\frac{1}{\tau_f + \tau_l + \tau_p} + \tilde{f}\right)|\tilde{s}_i, \tilde{s}_c\right] \\ & -E\left[\frac{1}{\delta}\exp\left(\tilde{f} + k\tilde{s}_c - g + \tilde{\xi}\right)|\tilde{s}_i, \tilde{s}_c\right]. \end{aligned}$$

As before, conditional on observing $\tilde{s}_i$ and $\tilde{s}_c$ speculator $i$ believes that $\tilde{f}$ is distributed normally with mean

$$\frac{\tau_f}{\tau_f + \tau_s + \tau_c}\bar{f} + \frac{\tau_s}{\tau_f + \tau_s + \tau_c}\tilde{s}_i + \frac{\tau_c}{\tau_f + \tau_s + \tau_c}\tilde{s}_c$$

and variance $1/(\tau_f + \tau_s + \tau_c)$. Hence, substituting for $z(P)$ (from (9)) and taking expectations, Equation (29) can be rewritten as:

$$\begin{aligned} & E\left[\tilde{F}I - P|\tilde{s}_i, \tilde{s}_c\right] \qquad (30) \\ = \; & \frac{1}{c}\exp\left(\begin{array}{c} \frac{(1-\beta)\tau_f \bar{f}}{\tau_f+\tau_l+\tau_p} + \frac{\left(\left(\frac{1-\beta}{1+k}+1\right)\right)\tau_p + \tau_f + (2-\beta)\tau_l}{\tau_f+\tau_l+\tau_p}\left(\frac{\tau_f \bar{f} + \tau_s \tilde{s}_i + \tau_c \tilde{s}_c}{\tau_f+\tau_s+\tau_c}\right) + \frac{1}{2}\left(\frac{\left(\left(\frac{1-\beta}{1+k}+1\right)\right)\tau_p + \tau_f + (2-\beta)\tau_l}{\tau_f+\tau_l+\tau_p}\right)^2 \frac{1}{\tau_f+\tau_s+\tau_c} \\ + \frac{(1-\beta)^2}{2}\frac{\tau_l + \tau_p^2\left(\frac{1}{1+k}\right)^2 \sigma_\xi^2}{\left(\tau_f+\tau_l+\tau_p\right)^2} + \frac{\tau_p \frac{k}{1+k}\tilde{s}_c}{\tau_f+\tau_l+\tau_p} + \frac{\left(1-\beta^2\right)}{2}\frac{1}{\tau_f+\tau_l+\tau_p} \end{array}\right) \\ & -\frac{1}{\delta}\exp\left(\frac{\tau_f \bar{f} + \tau_s \tilde{s}_i + \tau_c \tilde{s}_c + \frac{1}{2}}{\tau_f + \tau_s + \tau_c} + k\tilde{s}_c - g + \frac{1}{2}\sigma_\xi^2\right). \end{aligned}$$

In equilibrium, a speculator who receives a private signal $\tilde{s}_i = g - k\tilde{s}_c$ must be indifferent between shorting and buying the asset. That is,

$$E\left[\tilde{F}I - P|\tilde{s}_i = g - k\tilde{s}_c, \tilde{s}_c\right] = 0. \tag{31}$$

Substituting $\tilde{s}_i = g - k\tilde{s}_c$ into (30), and taking logs, the indifference condition for the marginal investor becomes:

$$\begin{aligned}
& \ln\frac{1}{\delta} + \frac{\tau_f \bar{f} + \tau_s (g - k\tilde{s}_c) + \tau_c \tilde{s}_c + \frac{1}{2}}{\tau_f + \tau_s + \tau_c} + k\tilde{s}_c - g + \frac{1}{2}\sigma_\xi^2 \\
= \; & \ln\frac{1}{c} + \frac{(1-\beta)\tau_f \bar{f}}{\tau_f + \tau_l + \tau_p} + \frac{\left(\left(\frac{1-\beta}{1+k} + 1\right)\right)\tau_p + \tau_f + (2-\beta)\tau_l}{\tau_f + \tau_l + \tau_p}\left(\frac{\tau_f \bar{f} + \tau_s (g - k\tilde{s}_c) + \tau_c \tilde{s}_c}{\tau_f + \tau_s + \tau_c}\right) \\
& + \frac{1}{2}\left(\frac{\left(\left(\frac{1-\beta}{1+k} + 1\right)\right)\tau_p + \tau_f + (2-\beta)\tau_l}{\tau_f + \tau_l + \tau_p}\right)^2 \frac{1}{\tau_f + \tau_s + \tau_c} \\
& + \frac{(1-\beta)^2}{2}\frac{\tau_l + \tau_p^2 \left(\frac{1}{1+k}\right)^2 \sigma_\xi^2}{(\tau_f + \tau_l + \tau_p)^2} + \frac{\tau_p \frac{k}{1+k}\tilde{s}_c}{\tau_f + \tau_l + \tau_p} + \frac{(1-\beta^2)}{2}\frac{1}{\tau_f + \tau_l + \tau_p}.
\end{aligned}$$

In a linear equilibrium the above equality must hold for all $\tilde{s}_c$. Therefore, the coefficient of $\tilde{s}_c$ must be zero. That is, the equilibrium $k$ in this case satisfies the following equation:

$$\frac{\tau_p}{\tau_f + \tau_l + \tau_p}\frac{k}{1+k} + \frac{\tau_f + (2-\beta)\tau_l + \left(\left(\frac{1-\beta}{1+k} + 1\right)\right)\tau_p}{\tau_f + \tau_l + \tau_p}\left(\frac{-\tau_s k + \tau_c}{\tau_f + \tau_s + \tau_c}\right) - \frac{-\tau_s k + \tau_c}{\tau_f + \tau_s + \tau_c} - k = 0.$$

Rearranging we obtain the following equation for the equilibrium $k$:

$$\begin{aligned}
& -(\tau_s \tau_l (1-\beta) + (\tau_f + \tau_l + \tau_c)(\tau_c + \tau_f + \tau_s))k^3 + \tau_c (\tau_l (1-\beta) - \tau_s (2-\beta) - \tau_f - \tau_c) k^2 \\
& + (1-\beta)\tau_c (\tau_c - \tau_s) k + \tau_c^2 (1-\beta) \\
& - \frac{\tau_c}{\tau_\xi}(\tau_s \tau_l (1-\beta) + (\tau_f + \tau_l)(\tau_s + \tau_c + \tau_f))k + \left(\frac{\tau_c^2 \tau_l}{\tau_\xi}(1-\beta)\right) = 0.
\end{aligned}$$

Let

$$\begin{aligned}
J(k) \;=\; & -(\tau_s \tau_l + (\tau_f + \tau_l + \tau_c)(\tau_c + \tau_f + \tau_s))k^3 + \tau_c (\tau_l - 2\tau_s - \tau_f - \tau_c)k^2 + \tau_c (\tau_c - \tau_s)k + \tau_c^2 \\
& - \frac{\tau_c}{\tau_\xi}(\tau_s \tau_l + (\tau_f + \tau_l)(\tau_s + \tau_c + \tau_f))k + \left(\frac{\tau_c^2 \tau_l}{\tau_\xi}\right)
\end{aligned}$$

and

$$G(k) = -\tau_s \tau_l k^3 + \tau_c (\tau_l - \tau_s) k^2 + \tau_c (\tau_c - \tau_s) k + \tau_c^2 - \frac{\tau_c}{\tau_\xi}\tau_s \tau_l k + \frac{\tau_c^2 \tau_l}{\tau_\xi}.$$

Thus the equilibrium condition is:

$$J(k(\beta)) - \beta G(k(\beta)) = 0. \tag{32}$$

From the proof of Proposition 1 we know that $J(k)$ has a unique strictly positive root. Thus for small enough $\beta$ (32) has a unique strictly positive root as well. and the equilibrium without policy intervention is given by $k(0)$ that solves $H(k(0)) = 0$. It is easy to see that

$$H(k) = G(k) - ((\tau_f + \tau_l + \tau_c)(\tau_c + \tau_f + \tau_s))k^3 + \tau_c(-\tau_s - \tau_f - \tau_c)k^2 - \frac{\tau_c}{\tau_\xi}((\tau_f + \tau_l)(\tau_s + \tau_c + \tau_f))k.$$

Therefore, $G(k(0)) > 0$.

Since

$$\frac{\partial k(\beta)}{\partial \beta} = \frac{G(k(\beta))}{H'(k(\beta)) - \beta G'(k(\beta))}$$

and $H'(k(0)) < 0$ (from the derivation of equilibrium $k(0)$ without policy intervention) we see that

$$\frac{\partial k(0)}{\partial \beta} = \frac{G(k(0))}{H'(k(0))} < 0.$$

Therefore, for $\beta$ close to zero, a counter-cyclical policy ($\beta > 0$) leads to less coordination among speculators and a pro-cyclical policy ($\beta < 0$) leads to more coordination among speculators. QED.