The Macroeconomic Effects of Housing Wealth, Housing Finance, and Limited Risk-Sharing in General Equilibrium*

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Abstract

We study a two-sector general equilibrium model of housing and non-housing production where heterogenous households face limited risk-sharing opportunities for insuring against idiosyncratic and aggregate risks, as a result of incomplete financial markets. The model generates variability in national house price-rent ratios, both because they fluctuate endogenously with the state of the economy and because they rise in response to a relaxation of credit constraints and decline in housing transaction costs (*financial market liberalization*). We find that a financial market liberalization plus an influx of foreign capital into domestic bond markets calibrated to match the increase in foreign ownership of U.S. Treasury and agency debt from 2000-2007 generates an increase in national price-rent ratios comparable to that observed in U.S. data over this period. Moreover, in a simulated transition for the period 2000-2009, the model generates a decline of greater than 16% in national house price-rent ratios in the two year period 2007 to 2009, driven by the economic contraction and by a presumed reversal of the financial market liberalization. A financial market liberalization drives risk premia in both the housing and equity market down, shifts the composition of wealth for all age and income groups towards housing, and leads to a short-run boom in aggregate consumption but a short-run bust in investment. By contrast, although an influx of foreign capital into the domestic bond market reduces interest rates, it increases risk-premia in both the housing and equity markets. Finally, the model implies that procyclical increases in equilibrium price-rent ratios reflect expectations of lower future housing returns, not higher future rents.

JEL: G11, G12, E44, E21

1 Introduction

Residential real estate is a large and volatile component of household wealth. Moreover, volatility in housing wealth is often accompanied by large swings in house prices relative to housing fundamentals. For example, Figure 1 shows that national house price-rent ratios climbed to unusual heights by the end of 2006, but have since exhibited sharp declines.

This paper studies the macroeconomic consequences of fluctuations in housing wealth and housing finance. To what extent can episodes of national house price appreciation be attributed to a liberalization in housing finance, such as declines in collateral constraints or reductions in the costs of borrowing and conducting transactions? How do movements in house prices affect expectations about future housing fundamentals and future home price appreciation? To what extent do changes in housing wealth and housing finance affect output and investment, risk premia in housing and equity markets, measures of cross-sectional risk-sharing, life-cycle wealth-savings patterns, and the size of housing wealth effects on consumer spending?

In this paper we address these questions by studying a two-sector general equilibrium model of housing and non-housing production where heterogenous households face limited risk-sharing opportunities as a result of incomplete financial markets. The goal of this research is to provide theoretical answers to the questions posed above using a model that is sufficiently general as to account for the endogenous interactions among financial and housing wealth, output and investment, rates of return and risk premia in both housing and equity assets, and consumption and wealth inequality.

A house in our model is a residential durable asset that provides utility to the household, is illiquid (expensive to trade), and can be used as collateral in debt obligations. The model economy is populated by a large number of overlapping generations of households who receive utility from both housing and nonhousing consumption and who face a stochastic life-cycle earnings profile. We introduce market incompleteness by modeling heterogeneous agents who face idiosyncratic and aggregate risks against which they cannot perfectly insure, and by imposing collateralized borrowing constraints on households.

Within the context of this model, we focus our theoretical investigation on the macroeconomic consequences of three systemic changes in housing finance. First, we investigate the impact of changes in housing collateral requirements. Second, we investigate the impact of changes in housing transactions costs. Third, we investigate the impact of an influx of foreign capital into the domestic bond market. We argue below that all three factors fluctuate over time and changed markedly during or preceding the period of rapid home price appreciation from 2000-2006. In particular, this period was marked by a widespread relaxation of collateralized borrowing constraints and declining housing transactions costs, a combination we refer to hereafter as *financial market liberalization.* The period was also marked by a sustained depression of long-term interest rates that coincided with an influx of foreign capital by governmental holders into U.S. bond markets. In the aftermath of the credit crisis that began in 2007, the sharp declines in credit standards and transactions costs have been reversed.[1] We use our framework as a laboratory for studying the impact of fluctuations in either direction of these features of housing finance.

We summarize the model's main implications as follows.

House prices relative to measures of fundamental value are volatile. The model generates substantial variability in national house price-rent ratios, both because they fluctuate procyclically with the state of the economy, and because they rise in response to a relaxation of credit constraints and decline in housing transaction costs. In an economic expansion, a financial market liberalization adds fuel to the fire in an already heated housing market, driving up price-rent ratios more than what would occur as the result of an economic boom alone. When we add to this an influx of foreign capital into domestic bond markets calibrated to match the increase in foreign ownership of U.S. Treasury and agency debt over the period 2000-2007, the model generates an increase in national house price-rent ratios that is comparable to that in the data over the 2000-2007 period. Moreover, in a simulated transition for the period 2000-2009, the model generates a decline of greater than 16% in national house price-rent ratios in the two years from 2007 to 2009, driven by the economic contraction and by a presumed reversal of the financial market liberalization (but not the foreign capital influx).

A financial market liberalization drives price-rent ratios up because it drives risk-premia down. The main driving force behind the rise in price-rent ratios after a financial market liberalization is an across-the-board decline in risk-premia in both housing and equity assets. Risk premia fall after a financial market liberalization for two reasons, both of which allow heterogeneous households to insure more of their idiosyncratic risks. First, lower collateral requirements directly increase access to credit. Second, lower transactions

[1] Some analysts have suggested that borrowing restrictions subsequently became even more strict than historical norms in the pre-boom period. Streitfeld (2009) reports that credit scores for mortgage loans were raised drastically in the aftermath of the credit crisis, and that government sponsored agencies have significantly increased the amount of non-housing collateral required to back mortgages.

costs make it cheaper to obtain the collateral required to increase borrowing capacity and provide insurance. These factors lead to an increase in risk-sharing and a decline in the cross-sectional variance of consumption growth.

It is important to note that the rise in price-rent ratios caused by a financial market liberalization must be attributed to a decline in risk premia and not to a fall in interest rates. Indeed, the very changes in housing finance that accompany a financial market liberalization drive the endogenous interest rate up, rather than down. It follows that price-rent ratios rise after a financial market liberalization because the decline in risk-premia more than offsets the rise in equilibrium interest rates. These findings underscore the crucial role of foreign capital in maintaining low interest rates during a financial market liberalization. Without an infusion of foreign capital, any period of looser collateral requirements and lower housing transactions costs (such as that which characterized the period of rapid home price appreciation from 2000-2006) would be accompanied by an increase in equilibrium interest rates, as households endogenously respond to the improved risk-sharing opportunities afforded by a financial market liberalization by reducing precautionary saving.

An influx of foreign capital into bond markets plays a central role in lower interest rates but a modest role in a housing booms. The influx of foreign governmental capital into the bond market generates a sharp decline in real interest rates in the model. In partial equilibrium analyses, a decline of such a magnitude would, by itself, generate a large increase in the price-rent ratio. When general equilibrium effects are taken into account, however, large declines in interest rates play a more modest role in driving up price-rent ratios. This occurs because the influx of foreign capital crowds domestic savers out of the safe bond market, exposing them to greater systematic risk in equity and housing markets. As a consequence, the housing risk-premium moves upward, in the opposite direction of interest rates, partially offsetting the effect of the latter on the price-rent ratio.

Procyclical increases in equilibrium price-rent ratios reflect lower future returns, not higher future rents. It is commonly assumed that increases in national house-price rent ratios reflect an expected increase in future housing fundamentals, such as rental growth. In partial equilibrium analyses where discount rates are held constant, this is the only outcome possible (e.g., Sinai and Souleles (2005), Campbell and Cocco (2007)). This reasoning, however, ignores the general equilibrium response of both residential investment and discount rates to economic growth. In the model here, positive economic shocks stimulate greater housing demand and greater residential investment. Under plausible parameterizations, the latter can lead to an equilibrium *decline* in future rental growth as the housing stock rises. Thus, high price-rent ratios in expansions must entirely reflect expectations of future house price depreciation (lower discount rates), driven by falling risk-premia as collateral values rise with the economy. Although future rental growth is expected to be lower, price-rent ratios still rise in response to positive economic shocks because the expected decline in future housing returns more than offsets the expected fall in future rental growth.

A financial market liberalization leads to a short-run boom in consumption, but a short-run bust in investment. A financial market liberalization leads to a short-run boom in aggregate consumption, consistent with common notions of housing "wealth effects." This result, however, occurs not for the usual partial equilibrium reason that a financial market liberalization allows credit-constrained households to borrow more against future income. On the contrary, we show that the sustained increase in consumption following a financial market liberalization is attributable to net lenders rather than net borrowers. A financial market liberalization is not stimulative for the economy as a whole, however, since the short-run boom in consumption drives up interest rates and crowds out investment.

Financial market liberalization plus foreign capital leads to a shift in the composition of wealth towards housing, increases financial wealth inequality, but has ambiguous affects on consumption inequality. A financial market liberalization plus an influx of foreign capital into the bond market leads households of all ages and incomes to shift the composition of their assets towards housing. Both the magnitude and age/income-distribution of these changes in the model are in line those observed in household-level data from 2000 to 2007. Such changes in housing finance also have implications for inequality. Although a financial market liberalization improves risk sharing and drives risk-premia down, an infusion of foreign governmental capital reduces risk sharing and drives risk premia up because it forces domestic savers out of the bond market, increasing their exposure to systematic risk in equity markets. We show that a financial market liberalization and foreign capital infusion have offsetting effects on consumption inequality but reinforcing upward effects on financial wealth inequality.

The paper is organized as follows. The next subsection briefly discusses related literature. Section 2 describes recent changes in the three key aspects of housing finance discussed above: collateral constraints, housing transactions costs, and foreign capital in U.S. debt markets. Section 3 presents the theoretical model. Section 4 presents our main findings, including benchmark business cycle and financial market statistics. Here we show the model generates a sizable equity premium and Sharpe ratio simultaneously with a plausible degree of variability in aggregate consumption. The model also generates forecastable variation both in long-horizon excess stock market returns and in excess returns on national house price indexes, consistent with statistical evidence, though it produces too much cash-flow predictability, as we discuss below. Section 5 concludes.

1.1 Related Literature

Our paper is related to a growing body of literature in finance that studies the asset pricing implications of incomplete markets models. The focus of this literature, however, is typically on the equity market implications of such models with no role for housing. The majority of this literature also does not model the production side of the economy, instead studying pure exchange economies with exogenous endowments.[2] Storesletten, Telmer, and Yaron (2007), Gomes and Michaelides (2008), and Favilukis (2008) explicitly model the production side of the economy, but focus on single-sector economies without housing.

Within the incomplete markets environment, our work is related to several papers that study questions related to housing and/or consumer durables more generally. These papers typically either do not model production (instead studying a pure exchange economy), and/or the portfolio choice problem underlying asset allocation between a risky and a risk-free asset, or are analyses of partial equilibrium environments. See for example, the general equilibrium exchange-economy analyses that embed bond, stock and housing markets of Ríos-Rull and Sánchez-Marcos (2006), Lustig and Van Nieuwerburgh (2007, 2008), Piazzesi and Schneider (2008), and the partial equilibrium analyses of Peterson (2006), Ortalo-Magné and Rady (2006), and Corbae and Quintin (2009).

Other researchers have studied the role of incomplete markets in housing decisions in models without aggregate risk. Fernández-Villaverde and Krueger (2005) study how consumption over the life-cycle is influenced by consumer durables in an incomplete markets model with production, but limit their focus to equilibria in which prices, wages and interest rates are constant over time. Kiyotaki, Michaelides, and Nikolov (2008) study a life-cycle model with housing and non-housing production, but focus their analysis on the perfect foresight equilibria of an economy without aggregate risk and an exogenous interest rate. One recent analysis that does combine aggregate risk, production, and incomplete markets is Iacoviello and Pavan (2009). These authors study the role of housing and debt for the volatility of the aggregate economy in a model with a single production and single saving

[2] See for example Aiyagari and Gertler (1991), Telmer (1993), Lucas (1994), Heaton and Lucas (1996), Basak and Cuoco (1998), Luttmer (1999)), for a study of single sector exchange economies, or Lustig and Van Nieuwerburgh (2005) for a two-sector exchange economy model.

technology. Because there is no risk-free asset, however, their model is silent about the role of risk-premia in the economy, a central focus of our paper.

Outside of the incomplete markets environment, a strand of the macroeconomic literature studies housing behavior in a two-sector, general equilibrium business cycle framework either with production (e.g., Davis and Heathcote (2005), Kahn (2008)) or without production (e.g., Piazzesi, Schneider, and Tuzel (2007)). The focus in these papers is on environments with complete markets for idiosyncratic risks and a representative agent representation. These models are silent on questions involving risk-sharing, inequality, and age and income heterogeneity.

It is important to note that our paper does not address the question of *why* credit market conditions changed so markedly in recent decades. It is widely understood that the financial market liberalization we discuss in the next section was preceded by a number of revolutionary changes in housing finance, notably by the rise in securitization. These changes initially decreased the risk of individual home mortgages and home equity loans, making it optimal for lenders to lower collateral requirements and reduce housing transactions fees (e.g. Green and Wachter (2008); Strongin, O'Neill, Himmelberg, Hindian, and Lawson (2009)). As these researchers note, however, these initially risk-reducing changes in housing finance were accompanied by government deregulation of financial institutions that ultimately increased risk, by permitting such institutions to alter the composition of their assets towards more high-risk securities, by permitting higher leverage ratios, and by presiding over the spread of complex financial holding companies that replaced the long-standing separation between investment bank, commercial bank and insurance company. The market's subsequent revised expectation upward of the riskiness of the underlying mortgage assets since 2007 appears, anecdotally, to have led to a reversal in collateral requirements and transactions fees. Embedding the optimal dynamic mortgage contracting problem into a general equilibrium model with limited risk-sharing remains a significant challenge for future research.

2 Changes in Housing Finance

We use the model of this paper to study the impact of changes in three features of housing finance. First, we investigate the impact of changes in housing collateral requirements, broadly defined. Collateral constraints can take the form of an explicit down payment requirement for new home purchases, but they also apply to home equity borrowing. Recent data suggests that down payment requirements declined for a range of mortgages categories in the period leading up to the broad decline in housing prices that began in 2006. Loan-to-value ratios on subprime loans rose from 79% to 86% over the period 2001-2005, while debt-income ratios rose (Demyanyk and Hemert (2008)). Other reports suggest that the increase in loan-to-value (LTV) ratios for prime mortgages was even greater, with one industry analysis finding that LTV ratios for such loans rose from 60.4% in 2002 to 75.2% in 2006.[3] There was also a surge in borrowing against existing home equity between 2002 and 2006 (Mian and Sufi (2009b)).

More generally, there was a widespread relaxation of underwriting standards in the U.S. mortgage market during the period leading up to the credit crisis of 2007. The loosening of standards can be observed in the marked rise in simultaneous second-lien mortgages and in no-documentation or low-documentation loans.[4] Looser underwriting standards provide a back-door means of reducing collateral requirements for home purchases. By the end of 2006 households routinely bought homes with 100% financing using a piggyback second mortgage or home equity loan. (See also Mian and Sufi (2009a).) Industry analysts indicate that LTV ratios for combined (first and second) mortgages have since returned to more normal levels of no greater than 75-80% of the appraised value of the home. We assess the impact of these changes collectively by modeling them as a reduction in collateralized borrowing constraints.

The period of rapid home price appreciation was also marked by a decline in the cost of conducting housing transactions. Costs associated with mortgage refinancing and home equity extraction fell sharply in the years leading up to the housing boom that ended in 2006/2007 (McCarthy and Steindel (2007)). Mortgage equity withdrawal rates surged 350% from 2000-2006.[5] The Federal Housing Financing Board reports monthly data on mortgage rates (based on a survey of the largest lenders). They report "contract rates", "initial fees and charges", and "effective rates." The latter add to the contract rate the discounted fees and charges. Figure 2 shows that initial fees and charges on mortgages have declined from 2.70% of the loan balance in January 1985 to 0.46% in April 2008. The difference between

[3]Source: UBS, April 16, 2007 Lunch and Learn, "How Did We Get Here and What Lies Ahead," Thomas Zimmerman, page 5.

[4]FDIC Outlook: Breaking New Ground in U.S. Mortgage Lending, December 18, 2006. <http://www.fdic.gov/bank/analytical/regional/ro20062q/na/2006_summer04.html#10A>. A simultaneous second-lien loan, also referred to as a "piggyback loan," is a lending arrangement where either a closed-end second lien or a home equity line of credit is originated at the same time as the first-lien mortgage loan, usually taking the place of a larger down payment.

[5]Figures based on updated estimates provided by James Kennedy of the mortgage analysis in Kennedy and Greenspan (2005).

the effective rate and the contract rate is also a measure of the initial fees and charges, but now expressed as an interest rate. This difference declined 90%, from 50 basis points to 5 basis points over the period 1985-2007. Anecdotal evidence suggests that these costs began moving back up in the aftermath of the credit crisis of 2007/2008.

A third key development in the housing market of recent years is the secular decline in interest rates coinciding with a surge in foreign ownership of U.S. bonds. Figure 3 shows that both 30-year FRMs and the 10-year Treasury bond yield have trended downward, with mortgage rates declining from around 18 percent in the early 1980s to near 6 percent by the end of 2007. This was not merely attributable to a decline in inflation: the real annual interest rate on the ten-year Treasury bond fell from 3.6% in 2000 to 0.93% in 2006 using the consumer price index as a measure of inflation. At the same time, foreign ownership of U.S. Treasuries (T-bonds and T-notes) increased from $118 billion in 1984, or 13.5% of marketable Treasuries outstanding, to $2.2 trillion in 2008, or 61% of marketable Treasuries (Figure 4). Foreign holdings of U.S. agency and Government Sponsored Enterprise-backed agency securities quintupled between 2000 and 2007, rising from $261 billion to $1.3 trillion, or from 7% to 21% of total agency debt. By pushing real interest rates lower, the rise in foreign capital has been directly linked to the surge in mortgage originations over this period (e.g., Strongin, O'Neill, Himmelberg, Hindian, and Lawson (2009)). The possible role of foreign capital in driving interest rates lower has also been emphasized by economic policymakers, such as Federal Reserve Chairman Ben Bernanke.[6]

In the model of this paper, interest rates are determined in equilibrium by a market clearing condition for bondholders. We consider one specification of the model in which we introduce an exogenous foreign demand for domestic bonds into the market clearing condition, referred to hereafter as *foreign capital*. By the end of 2008, Foreign Official Institutions held 70% of all foreign holdings of U.S. Treasuries. We therefore model foreign capital as supplied by foreign central banks and other governmental agencies who have specific regulatory and reserve currency motives for holding the safe asset. As explained in Kohn (2002), government entities face both legal and political restrictions on the type of assets that can be held, forcing them into safe securities. Krishnamurthy and Vissing-Jorgensen (2008) find that demand for U.S. Treasury securities by governmental holders is extremely inelastic, suggesting that when these holders receive funds to invest they buy U.S. Treasuries, regardless of their price relative to other U.S. assets. This motivates our modeling of foreign capital as both exogenous and as restricted to investments in the safe asset. In the model, we assume domestic borrowers may obtain credit at a fixed interest rate spread with the governmental rate. Because our model abstracts from default, we set this spread to zero in our calibration.

[6] For example, in 2005 Bernanke argued in 2005:

> I will argue that over the past decade a combination of diverse forces has created a significant increase in the global supply of saving–a global saving glut–which helps to explain both the increase in the U.S. current account deficit and the relatively low level of long-term real interest rates in the world today. [...] Because the dollar is the leading international reserve currency, and because some emerging-market countries use the dollar as a reference point when managing the values of their own currencies, the saving flow out of the developing world has been directed relatively more into dollar-denominated assets, such as U.S. Treasury securities.
>
> –Remarks by Governor Ben S. Bernanke at the Sandridge Lecture, Virginia Association of Economics, Richmond, Virginia, March 10, 2005.

In 2008, Bernanke tied the supply of foreign capital to the surge in U.S. house prices that peaked in 2006:

> The pressure of these net savings flows led to lower long-term real interest rates around the world, stimulated asset prices (including house prices), and pushed current accounts toward deficit in the industrial countries–notably the United States–that received these flows. –Remarks made by Federal Reserve Chairman Ben S. Bernanke to the International Monetary Conference, Barcelona, Spain (via satellite), June 3, 2008.

3 The Model

3.1 Firms

The production side of the economy consists of two sectors. One sector produces the non-housing consumption good, and the other sector produces the housing good. We refer to the first as the "consumption sector" and the second as the "housing sector." Time is discrete and each period corresponds to a year. In each period, a representative firm in each sector chooses labor (which it rents) and investment in capital (which it owns) to maximize the value of the firm to its owners.

3.1.1 Consumption Sector

Denote output in the consumption sector as

$$Y_{C,t} \equiv Z_{C,t} K_{C,t}^{\alpha} N_{C,t}^{1-\alpha}$$

where $Z_{C,t}$ is the stochastic productivity level at time t, K_j is the capital stock in the consumption sector, α is the share of capital, and N_C is the quantity of labor input in the consumption sector. Let I_C denote investment in the consumption sector. The firm's capital stock $K_{C,t}$ accumulates over time subject to proportional adjustment costs, $\phi_C \left(\frac{I_{C,t}}{K_{C,t}} \right) K_{C,t}$, modeled as a deduction from the earnings of the firm. The firm maximizes the present discounted value $V_{C,t}$ of a stream of earnings:

$$V_{C,t} = \max_{N_{C,t}, I_{C,t}} E_t \sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} \left(Y_{C,t+k} - w_{t+k} N_{C,t+k} - I_{C,t+k} - \phi_C \left(\frac{I_{C,t+k}}{K_{C,t+k}} \right) K_{C,t+k} \right), \quad (1)$$

where $\frac{\beta^k \Lambda_{t+k}}{\Lambda_t}$ is a stochastic discount factor discussed below, and w is the wage rate (equal across sectors in equilibrium). The evolution equation for the firm's capital stock is

$$K_{C,t+1} = (1-\delta) K_{C,t} + I_{C,t},$$

where δ is the depreciation rate of the capital stock.

The firm does not issue new shares and finances its capital stock entirely through retained earnings. The dividends to shareholders are equal to

$$D_{C,t} = Y_{C,t} - w_t N_{C,t} - I_{C,t} - \phi_K \left(\frac{I_{C,t}}{K_{C,t}} \right) K_{C,t} \geq 0.$$

3.1.2 Housing Sector

The housing firm's problem is directly analogous to the problem solved by the representative firm in the consumption sector. Denote output in the residential housing sector as

$$Y_{H,t} = Z_{H,t} K_{H,t}^{\nu} N_{H,t}^{1-\nu},$$

$Y_{H,t}$ represents construction of new housing (residential investment), where ν is the share of capital in housing output. Variables denoted with an "H" subscript are defined exactly as above for the consumption sector but now pertain to the housing sector, e.g., Z_H denotes the stochastic productivity level in the housing sector. The firm maximizes

$$V_{H,t} = \max_{N_{H,t}, I_{H,t}} E_t \sum_{k=0}^{\infty} \frac{\beta^k \Lambda_{t+k}}{\Lambda_t} \left(p_{t+k}^H Y_{H,t+k} - w_{t+k} N_{H,t+k} - I_{H,t+k} - \phi_H \left(\frac{I_{H,t+k}}{K_{H,t+k}} \right) K_{H,t+k} \right), \tag{2}$$

where p_{t+k}^H is the relative price of one unit of housing in units of the non-housing consumption good. Note that p_t^H is the time t price of a unit of housing of fixed quality and quantity. The dividends to shareholders in the housing sector are denoted

$$D_{H,t} = p_t^H Y_{H,t} - w_t N_{H,t} - I_{H,t} - \phi_H \left(\frac{I_{H,t}}{K_{H,t}} \right) K_{H,t} \geq 0.$$

Capital in the housing sector evolves:

$$K_{H,t+1} = (1-\delta) K_{H,t} + I_{H,t}.$$

Because $Y_{H,t}$ represents residential investment, the law of motion for the aggregate residential housing stock H_t is

$$H_{t+1} = (1-\delta_H) H_t + Y_{H,t},$$

where δ_H denotes the depreciation rate of the housing stock.

3.2 Risky Asset Returns

The firms' values $V_{H,t}$ and $V_{C,t}$ are the *cum* dividend values, measured before the dividend is paid out. Thus the *cum* dividend returns to shareholders in the housing sector and the consumption sector are defined, respectively, as

$$R_{Y_H,t+1} = \frac{V_{H,t+1}}{(V_{H,t} - D_{H,t})} \qquad R_{Y_C,t+1} = \frac{V_{C,t+1}}{(V_{C,t} - D_{C,t})}.$$

We define $V_{j,t}^e = V_{j,t} - D_{j,t}$ for $j = H, C$ to be the *ex* dividend value of the firm.[7]

[7] Using the *ex* dividend value of the firm the return reduces to the more familiar *ex* dividend definition: $R_{j,t+1}^e = \frac{V_{j,t+1}^e + D_{j,t+1}}{V_{j,t}^e}$.

3.3 Individuals

The economy is populated by A overlapping generations of individuals, indexed by $a = 1, ..., A$, with a continuum of individuals born each period. Individuals live through two stages of life, a working stage and a retirement stage. Adult age begins at age 21, so a equals this effective age minus 20. Agents live for a maximum of $A = 80$ (100 years). Workers live from age 21 ($a = 1$) to 65 ($a = 45$) and then retire. Retired workers die with an age-dependent probability calibrated from life expectancy data. The probability that an agent is alive at age $a+1$ conditional on being alive at age a is denoted $\pi_{a+1|a}$. Upon death, any remaining net worth of the individual in that period is counted as terminal "consumption," e.g., funeral and medical expenses.

Individuals have an intraperiod utility function given by

$$U(C_{a,t}, H_{a,t}) = \frac{\widetilde{C}_{a,t}^{1-\frac{1}{\sigma}}}{1-\frac{1}{\sigma}} \qquad \widetilde{C}_{a,t} = \left[\chi C_{a,t}^{\frac{\varepsilon-1}{\varepsilon}} + (1-\chi) H_{a,t}^{\frac{\varepsilon-1}{\varepsilon}}\right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where $C_{a,t}$ is non-housing consumption of an individual of age a, and $H_{a,t}$ is the stock of housing, σ is the coefficient of relative risk aversion, χ is the relative weight on non-housing consumption in utility, and ε is the constant elasticity of substitution between C and H. Implicit in this specification is the assumption that the service flow from houses is proportional to the stock $H_{a,t}$.

Financial market trade is limited to a one-period riskless bond and to risky capital, where the latter is restricted to be a mutual fund of equity in the housing and consumption sectors. The mutual fund is a value-weighted portfolio with return

$$R_{K,t+1} = \frac{V_{H,t}^e}{V_{H,t}^e + V_{C,t}^e} R_{Y_H,t+1} + \frac{V_{C,t}^e}{V_{H,t}^e + V_{C,t}^e} R_{Y_C,t+1}. \tag{3}$$

The gross bond return is denoted $R_{f,t} = \frac{1}{q_{t-1}}$, where q_{t-1} is the bond price known at time $t-1$. Individuals are born with no initial endowment of risky capital or bonds.

Individuals are heterogeneous in their labor productivity. To denote this heterogeneity, we index individuals i. Before retirement households supply labor inelastically. The stochastic process for individual income for workers is

$$Y_{a,t}^i = w_t L_{a,t}^i,$$

where $L_{a,t}^i$ is the individual's labor endowment (hours times an individual-specific productivity factor), and w_t is the aggregate wage per unit of productivity. Labor productivity is specified by a deterministic age-specific profile, G_a, and an individual shock $Z^i_{a,t}$:

$$\begin{aligned} L^i_{a,t} &= G_a Z^i_{a,t} \\ \log\left(Z^i_{a,t}\right) &= \log\left(Z^i_{a-1,t-1}\right) + \epsilon^i_{a,t}, \qquad \epsilon^i_{a,t} \sim i.i.d.\left(0, \sigma^2_t\right), \end{aligned}$$

where G_a is a deterministic function of age capturing a hump-shaped profile in life-cycle earnings and $\epsilon^i_{a,t}$ is a stochastic i.i.d. shock to individual earnings. To capture countercyclical variation in idiosyncratic risk of the type documented by Storesletten, Telmer, and Yaron (2004), we use a two-state specification for the variance of idiosyncratic earnings shocks:

$$\sigma^2_t = \begin{cases} \sigma^2_E & \text{if } Z_{C,t} \geq E\left(Z_{C,t}\right) \\ \sigma^2_R & \text{if } Z_{C,t} < E\left(Z_{C,t}\right) \end{cases}, \qquad \sigma^2_R > \sigma^2_E \tag{4}$$

This specification implies that the variance of idiosyncratic labor earnings is higher in "recessions" ($Z_{C,t} \leq E\left(Z_{C,t}\right)$) than in "expansions" ($Z_{C,t} \geq E\left(Z_{C,t}\right)$). The former is denoted with an "R" subscript, the latter with an "E" subscript. Finally, labor earnings are taxed at rate τ in order to finance social security retirement income.

At age a, agents enter the period with wealth invested in bonds, B^i_a, and shares θ^i_a of risky capital. The total number of shares outstanding of the risky asset is normalized to unity. We rule out short-sales in the risky asset,

$$\theta^i_{a,t} \geq 0.$$

If the individual chooses to invest in the mutual fund, it pays a fixed, per-period participation cost, $F_{K,t}$.

We assume that the housing owned by each individual depreciates at rate δ_H, the rate of depreciation of the aggregate housing stock. Households may choose to increase the quantity of housing consumed at time $t+1$ by making a net investment $H^i_{a,t+1} - (1-\delta_H)\, H^i_{a,t} > 0$. Because houses are illiquid, it is expensive to change housing consumption. If the individual chooses to change its housing consumption, it pays a transaction cost $F^i_{H,a,t}$. Denote the sum of the per period equity participation cost and housing transaction cost for individual i as

$$F^i_{a,t} \equiv F^i_{H,a,t} + F_{K,t}.$$

Define the individual's gross financial wealth at time t as

$$W^i_{a,t} \equiv \theta^i_{a,t}\left(V^e_{C,t} + V^e_{H,t} + D_{C,t} + D_{H,t}\right) + B^i_{a,t}.$$

The budget constraint for an agent of age a who is not retired is

$$C^i_{a,t} + B^i_{a+1,t+1} q_t + \theta^i_{a+1,t+1}\left(V^e_{C,t} + V^e_{H,t}\right) \leq W^i_{a,t} + (1-\tau)\, w_t L^i_{a,t} + p^H_t\left((1-\delta_H) H^i_{a,t} - H^i_{a+1,t+1}\right) - F^i_{a,t} \tag{5}$$

$$\begin{aligned} W^i_{a+1,t+1} &\geq -(1-\varpi)\, p^H_t H^i_{a,t+1}, \qquad \forall a, t \\ \theta^i_{a,t} &\geq 0 \qquad \forall a, t \end{aligned} \tag{6}$$

where τ is a social security tax rate and where

$$F^i_{H,a,t} = \begin{cases} 0 & \text{if } H^i_{a+1,t+1} = (1-\delta_H)\, H^i_{a,t} \\ \psi_0 + \psi_1 p^H_t H^i_{a,t} & \text{if } H^i_{a+1,t+1} \neq (1-\delta_H)\, H^i_{a,t} \end{cases}.$$

$$F_{K,t} = \begin{cases} 0 & \text{if } \theta^i_{a+1,t+1} = 0 \\ \overline{F} & \text{if } \theta^i_{a+1,t+1} > 0 \end{cases}.$$

$F^i_{H,a,t}$ is the housing transactions cost which contains both a fixed and variable component. Equation (6) is the collateral constraint, where $0 \leq \varpi \leq 1$. It says that households may borrow no more than a fraction $(1-\varpi)$ of the value of housing, implying that they must post collateral equal to a fraction ϖ of the value of the house. This constraint can be thought of as a down-payment constraint for new home purchases, but it also encompasses collateral requirements for home equity borrowing against existing homes. The constraint gives the maximum combined LTV ratio for first and second mortgages and home equity withdrawal. Notice that if the price p^h_t of the house rises and nothing else changes, the individual can finance a greater level of consumption of both housing and nonhousing goods and services.

Two points about the collateral constraint above are worth noting. First, it applies to any borrowing against home equity, not just to mortgages. Second, borrowing takes place using one-period debt. Thus, an individual's borrowing capacity fluctuates period-by-period with the value of the house.

We also prevent individuals from buying stock on margin. If the individual is a net borrower, this means we restrict holdings of the risky asset to be zero, $\theta^i_{a+1,t+1} = 0$. This restriction is stated mathematically as follows:

$$\begin{aligned} &\text{if } W^i_{a,t} + (1-\tau)\, w_t L^i_{a,t} - \left(C^i_{a,t} + p^H_t\left(H^i_{a+1,t+1} - (1-\delta_H) H^i_{a,t}\right) - F^i_{a,t}\right) < 0 \\ &\qquad \text{then} \qquad B^i_{a+1,t+1} < 0, \qquad \theta^i_{a+1,t+1} = 0. \end{aligned} \tag{7}$$

Net lenders may take a positive position in the risky asset but may not short the bond to do so:

$$\text{if } W^i_{a,t} + (1-\tau)\, w_t L^i_{a,t} - \left(C^i_{a,t} + p^H_t \left(H^i_{a+1,t+1} - (1-\delta_H) H^i_{a,t}\right) - F^i_{a,t}\right) \geq 0 \quad (8)$$
$$\text{then} \qquad B^i_{a+1,t+1} \geq 0, \qquad \theta^i_{a+1,t+1} \geq 0.$$

Let Z^i_{ar} denote the value of the stochastic component of individual labor productivity, $Z^i_{a,t}$, during the last year of working life. Each period, retired workers receive a government pension $PE^i_{a,t} = Z^i_{ar} X_t$ where $X_t = \tau \frac{N^W}{N^R}$ is the pension determined by a pay as you go system, and N^W and N^R are the numbers of working age and retired households.[8] For agents who have reached retirement age, the budget constraint is identical to that for workers (5) except that wage income $(1-\tau)\, w_t L^i_{a,t}$ is replaced by pension income $PE^i_{a,t}$.

Let $Z_t \equiv (Z_{C,t}, Z_{H,t})'$ denote the aggregate shocks. The state of the economy is a pair, (Z, μ), where μ is a measure defined over $\mathcal{S} = (\mathcal{A} \times \mathcal{Z} \times \mathcal{W} \times \mathcal{H})$, where $\mathcal{A} = \{1, 2, ...A\}$ is the set of ages, where $\mathcal{Z}$ is the set of all possible idiosyncratic shocks, where $\mathcal{W}$ is the set of all possible beginning-of-period financial wealth realizations, and where $\mathcal{H}$ is the set of all possible beginning-of-period housing wealth realizations. That is, μ is a distribution of agents across ages, idiosyncratic shocks, financial and housing wealth. The presence of aggregate shocks implies that μ evolves stochastically over time. We specify a law of motion, Γ, for μ,

$$\mu_{t+1} = \Gamma\left(\mu_t, Z_t, Z_{t+1}\right).$$

3.4 Stochastic Discount Factor

The stochastic discount factor (SDF), $\frac{\beta \Lambda_{t+1}}{\Lambda_t}$, appears in the dynamic value maximization problem (1) and (2) undertaken by each representative firm. As an alternative, we could assume that firms rent capital from households on a period-by-period basis and solve a static optimization problem (hence face no adjustment costs to changing capital). In this case, to make the volatility of the equity return realistic we would also need to assume stochastic depreciation in the rented capital stocks (e.g., Storesletten, Telmer, and Yaron (2007), Gomes and Michaelides (2008)). Here we instead keep depreciation deterministic and model dynamic firms that own capital and face adjustment costs when changing their

[8] The decomposition of the population into workers and retirees is determined from life-expectancy tables as follows. Let X denote the total number of people born each period. (In practice this is calibrated to be a large number in order to approximate a continuum.) Then $N^W = 45 \cdot X$ is the total number of workers. Next, from life expectancy tables, if the probability of dying at age $a > 45$ is denoted p_a then $N^R = \sum_{a=46}^{80} (1 - p_a)\, X$ is the total number of retired persons.

capital stocks. We do this for several reasons. First, in our own experimentation we found that the amount of stochastic depreciation required to achieve reasonable levels of stock market volatility produced excessive volatility in investment. Second, it is difficult to know what amount of stochastic depreciation, if any, is reasonable. Third, an economy populated entirely of static firms is unrealistic. In the real world, firms own their own capital stocks and must think dynamically about shareholder value.

For these reasons, we assume that the representative firm in each sector solves the dynamic problem presented above and discount future profits using a weighted average of the individual shareholders' intertemporal marginal rates of substitution (IMRS) in non-housing consumption, $\frac{\beta \partial U / \partial C^i_{a+1,t+1}}{\partial U / \partial C^i_{a,t}}$, where the weights, $\theta^i_{a,t}$, correspond to the shareholder's proportional ownership in the firm. Let $\frac{\beta \Lambda_{t+1}}{\Lambda_t}$ denote this weighted average. Recalling that the total number of shares in the risky portfolio is normalized to unity, we have

$$\frac{\beta \Lambda_{t+1}}{\Lambda_t} \equiv \int_{\mathcal{S}} \theta^i_{a+1,t+1} \frac{\beta \partial U / \partial C^i_{a+1,t+1}}{\partial U / \partial C^i_{a,t}} d\mu \tag{9}$$

$$\frac{\beta \partial U / \partial C^i_{a+1,t+1}}{\partial U / \partial C^i_{a,t}} = \beta \left[\left(\frac{C^i_{a+1,t+1}}{C^i_{a,t}} \right)^{-\frac{1}{\sigma}} \left[\frac{\chi + (1-\chi) \left(\frac{H^i_{a+1,t+1}}{C^i_{a+1,t+1}} \right)^{\frac{\varepsilon-1}{\varepsilon}}}{\chi + (1-\chi) \left(\frac{H^i_{a,t}}{C^i_{a,t}} \right)^{\frac{\varepsilon-1}{\varepsilon}}} \right]^{\frac{\sigma-\varepsilon}{\sigma(\varepsilon-1)}} \right]. \tag{10}$$

Since we weight each individual's IMRS by its proportional ownership (and since short-sales in the risky asset are prohibited), only those households who have taken a positive position in the risky asset (shareholders) will receive non-zero weight in the SDF.

This specification of the stochastic discount factor leads to an equilibrium that depends on the control of the firm being fixed according to the proportional ownership structure described above. The equilibrium is not necessarily sensitive to this assumption on ownership control, however. For example, Carceles Poveda and Coen-Pirani (2009) show that, given the firm's objective of value maximization, the equilibrium allocations in their incomplete markets models are invariant to the choice of stochastic discount factor within the set that includes the IMRS of any household (or any weighted average of these) for whom the Euler equation for the risky asset return is satisfied. They show in addition that the equilibrium allocations of such economies are the same as the allocations obtained in otherwise identical economies with "static" firms that rent capital from households on a period-by-period basis.[9] Although these results have been formally proved only in an environment without adjustment

[9] "Otherwise identical" means that the two economies are identical with respect to the specification of preference orderings, initial endowments, probability laws governing stochastic shocks, and borrowing limits.

costs, we note that our calibration of adjustment costs (discussed below) implies that they are quantitatively small, amounting to less than one percent of investment per year. We have checked that our results are not affected by the following variants of the SDF above: (i) equally weighting the IMRS of shareholders (gives proportionally more weight to small stakeholders), (ii) weighting the IMRS of shareholders by the squares of their ownership stakes, $\left(\theta^{i}_{a+1,t+1}\right)^{2}$, (gives proportionally more weight to big stakeholders), (iii) using the IMRS of the largest shareholder.

3.5 Equilibrium

An equilibrium is defined as a set of endogenously determined prices (bond prices, wages, risky asset returns) given by time-invariant functions $q_t = q\left(\mu_t, Z_t\right)$, $p^H_t = p^H\left(\mu_t, Z_t\right)$, $w_t = w\left(\mu_t, Z_t\right)$, and $R_{K,t} = R_K\left(\mu_t, Z_t\right)$, respectively, a set of cohort-specific value functions and decision rules for each individual i, $\left\{V_a, H^i_{a+1,t+1}, \theta^i_{a+1,t+1} B^i_{a+1,t+1}\right\}^{A}_{a=1}$ and a law of motion for μ, $\mu_{t+1} = \Gamma\left(\mu_t, Z_t, Z_{t+1}\right)$ such that:

1. Households optimize:

$$\begin{aligned} V_a(\mu_t, Z_t, Z^i_{a,t}, W^i_{a,t}, H^i_{a,t}) &= \max_{H^i_{a+1,t+1}, \theta^i_{a+1,t+1} B^i_{a+1,t+1}} \{U(C^i_{a,t}, H^i_{a,t}) \\ &\quad + \beta \pi_{a+1|a} E_t[V_{a+1}(\mu_{t+1}, Z_{t+1}, Z^i_{a,t+1}, W^i_{a+1,t+1}, H^i_{a+1,t+1})]\} \end{aligned} \tag{11}$$

   subject to (5), (6), (7), and (8) if the individual of working age, and subject to the analogous versions of (5), (6), (7), and (8) (using pension income in place of wage income), if the individual is retired.

2. Firm's maximize value: $V_{C,t}$ solves (1), $V_{H,t}$ solves (2).

3. Wages $w_t = w\left(\mu_t, Z_t\right)$ satisfy

$$w_t = (1-\alpha) Z_{C,t} K^{\alpha}_{C,t} N^{-\alpha}_{C,t} \tag{12}$$

$$w_t = (1-\nu) p^H_t Z_{H,t} K^{\nu}_{H,t} N^{-\nu}_{H,t}. \tag{13}$$

4. The housing market clears: $p^H_t = p^H\left(\mu_t, Z_t\right)$ is such that

$$Y_{H,t} = \int_{\mathcal{S}} \left(H^i_{a,t+1} - H^i_{a,t}\left(1-\delta_H\right)\right) d\mu. \tag{14}$$

5. The bond market clears: $q_t = q(\mu_t, Z_t)$ is such that

$$\int_{\mathcal{S}} B^i_{a,t} d\mu + B^F_t = 0, \tag{15}$$

where $B^F_t \geq 0$ is an exogenous supply of foreign capital discussed below.

6. The risky asset market clears:

$$1 = \int_{\mathcal{S}} \theta^i_{a,t} d\mu. \tag{16}$$

7. The labor market clears:

$$N_t \equiv N_{C,t} + N_{H,t} = \int_{\mathcal{S}} L^i_{a,t} d\mu. \tag{17}$$

8. The social security tax rate is set so that total taxes equal total retirement benefits:

$$\tau N_t w_t = \int_{\mathcal{S}} PE^i_{a,t} d\mu, \tag{18}$$

9. The presumed law of motion for the state space $\mu_{t+1} = \Gamma(\mu_t, Z_t, Z_{t+1})$ is consistent with individual behavior.

Equations (12), (13) and (17) determine the $N_{C,t}$ and therefore determine the allocation of labor across sectors:

$$(1-\alpha) Z_{C,t} K^{\alpha}_{C,t} N^{-\alpha}_{C,t} = (1-\nu) p^H_t Z_{H,t} K^{\nu}_{H,t} (N_t - N_{C,t})^{-\nu}. \tag{19}$$

Also, the aggregate resource constraint for the economy must take into account the housing and risky capital market transactions/participation costs, which reduce consumption, the adjustment costs in productive capital, which reduce firm profits, and the net foreign supply of capital in the bond market, which finances domestic consumption and investment. Thus, non-housing output minus non-housing consumption equals aggregate investment (gross of adjustment costs) less the net change in the value of foreign capital:

$$Y_{C,t} - C_t - F_t = \left(I_{C,t} + \phi_C \left(\frac{I_{C,t}}{K_{C,t}} \right) K_{C,t} \right) + \left(I_{H,t} + \phi_H \left(\frac{I_{H,t}}{K_{H,t}} \right) K_{H,t} \right) - \left(B^F_{t+1} q(\mu_t, Z_t) - B^F_t \right), \tag{20}$$

where C_t and F_t are aggregate quantities defined as[10]

$$C_t \equiv \int_{\mathcal{S}} C^i_{a,t} d\mu \tag{21}$$

$$F_t \equiv \int_{\mathcal{S}} F^i_{a,t} d\mu. \tag{22}$$

[10]Note that (20) simply results from aggregating the budget constraints across all households, imposing all market clearing conditions, and using the definitions of dividends as equal to firm revenue minus costs.

To solve the model, it is necessary to approximate the infinite dimensional object μ with a finite dimensional object. The appendix explains the solution procedure and how we specify a finite dimensional vector to represent the law of motion for μ.

3.6 Model Calibration

This section discusses our calibration of the model's primitive parameters under three alternative set of parameterizations. *Model 1* is our benchmark calibration, with "normal" collateral requirements and housing transactions costs calibrated to roughly match the data prior to the housing boom of 2000-2006. *Model 2* is an alternative calibration designed to match an economy that is otherwise identical to Model 1 but has undergone a financial market liberalization, where a liberalization is defined by a decline in both collateral requirements and housing transactions costs. In both Model 1 and Model 2, trade in the risk-free asset is entirely conducted between domestic residents: $B_t^F = 0$. *Model 3* is calibration that is identical to that of Model 2 except that we add an exogenous foreign demand for the risk-free bond: $B_t^F > 0$.

3.6.1 Calibration of Parameters

For convenience, the model's parameters and their numerical calibration are summarized in Table 1. We describe this calibration next.

The technology shocks Z_C and Z_H are assumed to follow two-state independent Markov chains; the calibration is described in the Appendix. The Appendix also describes our calibration of the individual productivity shocks.

Parameters pertaining to the firms' decisions are set as follows. The capital depreciation rate, δ is set to 0.12, which corresponds to the average Bureau of Economic Analysis (BEA) depreciation rates for equipment and structures. The housing depreciation rate δ_H, is set to 0.025 following Tuzel (2009). Following Kydland and Prescott (1982) and Hansen (1985), the capital share for the non-housing sector is set to $\alpha = 0.36$. For the residential investment sector, the value of the capital share in production is taken from a BEA study of gross product originating, by industry. The study finds that the capital share in the construction sector ranges from 29.4% and 31.0% over the period 1992-1996. We therefore set the capital share in the housing sector to $\nu = 0.30$.[11] The adjustment costs for capital in both sectors are assumed

[11] From the November 1997 SURVEY OF CURRENT BUSINESS, "Gross Product by Industry, 1947–96, "by Sherlene K.S. Lum and Robert E. Yuskavage.

to be the same quadratic function of the investment to capital-ratio, $\varphi\left(\frac{I}{K}-\delta\right)^2$, where the constant φ is chosen to represent a tradeoff between the desire to match aggregate investment volatility simultaneously with the volatility of asset returns. Under this calibration, firms pay a cost only for net new investment; there is no cost to replace depreciated capital. This implies that the total adjustment cost $\varphi\left(\frac{I}{K}-\delta\right)^2 K_t$ under our calibration is quite small: on average less than one percent of investment, I_t.

Parameters of the individual's problem are set as follows. The subjective time discount factor is set to $\beta = 0.923$ at annual frequency, to allow the model to match the mean of a short-term Treasury rate in the data. The survival probability $\pi_{a+1|a} = 1$ for $a+1 \leq 65$. For $a+1 > 65$, we set $\pi_{a+1|a}$ equal to the fraction of households over 65 born in a particular year alive at age $a+1$, as measured by the U.S. Census Bureau. From these numbers, we obtain the stationary age distribution in the model, and use it to match the average earnings over the life-cycle, G_a, to that observed from the Survey of Consumer Finances. Risk aversion is set to $\sigma = 8$, to help the models match the high Sharpe ratio for equity observed in the data. The static elasticity of substitution between C and H is set to $\varepsilon = 1$ (Cobb-Douglas utility). In future work, we plan to explore lower values.[12] The weight, χ on C in the utility function is set to 0.70, corresponding to a housing expenditure share of 0.30. The regime-switching conditional variance in the unit root process in idiosyncratic earnings is calibrated following Storesletten, Telmer, and Yaron (2007) to match their estimates from the Panel Study of Income Dynamics. These are $\sigma_E = 0.0768$, and $\sigma_R = 0.1296$.

The other parameters of the individual's problem are less precisely pinned down from empirical observation. The costs of stock market participation could include non-pecuniary costs as well as explicit transactions fees. Vissing-Jorgensen (2002) finds support for the presence of a fixed, per period participation cost, but not for the hypothesis of variable costs. She estimates the size of these costs and finds that they are small, less than 50 dollars per year in year 2000 dollars. These findings motivate our calibration of these costs so that

http://www.bea.gov/scb/account_articles/national/1197gpo/maintext.htm

Gross Product Originating is equal to gross domestic income, whose components can be grouped into categories that approximate shares of labor and capital. Under a Cobb-Douglas production function, these equal shares of capital and labor in output.

[12] Ogaki and Reinhart (1998) estimate a value of 1.167 for the elasticity of substitution between durables and nondurables in macro-level data, though without housing. Yogo (2006) estimates a value of 0.790 for the same elasticity again for durables that exclude housing. Estimates using household-level data on housing and nonhousing consumption are often lower than unity. Li, Liu, and Yao (2008), for example, estimate this elasticity to be 0.58.

they are no greater than 1% of per capita, average consumption, denoted $\overline{C}^i$ in the table above.

There are no readily available data on average collateral requirements for mortgages and home equity loans. Our own conversations with government economists and analysts who follow the housing sector, however, indicated that prior to the housing boom that ended in 2006/2007, the combined LTV for first and second conventional mortgages (mortgages without mortgage insurance) typically was not allowed to exceed 75 to 80% of the appraised value of the home. Moreover, home equity lines of credit were not widely available until relatively recently (McCarthy and Steindel (2007)). By contrast, these same analysts suggested, during the boom years, households routinely bought homes with 100% financing using a piggyback second or home equity loan. Loans for 125% of the home value were even available if the borrower used the top 25% to pay off existing debt. Our Model 1 sets the maximum combined LTV (first and second mortgages) to be 75%, corresponding to $\varpi = 25\%$. In Model 2, we lower this to $\varpi = 1\%$.

There are also no data on the fixed and variable transactions costs for housing consumption, governed by the parameters ψ_0, and ψ_1. For home purchases, these costs vary considerably by region, over time, by appraised value, and by type of sale (owner versus broker). But the housing transactions costs in the model are more comprehensive than the costs of buying and selling existing homes. They include costs of any change in housing consumption, such as home improvements and additions, that may be associated with mortgage refinancing and home equity extraction. As discussed above, fees and costs associated with home purchases and home equity finance eroded considerably in the housing boom, in many cases declining 90% or more. As a crude way of anchoring the level of these costs, in the baseline Model 1 we set fixed costs ψ_0 and variable costs ψ_1 to match the average number of years individuals in the model go without changing housing consumption equal to the average length of residency (in years) for home owners in the Survey of Consumer Finances across the 1989-2001 waves of the survey. In the equilibrium of our model, this amount corresponds to a value for ψ_0 that is approximately 3.2% of annual per capita consumption, and a value for ψ_1 that is approximately 5.5% of the value of the house $p_t^H H_{a,t}^i$. In Models 2 and 3 we decrease the fixed cost by 31%, setting it to approximately 2.2% of per capita aggregate consumption, and we decrease the variable cost by 36%, setting it to 3.5% of home value $p_t^H H_{a,t}^i$. Because the housing transactions costs in our model include many non-pecuniary costs that may not have changed with the financial market liberalization, as well as those that have (all of which contribute to the illiquidity of housing), it is not possible to observe directly the transactions costs captured by our model. As a result, our calibration of the Model 2 and 3 decline in costs (admittedly arbitrary), is intended to be conservative compared to the percentage decline in observable transactions costs associated with mortgage refinancing and home equity extraction, many of which fell more than 90% during or preceding the housing boom.

Finally, we calibrate foreign ownership of U.S. debt, B_t^F, by targeting a value for foreign bond holdings relative to GDP. Specifically, when we add foreign capital to the economy in Model 3, we experiment with several constant values for $B_t^F \equiv B^F$ until the model solution implies a value equal to 18% of average total output, $\overline{Y}$, an amount that is approximately equal to the rise in foreign ownership of U.S. Treasuries and agency debt over the period 2000-2008. Figure 5 shows that, as of the middle of 2008, foreign holdings of long-term Treasuries alone represent 15% of GDP. Higher values are obtained if one includes foreign holdings of U.S. agency debt and/or short-term Treasuries. Depending on how many of these categories are included, the fraction of foreign holdings in 2008 ranges from 15-30%.

3.6.2 Model Returns

Housing Return Abstracting from transactions costs and borrowing constraints, the first-order condition for optimal housing choice is

$$\frac{\partial U}{\partial C_{a,t}^i} = \frac{1}{p_t^H}\beta E_t\left[\frac{\partial U}{\partial C_{a+1,t+1}^i}\left(\frac{\frac{\partial U}{\partial H_{a+1,t+1}^i}}{\frac{\partial U}{\partial C_{a+1,t+1}^i}} + p_{t+1}^H\left(1-\delta_H\right)\right)\right], \tag{23}$$

implying that each individual's housing return is given by $\frac{\partial U/\partial H_{a+1,t+1}^i}{\partial U_{t+1}/\partial C_{a+1,t+1}^i} + p_{t+1}^H\left(1-\delta_H\right)$ where $\frac{\partial U/\partial H_{a+1,t+1}^i}{\partial U_{t+1}/\partial C_{a+1,t+1}^i}$ is the implicit rental price for housing services, referred to hereafter as "rent." For the national housing return, we define national rent, $\mathcal{R}_{t+1}$, as the average of $\frac{\partial U/\partial H_{a+1,t+1}^i}{\partial U_{t+1}/\partial C_{a+1,t+1}^i}$ across individuals. Given this definition of national rent, we define the corresponding national housing return as

$$R_{H,t+1} \equiv \frac{p_{t+1}^H\left(1-\delta_H\right) + \mathcal{R}_{t+1}}{p_t^H}, \tag{24}$$

$$\mathcal{R}_{t+1} \equiv \int_{\mathcal{S}} \frac{\partial U/\partial H_{a+1,t+1}^i}{\partial U_{t+1}/\partial C_{a+1,t+1}^i} d\mu. \tag{25}$$

In the model, p_t^H is the price of a unit of housing stock, which holds fixed the composition of housing (quality, square footage, etc.) over time.

We compare our model results with three different measures of single-family residential price-rent ratios and associated housing returns. These are (i) a measure based on housing wealth for the household sector from the Flow of Funds, hereafter *FoF*, (ii) a measure based on the Freddie Mac Conventional Mortgage House Price index, hereafter *Freddie Mac*, (iii) a measure based on the Case-Shiller national house price index, hereafter *CS*. The FoF data are combined with a measure of housing services from the national income and product accounts (NIPA) to measure rent, or housing services, and compute a national price-rent ratio and housing return. The Freddie Mac and CS price indexes are combined with the bureau of labor statistics (BLS) rental index for shelter to do the same. The Appendix details our construction of these variables.

It is important to bear in mind a caveat with these measures: the *level* of the average price-rent ratio in the data is, for practical purposes, unidentified. For Freddie Mac and CS, the price-rent ratio cannot be identified, since both price in the numerator and rent in the denominator are given by indexes. For FoF, we observe the stock of housing wealth and the flow of housing consumption from NIPA, where the latter is a measure of housing expenses for renters aggregated with an imputed rent measure for owner-occupiers. We normalize the first observations of the Freddie Mac and CS price-rent ratio to be the same as the FoF ratio for that year. However, it is notoriously difficult to impute rents for owner-occupiers from rental data for non-homeowners, a potentially serious difficulty for obtaining an aggregate rent measure since owners represent two-thirds of the population. Moreover, because owners are on average wealthier than non-homeowners, the NIPA imputed rent measure for owner-occupiers is likely to be biased down, implying that the level of the price-rent ratio is likely to be biased up and the average housing return biased down. For this reason, we do not attempt to match our model to the levels of the price-rent ratios and housing returns in the data, instead focusing on the changes in these ratios over time.

Equity Return The risky capital return $R_{K,t}$ in the model is not comparable to a realistic equity market return because it is unlevered. To make our results comparable to a stock market return, we adjust our risky capital return to account for leverage in a simple way. Specifically, we define the equity return, $R_{E,t}$,to be

$$R_{E,t} \equiv R_{f,t} + (1 + B/E)(R_{K,t} - R_{f,t}),$$

where B/E is the fixed debt-equity ratio and where $R_{K,t}$ is the portfolio return for risky capital given in (3).[13] Note that this calculation explicitly assumes that corporate debt in

[13]The cost of capital R_K is a portfolio weighted average of the return on debt R_f and the return on equity R_e: $R_K = aR_f + (1-a)R_e$, where $a \equiv \frac{B}{B+E}$.

the model is completely exogenous, and must be held in fixed proportion to the value of the firm. (There is no financing decision.) For the results reported below, we set $B/E = 2/3$ to match debt-equity ratios computed in Benninga and Protopapadakis (1990).

4 Results

This section presents some of the models main implications. Much of our analysis consists of a comparison of stochastic steady states across Models, 1, 2 and 3.[14] We also study a transition path for house prices and national price-rent ratios designed to mimic the state of the economy and housing market conditions over the period 2000-2009, as explained below. We being by presenting a set of benchmark business cycle and life-cycle wealth profile results.

4.1 Benchmark Results

4.1.1 Business Cycle Variables

We begin by presenting a set of benchmark results for Hodrick-Prescott (Hodrick and Prescott (1997)) detrended aggregate quantities. Panel A of Table 2 presents business cycle moments from U.S. annual data over the period 1953 to 2008. Panel B of Table 2 uses simulated data to summarize the implications for these same moments in our benchmark Model 1, (with "normal" collateral constraints and housing adjustment costs, but no foreign capital). Panel C presents the same results for Model 2, where collateral constraints and housing adjustment costs are low, but where there is still no foreign capital. We report statistics for total output, or GDP, defined $GDP \equiv Y_C + p^H Y_H + C_H$, for non-housing consumption (inclusive of expenditures on financial services), equal to $C + F$, for housing consumption, defined as price per unit of housing services times quantity of housing $C_H \equiv \mathcal{R}_t H_t$, for total consumption (housing and non-housing), denoted $C_T = C + F + C_H$, for non-housing investment, $I = I_{C,t} + I_{H,t}$, for residential investment $p^H Y_{H,t}$ and for total investment $I_T = I + p^H Y_H$. Because Model 1 and Model 2 generate similar results for these statistics; for brevity, we primarily discuss only the results for Model 1 (Panel A).

The standard deviation of total aggregate consumption divided by the standard deviation of GDP is 0.77 in Model 1 and 0.69 in Model 2, close to the 0.70 value found in the data.

[14] Note that with all shocks in the model set to zero, the portfolio choice problem is indeterminant since all assets earn the risk-free return. Thus, there is no deterministic steady state in this model. We define stochastic steady state as the average equilibrium allocation over a large number of simulated sample paths.

Also, the level of GDP volatility in the model is close to that in the data. Thus the model produces a plausible amount of aggregate consumption volatility. Broken down by type of consumption, both the model and the data imply that housing and non-housing consumption have about the same volatility.[15] Total investment is more volatile than output, both in the model and in the data, but the model produces too little relative volatility: the ratio of the standard deviation of investment to that of output is 1.7 in Model 1 but is 2.9 in the data.[16] The model does a good job of matching the relative volatility of residential investment to output: in the data the ratio of these volatilities is 4.6, while it is 5.4 in Model 1 and 5.1 in Model 2. Finally, both in the model and the data, residential investment is less correlated with output than is consumption and total investment.

Table 3 shows the model's implications for the cyclical properties of national house prices. The housing price indexes in the data are all procyclical, but not as strongly so as in the model. This may be partly attributable to the fact that the national house price indexes in the data are measured with error, whereas in the model they are not. As in the data, the model implies that both the level of house prices and price-rent ratios are procyclical, regardless of the calibration (Model 1, 2, or 3). Price-rent ratios are less procyclical than the level of prices because rents, in the denominator, are also procyclical. The correlation between GDP and the national price-rent ratio ranges from 0.17 to 0.62 across the three models, whereas, in the data, these correlations vary substantially by data source and sample, ranging from 0.29 to 0.10.

4.1.2 Life Cycle Profiles

Turning to individual-level implications, Figure 6 presents the age and income distribution of wealth, both in the model and in the historical data as given by the Survey of Consumer Finance (SCF). The figure shows total non-housing wealth, by age, divided by average wealth across all households, for three income groups (low, medium and high earners).

In both the model and the data, total non-housing wealth is hump-shaped over the life-cycle, and is close to zero early in life when households borrow to finance home purchases.

[15] With Cobb-Douglas utility, $\varepsilon = 1$, housing and non-housing consumption are proportional. The standard deviations of housing and non-housing consumption are identical in the table because we report moments for Hodrick-Prescott (Hodrick and Prescott (1997)) detrended data.

[16] Volatility of investment could be increased by adding stochastic depreciation in capital as in Storesletten, Telmer, and Yaron (2007) and Gomes and Michaelides (2008), or by adding investment-specific technology shocks. We abstract from these additional features in order to maintain a manageable level of complexity in the model.

As agents age, wealth slowly accumulates. In the data, it peaks between 60 and 70 years old (depending on the income level). In the model, the peak for all three income groups is about 65 years. After retirement, wealth is drawn down until death. Households in the model continue to hold some net worth in the final years of life to insure against the possibility of living long into old age. A similar observation holds in the data. For low and medium earners, the model gets the average amount of wealth about right, but it under-predicts the wealth of high earners.

The right-hand panels in Figure 6 plot the age distribution of housing wealth alone. Up to age 65, the model produces about the right level of housing wealth for each income group, as compared to the data. In the data, however, housing wealth peaks around age 60 for high earners and around age 67 for low and medium earners, and then declines. The model misses this hump-shape: housing wealth remains high until death. In the absence of a rental market, owning a home is the only way to generate housing consumption. For this reason, agents in the model continue to maintain a high level of housing wealth later in life even as they drawn down financial wealth.

What is the effect of a financial market liberalization and foreign capital infusion on the optimal portfolio decisions of individuals? Table 4 exhibits the age and income distribution of housing wealth relative to total net worth, both over time in the SCF data and in Models, 1, 2 and 3. The benchmark model captures an empirical stylized fact emphasized by Fernández-Villaverde and Krueger (2005), namely that young households hold most of their wealth in consumer durables (primarily housing) and hold very little in financial assets. Indeed, our calibrations imply that young households (age 35 and under), hold slightly more of their wealth as durables than do households in the data.[17]

The model also predicts that a financial market liberalization plus an influx of foreign capital leads households of all ages and income groups to shift the composition of their wealth towards housing (Model 1 to Model 3). The combination of lower interest rates, lower collateral constraints, and lower housing transactions costs makes possible greater housing investment by the young, whose incomes are growing and who rely on borrowing to expand their housing consumption. But the decline in housing transactions costs also has important effects on the asset allocation of net savers (primarily older, higher income individuals), consistent with the findings of Stokey (2009) who shows that such costs can have large

[17] This is likely attributable to the fact that young households in the model borrow more than young households in most waves of the SCF data, so that housing wealth exceeds net worth by an amount that is larger in the model than in the data.

effects on portfolio decisions. Here, a decline in housing transactions costs makes housing relatively less risky as compared to equity, which leads even unconstrained individuals to shift the composition of their wealth towards housing. Because of the simultaneous relaxation in credit constraints, the increase in housing is still largest for the young and for low income earners, where the housing wealth-total wealth ratio rises by 19% and 12%, respectively, between Model 1 and Model 3. But, the housing wealth-total wealth ratio also rises by 13% for households above age 35 and by 14% for high income individuals. Table 4 shows that these changes are in line with those in individual-level data from 2001 to 2007.

4.2 Asset Pricing

4.2.1 Return Moments

Table 5 presents asset pricing implications of the model, for the calibrations represented by Models 1, 2 and 3. The statistics reported are averages over 1000 periods. We first discuss the implications of the benchmark Model 1 with normal collateral constraints and transactions costs and no foreign capital. We see that this benchmark matches the historical mean return for the risk-free rate and only slightly overstates the volatility of the risk-free rate. In addition, the model produces a sizable equity return of 4% per annum and an annual Sharpe ratio of 0.31, compared to 0.34 in the data. Two factors related to the cyclicality of the cross-sectional distribution of consumption contribute to the model's high average Sharpe ratio. First, idiosyncratic income risk is countercyclical. Second, house prices and therefore collateral values are procyclical, making collateral constraints tighter in recessions than in booms. Risk-sharing/insurance opportunities are reduced at the very time when households need them most (in recessions), resulting in a high risk-premium and Sharpe ratio.

Turning to the implications for housing assets, the average housing return in the benchmark Model 1 is 13% per annum; the standard deviation of the housing return in the model is 6.2% per annum. The housing return Sharpe ratio for Model 1 is 1.52. Finally, the far right-hand column of Table 5 gives the mean price-rent ratio in Model 1 as 7.56. These values could be compared with the data, subject to the caveat discussed above that the levels of the price-rent ratio and housing return are poorly identified in the data with measured $P/\mathcal{R}$ likely to be biased up and average returns biased down. The average annual housing return from the FoF and Freddie Mac data, equal to 9.89% and 9.11%, respectively. The standard deviation of the housing returns range from 4.9% to 5.9% in FoF data, depending on sample, and is 4.32% according to the Freddie Mac measure. The FoF Sharpe ratio is between 1.2 and 1.5, while the Freddie Mac Sharpe ratio is 1.4. In the historical data, average price-rent ratios range from 14.7-15.2 for FoF, and are equal to 13.7 according to the Freddie Mac measure.

The Financial Market Liberalization and the Housing Boom How are asset prices affected by a financial market liberalization? We first answer this question by comparing stochastic steady states for Model 1 and Model 1. We see that both the equity premium and the equity Sharpe ratio fall in an economy that has undergone a financial market liberalization. Specifically, the equity premium falls from 4% to 3.6% from Model 1 to Model 2, while the Sharpe ratio falls from 0.31 to 0.23, a 26% decline. A financial market liberalization lowers the risk-premium on housing assets even more. The housing risk premium is cut by 40 percent from Model 1 to Model 2, from 11.39% per annum to 6.86%, while the housing Sharpe ratio declines by 47% from 1.52 to 0.8. This decline in the riskiness of both housing and equity assets reflects the greater amount of risk-sharing possible after a financial market liberalization, discussed further below. The housing Sharpe ratio declines more because there is an additional factor pushing down the housing risk premium that is inoperative for the equity market: a financial market liberalization is accompanied by a decline in transactions costs for housing but not for equity (or the risk-free asset).

The national price-rent ratio $p^H/\mathcal{R}$ is about 23% higher in Model 2 than it is in the benchmark Model 1. Recalling that price-rent ratios are procyclical (Table 3), these results imply that a financial market liberalization adds fuel to the fire in the housing market during an economic expansion, driving up price-rent ratios more than what would occur as the result of the boom alone. (Below we study a transition that includes economic shocks.) But a financial market liberalization also leads to a sharp increase in equilibrium interest rates, which by itself decreases $p^H/\mathcal{R}$. Indeed, the endogenous risk-free interest rate more than doubles in Model 2 to 3.56% per annum, from 1.63% in Model 1. This occurs because the relaxation of borrowing constraints and housing transactions costs reduces precautionary savings, as households endogenously respond to the improved risk-sharing/insurance opportunities afforded by financial market liberalization. Note also that there are no differences in average annual rental growth rates across Models 1, and 2 and Model 3.[18] It follows that the

[18] Because the statistics for each model are computed from averages across 1000 periods, they give the long-run annualized values of rental growth. This is the same across all three models because it is pinned down by the steady state growth of technology, which is the same in each model, assumed to be two percent.

increase in price-rent ratios following a financial market liberalization is entirely attributable to the decline in the risk-premium, which more than offsets the rise in equilibrium interest rates.

The Role of Foreign Capital in the Housing Boom In Model 3 we add an infusion of foreign capital calibrated to match the increase in foreign ownership of U.S. Treasuries and U.S. agency debt over the period 2000-2007. Table 5 shows that such an increase has a large impact on equilibrium interest rates: the risk-free rate falls by 100% from Model 2 (financial market liberalization with no foreign capital influx) to Model 3, where foreign demand for the safe asset is added. The magnitude of this decline is close to the roughly 75% reduction in real interest rates observed in U.S. data over the period 2000-2007.[19] But although the foreign capital influx plays a central role in reducing interest rates (without which the looser collateral requirements and lower housing transactions costs would generate an increase in equilibrium interest rates), its role in increasing price-rent ratios is modest. The last column of Table 5 shows that the average price-rent ratio is 31 percent higher in the steady state of Model 3 than in the benchmark Model 1. (As a comparison, this value represents more than all of the increase in two measures of national house price-rent ratios over the 2000-2007 period–FoF and Freddie Mac, which increased 31%–and 84 percent of the increase in the Case-Shiller index, which rose 43%.) But the increase from Model 2 to Model 3 represents only 6% of the total increase from Model 1 to Model 3. Why does a 100% decline in interest rates lead to such a small increase in the price-rent ratio?

The answer is found in the endogenous response of the housing risk-premium to an increase in foreign demand for the safe asset. Comparing Model 2 and Model 3, we see that the infusion of foreign capital makes both equity and housing assets more *risky*. Both the risk-premium and Sharpe ratio for equity and housing rise substantially from Model 2 to Model 3. This occurs because the exogenous supply of capital in the bond market that is included in Model 3 drives up leverage in the domestic economy, which increases the equity premium. In addition, the rise in foreign demand for the safe security means that more domestic saving must take place in risky assets, increasing the exposure of domestic households to systematic risk in the equity and housing markets. Domestic savers are, in effect, "crowded out" of the bond market by foreign governmental holders who are willing to hold the safe asset at any price. In equilibrium, the equity and housing risk-premia and

[19] Figure 3 shows the decline in nominal rates; subtracting off inflation to compute a real rate, we observe that the 10-year real Treasury bond rate fell from 3.6% to 0.93% from December 1999 to June 2006.

Sharpe ratios rise from Model 2 to Model 3, as domestic savers shift the composition of their financial wealth towards risky securities.[20] This generates an increase in volatility of the SDF, $\frac{\beta\Lambda_{t+1}}{\Lambda_t}$, driven, as discussed below, by a decrease in risk-sharing.

These findings illustrate the importance of general equilibrium considerations for understanding the role low interest rates play in a housing boom. In partial equilibrium models of the housing market (e.g., Titman (1982)), or in small open-economy models without aggregate risk (e.g., Kiyotaki, Michaelides, and Nikolov (2008)), the risk-premium is exogenously held fixed. By contrast, the results above show that, in general equilibrium, a foreign capital infusion causes the endogenous housing risk-premium to rise at the same time that interest rates fall. These two factors have offsetting effects on the price-rent ratio, with the increase in housing risk-premium strongly mitigating (though not fully offsetting) the rise in resulting from lower interest rates. This offsetting effect is ignored by partial equilibrium analyses where the risk-premium is held fixed while the interest rate is exogenously decreased, underscoring the importance of general equilibrium considerations when investigating the affect on house prices of changes in interest rates. When general equilibrium effects are taken into account, large declines in interest rates, often presumed to have played a predominant role the housing boom of 2000-2006, are found instead to play a modest role.

4.2.2 Transition Dynamics: Housing Boom to Bust

Above we studied the influence of lower collateral constraints, lower transactions costs, and lower interest rates by comparing stochastic steady states. The differences between models show long-run changes only and do not account for business cycle fluctuations. In this section we study a simple transition path for house prices and price-rent ratios, in response to a series of shocks designed to mimic both the state of the economy and housing market conditions over the period 2000-2009. Ideally, we would study such a path after solving a larger model that specified a probability law over parameters corresponding to the different models (1 through 3) defined above. Unfortunately, solving such a specification would be computationally infeasible. We therefore pursue a simpler strategy: We assume that, at time 0 (taken to be the year 2000), the economy begins in the stochastic steady state of Model

[20] The equity Sharpe ratio, though lower in Model 2 than in Model 1, rises substantially from Model 2 to Model 3, so much so that their values in Model 3 now exceed those in Model 1. As noted, the housing risk premium and housing Sharpe ratio also rise with the infusion of foreign capital (compare Model 3 to Model 2). Unlike the case for equity, however, the rise in risk premia from Model 2 to Model 3 is not enough to fully offset the decline in risk premia from Model 1 to Model 2.

1. In 2001, the economy undergoes an unanticipated shift to Model 3 (financial market liberalization and foreign holdings of U.S. bonds equal to 18% of GDP), at which time the policy functions and beliefs of Model 3 are applied.[21] The adjustment to the new stochastic steady state of model 3 is then traced out over the seven year period from 2001 to 2006 as the state variables evolve. Starting in 2007 and continuing through 2009, the economy is presumed to undergo a reversal of the financial market liberalization, and unexpectedly shifts to a state in which the parameters of Model 1 apply but foreign capital remains equal to 18% of GDP, as in Model 3. This hybrid of Models 1 and 3 is referred to as *Model 4*.

In addition, we feed in a specific sequence of aggregate shocks designed to mimic the business cycle over this period. The aggregate technology shock processes Z_C and Z_H follow Markov chains, with two possible values for each shock, "low" and "high" (see the Appendix). Denote these possibilities with the subscripts "l" and "h":

$$Z_C = \{Z_{Cl}, Z_{Ch}\}, \; Z_H = \{Z_{Hl}, Z_{Hh}\}.$$

As the general economy began to decline in 2000, construction relative to GDP in U.S. data continued to expand, and did so in every quarter until the end of 2005. Thus, the recession of 2001 was a non-housing recession. Starting in 2006, construction relative to GDP fell and has done so in every quarter through the most recent data at the time of this writing (2009:Q2). Thus, in contrast to the 2001 recession, housing led the recession of 2007-2009. To capture these cyclical dynamics, we feed in the following sequence of shocks for the period 2000-2009: $\{Z_{Cl}, Z_{Hh}\}_{t=2000}$, $\{Z_{Cl}, Z_{Hh}\}_{t=2001}$, $\{Z_{Ch}, Z_{Hh}\}_{t=2002}$, $\{Z_{Ch}, Z_{Hh}\}_{t=2003}$, $\{Z_{Ch}, Z_{Hh}\}_{t=2004}$, $\{Z_{Ch}, Z_{Hh}\}_{t=2005}$, $\{Z_{Ch}, Z_{Hl}\}_{t=2006}$, $\{Z_{Cl}, Z_{Hl}\}_{t=2007}$, $\{Z_{Cl}, Z_{Hl}\}_{t=2008}$, $\{Z_{Cl}, Z_{Hl}\}_{t=2009}$.

Figure 7 shows that the price-rent ratio, $p_t^H/\mathcal{R}_t$, (shown on the right scale) rises by 41% over the period 2000-2006, boosted by economic growth, the financial market liberalization, and lower interest rates. House prices themselves (shown on the left scale) rise 18%, both initially in 2002 as the broader economy begins expanding, and again in 2006. The increase in 2006 occurs because there is a negative shock to the housing sector that leads the recession of 2007-2009 and drives construction down. Since the rest of the economy is still booming in 2006, and since foreign demand for the safe asset is still holding interest rates down, the expected relative scarcity of housing causes a rise in house prices, p_t^H, in 2006. The increase in $p_t^H/\mathcal{R}_t$ from 2000-2006 is larger than the increase in p_t^H because, in the model, rents fall modestly over this period as the housing stock expands in response to positive economic

[21] Along the transition path, foreign holdings of bonds are increased linearly from 0% to 18% of GDP from 2000 to 2009.

shocks. The model generates a decline of greater than 16% in national house price-rent ratios in the two year period 2007 to 2009, driven by the economic contraction and by a presumed reversal of the financial market liberalization. From 2007 to 2009, the broader economic contraction reduces house prices by more than 12%.

4.2.3 Cyclical Dynamics of Housing: What Do Changes in House Price-Rent Ratios Forecast?

How do increases in price-rent ratios affect expectations of future rental growth rates and future home price appreciation? The left panels of Tables 6 and 7 show the predictability results for housing returns. Both excess and raw housing returns are forecastable over long-horizons. In particular, high price-rent ratios forecast low future housing returns, consistent with empirical evidence in the bottom left panels of Table 6 and Table 7 (see also Campbell, Davis, Gallin, and Martin (2010)). High price-rent ratios in an expansion also forecast lower future *excess* returns to housing assets, or risk-premia (Table 7). Risk-premia fall as the economy grows, for two reasons. First, economic growth reduces idiosyncratic income risk via (4). Second, as price-rent ratios rise with the economy so do collateral values, which expands risk-sharing and insurance opportunities and lowers risk-premia.

High price-rent ratios forecast *lower* future rental growth. It is often suggested that increases in price-rent ratios reflect an expected increase in rental growth. For example, in a partial equilibrium setting where discount rates are constant, higher house prices relative to fundamentals can only be generated by higher implicit rental growth rates in the future (Sinai and Souleles (2005), Campbell and Cocco (2007)).[22] The partial equilibrium setting, however, ignores the endogenous response of both discount rates and residential investment to economic growth. In general equilibrium, positive economic shocks can simultaneously drive discount rates down and residential investment up. As the housing supply expands, the cost of future housing services (rent) is forecast to be lower. It follows that high price-rent ratios in expansions must entirely reflect expectations of future home price depreciation (lower future returns), in part driven by lower risk-premia as collateral values rise with the economy. Although future rental growth is expected to be lower, price-rent ratios are still high because the decline in future housing returns more than offsets the expected fall in future rental growth.[23]

[22] See also the discussion in Campbell, Davis, Gallin, and Martin (2010) using the Gordon growth model as a motivation.

[23] Predictable variation in housing returns must therefore account for more than 100 percent of the vari-

For completeness, Tables 6 and 7 report predictability results for equity returns. In model generated data, both the raw equity return and the excess return are forecastable over long horizons, consistent with evidence from U.S. stock market returns.[24] High price-dividend ratios forecast low future equity returns (Table 6, right column) and low excess returns (Table 7) over horizons ranging from 1 to 30 years. Compared to the data, the model produces about the right amount of forecastability in excess equity returns (Table 7), but produces too much forecastability of dividend growth. This is not surprising since, unlike an endowment/exchange economy where dividends are set exogenously, in the model here both profits and the value of the firm respond endogenously to aggregate shocks.[25]

4.3 Macroeconomic Effects of Financial Market Liberalization

A growing body of academic work has argued that house price increases and financial liberalization are likely to stimulate a boom in consumption, and therefore have a stimulative affect on the economy as a whole (for example, Muellbauer and Murphy (1990), Mishkin (2007), and Muellbauer (2007)). Others have studied the effect of house price changes on consumption in household-level data and found a positive correlation (e.g., Campbell and Cocco (2007)). These conclusions are drawn from partial equilibrium life-cycle models.

Causal relationships between housing wealth and consumption are difficult to assess empirically because housing wealth is not an exogenous variable to which consumption responds, though it is often treated as such in empirical analysis. The model environment studied here offers an advantage in this regard because we can control for this endogeneity explicitly by studying how consumption is influenced by factors exogenous to our model, such as changes in collateralized borrowing constraints and housing transactions costs. These experiments give us some idea of the causality running from wealth to consumption and not the other way around. Here we focus on changes in housing wealth that arise from a financial market

ability in price-rent ratios.

[24] A large body of research in asset pricing finds evidence that stock returns are predictable over long horizons. See, for example, the summary evidence in Cochrane (2005), Chapter 20, and Lettau and Ludvigson (2009).

[25] For this same reason, the model also produces too much predictability in raw returns (Table 6). This happens because, although the model generates about the right amount of predictability in excess returns, it generates too much predictability in interest rates. Positive economic shocks increase consumption but not as much as income, thus saving and investment also rise. This pushes down expected rates of return to saving, implying that procyclical increases in price-dividend ratios forecast lower future interest rates, as well as lower future excess returns.

liberalization.

Figure 8 presents three panels that illustrate how a financial market liberalization affects macroeconomic variables by investigating a transition from Model 1 to Model 2. The transition is modeled in the same way as described above, except that we look at an arbitrary 50 year transition and do not feed in a specific sequence of shocks. Thus the transition paths plotted in Figure 8 are the average over 40 sample paths.

As Figure 8 shows, a financial market liberalization leads to a short-run boom in aggregate consumption, consistent with the implications of partial equilibrium life-cycle models. The general equilibrium framework studied here, however, does not imply that a financial market liberalization is stimulative for the economy as a whole. This is because the decline in collateralized borrowing constraints and housing transactions costs drives the endogenous interest rate up (Table 5), which chokes off investment. As a consequence, the immediate impact on investment is negative and on GDP is approximately zero. Moreover, in the long-run, a financial market liberalization leads to lower consumption as capital accumulation declines in the wake of lower aggregate saving rates.

The middle panel of Figure 8 shows that the youngest households increase their consumption the most, immediately upon the onset of a financial market liberalization. By contrast, retirees increase consumption very little. At first glance, these results may appear to differ from the findings of Campbell and Cocco (2007) who report that changes in house prices have their smallest impact on young households in UK household-level data. As these authors emphasize, however, many young households are renters, in contrast to older households. When Campbell and Cocco (2007) study simulated data from a life-cycle model and control for the selection bias attributable to the endogeneity of homeowner status, the model predicts that house price changes have a larger effect on the consumption of young homeowners than on old homeowners. Young households are relatively more constrained, and looser collateral constraints and lower housing transactions costs have the greatest influence on their spending.

The third panel of Figure 8 shows the differential consumption response of net savers and net borrowers to a financial market liberalization. Immediately following the onset of the financial market liberalization, net borrowers and net lenders raise their consumption by about the same percentage amount. All households raise their consumption initially as part of an endogenous response to improved risk-sharing opportunities, which leads to less precautionary saving. Unlike partial equilibrium life-cycle models, however, as the transition proceeds the stimulative effect of the financial liberalization is entirely attributable to the higher consumption of savers. Savers benefit from the rise in endogenous interest rates throughout the transition, while borrowers suffer for the same reason. Twenty years out, there is a switch: the consumption of borrowers is about the same as it was in Model 1, while the consumption of lenders is lower than in Model 1. This is because wealth is lower in Model 2 than in Model 1, which reduces the total asset cash-flow of savers more than borrowers.

4.4 The Role of Housing Finance in Risk Sharing and Inequality

Table 4 showed that a financial market liberalization lowers risk premia in both housing and equity assets. Let C_T denote total (housing plus non-housing) consumption. Table 8 presents several measures of risk-sharing for Models 1, 2 and 3: the cross-sectional standard deviation in the individual consumption share in aggregate consumption, the cross-sectional standard deviation of the intertemporal marginal rates of substitution in consumption $\frac{\beta \partial U / \partial C^i_{a+1,t+1}}{\partial U / \partial C^i_{a,t}}$, and the Gini and variance of log consumption. Note that in a model of perfect risk-sharing, the cross-sectional variance of the individual IMRS would be zero. Changes in this statistic therefore capture changes in the degree of risk-sharing. Table 8 shows that the decline in risk premia from Model 1 to Model 2 coincides increase in risk-sharing and a decline in consumption inequality. Moreover, the age dispersion in the consumption-GDP ratio also declines (bottom panel). Risk-sharing improves both because a financial liberalization increases access to credit, and because lower transactions costs reduce the expense of acquiring additional collateral, which increases borrowing capacity. Both factors allow heterogeneous households to insure more of their idiosyncratic risks. Consumption inequality falls from Model 1 to Model 2.

By contrast, these same measures of risk-sharing and consumption inequality rise from Model 2 to Model 3. The rise in foreign capital, in effect, makes existing financial markets more incomplete because the foreign governmental holders' perfectly inelastic demand for the risk-free asset reduces the availability of this asset to domestic savers for insurance. Thus, the increase in risk-sharing and fall in consumption inequality resulting from a financial market liberalization is offset by a fall in risk-sharing and a rise in inequality resulting from foreign demand for the risk-free asset. In the calibration here, the former more than offsets the latter so that the net change in consumption inequality is small but negative moving from Model 1 (benchmark) to Model 3 (financial liberalization plus foreign capital).

What about wealth inequality? Unlike consumption inequality, a financial market liberalization and foreign demand for the risk-free asset have *reinforcing* effects on financial wealth inequality. Figure 9 shows the Gini Index for inequality in total net worth, decomposed into financial wealth and housing wealth, for Models 1, 2, and 3 (right scale), as well as the Gini indexes based on the SCF data for the years 2001, 2004 and 2007 (left scale). The Figure compares the change in the wealth Gini index from 2001 to 2007 with the change in the model Gini index between Models 1, 2 and 3.

The present model does not explain the degree of wealth inequality in the data.[26] (The level of the Gini index in the model is lower than that in the data.) But the model captures some recent trends in wealth inequality. In the data, the Gini index for financial wealth rises by almost 20 percent between 2001 and 2007. In the model, the Gini for financial wealth increases by about 10 percent as a result of financial market liberalization (Model 1 to Model 2), and by another 5.4 percent as a result of foreign governmental demand for the safe asset (Model 2 to Model 3). In addition, both in the model and in the data, housing wealth inequality increases far less than financial wealth inequality: the Gini index for housing wealth in the SCF data is flat from 2001 to 2007, while in the model it falls slightly from Model 1 to Model 3.

Why do a financial market liberalization and a foreign capital infusion have reinforcing affects on financial wealth inequality but offsetting affects on consumption inequality? A financial market liberalization relaxes the constraints of households, both by making it easier to borrow against home equity and by making it less costly to transact. This improves risk-sharing and reduces consumption inequality and housing inequality. But financial wealth inequality rises because, as domestic borrowers (mostly young individuals) take advantage of the market liberalization to increase current consumption, their net worth position is driven more negative. The foreign capital influx further raises financial wealth inequality because domestic savers as a whole (older households primarily concerned about retirement) are now effectively in a leveraged position as a result of foreign demand for the safe asset. They therefore earn a higher rate of return on the risky asset and on their savings, as compared to Model 2, which drives their wealth more positive and further increases wealth inequality. Because a financial market liberalization and a foreign capital infusion have reinforcing effects on financial wealth inequality but offsetting effects on consumption inequality, the model

[26] It is understood that general equilibrium, incomplete markets models without preference heterogeneity cannot explain the extreme concentration of wealth in the upper tail of the distribution. Following Krusell and Smith (1997, 1998), the wealth distribution could be better approximated by introducing heterogeneity in the subjective time discount factor.

has the potential to help explain why wealth inequality has risen more than consumption inequality in recent decades (Heathcote, Perri, and Violante (2009)).[27]

5 Conclusion

In this paper we have studied the macroeconomic and individual-level consequences of fluctuations in housing wealth and housing finance. We have focused much of our investigation on studying the macroeconomic impact of systemic changes in housing finance that were a key feature of the period of rapid home price appreciation from 2000-2006. Aspects of the larger questions posed here have been studied elsewhere, often in partial equilibrium settings or in general equilibrium settings without production, and/or aggregate risk, and/or without embedding the portfolio choice aspects required to study risk premia. The framework studied here endogenizes the interaction among financial and housing wealth, output and investment, rates of return and risk premia in both housing and equity assets, and consumption and wealth inequality.

There has been much discussion, both in the popular press and among academic economists, of bubbles in explaining the recent housing boom. The model studied here has no role for a bubble, yet implies that national house price-rent ratios may fluctuate considerably in response to a financial market liberalization or an increase in foreign demand for the safe asset, as well as in response to movements in the aggregate economy. In a simulated transition for the period 2000-2009, the model captures all of the run-up observed in U.S. national house price rent ratios from 2000-2006 and predicts a sharp decline in housing markets starting in 2007. Price-rent ratios fluctuate because both risk-premia and interest rates respond endogenously to changes in housing finance and to the state of the economy. We found that the general equilibrium environment is particularly important for understanding some features of these results. For example, the model implies that procyclical increases in national house price-rent ratios must reflect lower future housing returns rather than higher future rents, a finding that is difficult to understand without taking into account the endogenous response of residential investment and discount rates to positive economic shocks.

A financial market liberalization increases house prices because it drives risk premia in

[27] Heathcote, Perri, and Violante (2009) study income and consumption inequality directly, and show that consumption inequality has risen far less than income inequality (see also Krueger and Perri (2006)). But their results for saving and income inequality suggest that wealth inequality has risen more than consumption inequality over time.

both the housing and equity market down and shifts the composition of wealth for all age and income groups towards housing. It also leads to a short-run boom in aggregate consumption, but is not necessarily stimulative for the economy as a whole because the higher equilibrium interest rates that accompany a financial market liberalization lead to a short-run bust in investment.

We also found that–in contrast to a financial market liberalization–an influx of foreign capital by governmental holders lowers interest rates but raises consumption and wealth inequality, as well as risk-premia in both housing and equity assets. As a result, although an influx of foreign capital into the domestic bond market plays a central role in reducing interest rates, it has a modest role in raising house prices. The foreign capital influx pushes interest rates down, but it simultaneously pushes risk-premia up, two effects that have offsetting influences on national price-rent ratios.

Appendix

This appendix describes how we calibrate the stochastic shock processes in the model, describes the historical data we use to measure house price-rent ratios and returns, and describes our numerical solution strategy.

Calibration of Shocks

The aggregate technology shock processes Z_C and Z_H are calibrated following a two-state Markov chain, with two possible values for each shock, $\{Z_C = Z_{Cl}, Z_C = Z_{Ch}\}, \{Z_H = Z_{Hl}, Z_H = Z_{Hh}\}$, implying four possible combinations:

$$
\begin{aligned}
Z_C &= Z_{Cl,} \qquad Z_H = Z_{Hl} \\
Z_C &= Z_{Ch,} \qquad Z_H = Z_{Hl} \\
Z_C &= Z_{Cl,} \qquad Z_H = Z_{Hh} \\
Z_C &= Z_{Ch,} \qquad Z_H = Z_{Hh.}
\end{aligned}
$$

Each shock is modeled as,

$$
\begin{aligned}
Z_{Cl} &= 1 - e_C, \qquad Z_{Ch} = 1 + e_C \\
Z_{Hl} &= 1 - e_H, \qquad Z_{Ch} = 1 + e_H,
\end{aligned}
$$

where e_C and e_H are calibrated to match the volatilities of GDP and residential investment in the data.

We assume that Z_C and Z_H are independent of one another. Let $\mathbf{P}^C$ be the transition matrix for Z_C and $\mathbf{P}^H$ be the transition matrix for Z_H. The full transition matrix equals

$$\mathbf{P} = \begin{bmatrix} p_{ll}^H \mathbf{P}^C & p_{lh}^H \mathbf{P}^C \\ p_{hl}^H \mathbf{P}^C & p_{hh}^H \mathbf{P}^C \end{bmatrix},$$

where

$$\mathbf{P}^H = \begin{bmatrix} p_{ll}^H & p_{lh}^H \\ p_{hl}^H & p_{hh}^H \end{bmatrix} = \begin{bmatrix} p_{ll}^H & 1 - p_{ll}^H \\ 1 - p_{hh}^H & p_{hh}^H \end{bmatrix},$$

and where we assume $\mathbf{P}^C$, defined analogously, equals $\mathbf{P}^H$. We calibrate values for the matrices as

$$\begin{aligned} \mathbf{P}^C &= \begin{bmatrix} .60 & .40 \\ .25 & .75 \end{bmatrix} \\ \mathbf{P}^H &= \begin{bmatrix} .60 & .40 \\ .25 & .75 \end{bmatrix} => \\ \mathbf{P} &= \begin{bmatrix} .36 & .24 & .24 & .16 \\ .15 & .45 & .10 & .30 \\ .15 & .10 & .45 & .30 \\ .0625 & .1875 & .1875 & .5625 \end{bmatrix}. \end{aligned}$$

With these parameter values, we roughly match the average length of expansions divided by the average length of recessions (equal to 2.2 in NBER data from over the period 1854-2001). We define a recession as the event with joint probability $p_{ll}^H p_{ll}^C = 0.36$, so that a recession persists on average for $1/(1 - .36) = 1.56$ years. If we define an expansion as the event given by the sum of joint probabilities $p_{hh}^H p_{hh}^C + p_{hl}^H p_{hh}^C = .75$, so that an expansion will persist on average for $1/(1 - .75) = 4$ years. Thus the average length of expansions relative to that of recessions is then $4/(1.46) = 2.56$ years.

Idiosyncratic income shocks follow the first order Markov process $\log\left(Z_{a,t}^i\right) = \log\left(Z_{a-1,t-1}^i\right) + \epsilon_{a,t}^i$, where $\epsilon_{a,t}^i$ takes on one of two values in each aggregate state:

$$\begin{aligned} \epsilon_{a,t}^i &= \begin{cases} \sigma_E & \text{with } \Pr = 0.5 \\ -\sigma_E & \text{with } \Pr = 0.5 \end{cases}, \qquad \text{if } Z_{C,t} \geq E\left(Z_{C,t}\right) \\ \epsilon_{a,t}^i &= \begin{cases} \sigma_R & \text{with } \Pr = 0.5 \\ -\sigma_R & \text{with } \Pr = 0.5 \end{cases}, \qquad \text{if } Z_{C,t} < E\left(Z_{C,t}\right) \\ \sigma_R &> \sigma_E. \end{aligned}$$

Housing Price and Return Data

Our first measure of house prices uses aggregate housing wealth for the household sector from the Flow of Funds (FoF) (which includes the part of private business wealth which is residential real estate wealth) and housing consumption from the National Income and Products Accounts. The price-rent ratio is the ratio of housing wealth in the fourth quarter of the year divided by housing consumption summed over the year. The return is constructed as housing wealth in the fourth quarter plus housing consumption over the year divided by housing wealth in the fourth quarter of the preceding year. We subtract CPI inflation to express the return in real terms and population growth in order to correct for the growth in housing quantities that is attributable solely to population growth. (Since the return is based on a price times quantity, it grows mechanically with the population. In the model, population growth is zero.) The advantage of this housing return series is that it is for residential real estate and for the entire population. The disadvantages are that it is not a per-share return (it has the growth in the housing stock in it, which we only partially control for by subtracting population growth), it is not an investable asset return, and it does not control for quality changes in the housing stock. There is also substantial measurement error in how the Flow of Funds imputes market prices to value the housing stock as well as in how the BEA imputes housing services consumption for owners. These errors, however, may be more likely to affect the level of the price-rent ratio more than the change in the ratio.

Our second series combines the Freddie Mac Conventional Mortgage House Price index for home purchases (Freddie Mac) and the rental price index for shelter from the Bureau of Labor Statistics (BLS). The price-rent ratio is the ratio of the price index in the last quarter of the year, divided by the rent index averaged over the quarters in the year. Since the level of the price-rent ratio is indeterminate (given by the ratio of two indexes), we normalize the level of the series by assuming that the 1970 Freddie Mac price-rent ratio is the same as that of the FoF price-rent ratio in 1970. The return is the price index plus the rent divided by the price index at the end of the previous year. We subtract CPI inflation to express the return in real terms. The FoF return has a correlation of 82% with the Freddie Mac return over 1973-2008. Since the Freddie Mac price index is a repeat-sales price index, it controls for quality changes in the housing stock (price changes are computed on the same house). It also is a per-share return (no quantities). Alternative repeat-sale price indices such as the Freddie Mac CMHPI which includes refinancing and purchases, or the OFHEO house price index, deliver similar results. The same is true if we use the BLS rental index for housing instead of shelter. (The rental index for housing includes utilities while the rental price index for shelter excludes them).

The third series is the ratio of the Case-Shiller national house price index to the Bureau of Labor Statistics's price index of shelter (CS). The Case-Shiller price index is also a repeat-sales price index, which receives a lot of attention in the literature. It is available from 1987 on a quarterly basis.

Numerical Solution Procedure

This section describes our numerical solution strategy, which is related to strategies used in Krusell and Smith (1998) and Storesletten, Telmer, and Yaron (2007). The strategy consists of solving the individual's problem taking as given her beliefs about the evolution of the aggregate state variables. With this solution in hand, the economy is simulated for many individuals and the simulation is used to compute the equilibrium evolution of the aggregate state variables, given the assumed beliefs. If the equilibrium evolution differs from the beliefs individuals had about that evolution, a new set of beliefs are assumed and the process is repeated. Individuals' expectations are rational once this process converges and individual beliefs coincide with the resulting equilibrium evolution.

The state of the economy is a pair, (Z_t, μ_t), where μ_t is a measure defined over

$$\mathcal{S} = (\mathcal{A} \times \mathcal{Z} \times \mathcal{W} \times \mathcal{H}),$$

where $\mathcal{A} = \{1, 2, ...A\}$ is the set of ages, where $\mathcal{Z}$ is the set of all possible idiosyncratic shocks, where $\mathcal{W}$ is the set of all possible beginning-of-period financial wealth realizations, and where $\mathcal{H}$ is the set of all possible beginning-of-period housing wealth realizations. That is, μ_t is a distribution of agents across ages, idiosyncratic shocks, financial, and housing wealth. Given a finite dimensional vector to approximate μ_t, and a vector of individual state variables

$$\mu_t^i = (Z_t^i, W_t^i, H_t^i),$$

the individual's problem is solved using dynamic programming.

An important step in the numerical strategy is approximating the joint distribution of individuals, μ_t, with a finite dimensional object. The resulting approximation, or "bounded rationality" equilibrium has been used elsewhere to solve overlapping generations models with heterogenous agents and aggregate risk, including Krusell and Smith (1998); Ríos-Rull and Sánchez-Marcos (2006); Storesletten, Telmer, and Yaron (2007); Gomes and Michaelides (2008); Favilukis (2008), among others. For our application, we approximate this space with a vector of aggregate state variables given by

$$\mu_t^{AG} = (Z_t, K_t, S_t, H_t, p_t^H, q_t),$$

where

$$K_t = K_{C,t} + K_{H,t}$$

and

$$S_t = \frac{K_{C,t}}{K_{C,t} + K_{H,t}}.$$

The state variables are the observable aggregate technology shocks, the first moment of the aggregate capital stock, the share of aggregate capital used in production of the consumption good, the aggregate stock of housing, and the relative house price and bond price, respectively. The bond and the house price are natural state variables because the joint distribution of all individuals only matters for the individual's problem in so far as it affects asset prices. Note that knowledge of K_t and S_t is tantamount to knowledge of $K_{C,t}$ and $K_{H,t}$ separately, and vice versa ($K_{C,t} = K_t S_t$; $K_{H,t} = K_t(1 - S_t)$).

Because of the large number of state variables and because the problem requires that prices in two asset markets (housing and bond) must be determined by clearing markets every period, the proposed problem is highly numerically intensive. To make the problem tractable, we obviate the need to solve the dynamic programming problem of firms numerically by instead solving analytically for a recursive solution to value function taking the form $V(K_t) = Q_t K_t$, where Q_t is a recursive function. We discuss this below.

In order to solve the individual's dynamic programming problem, the individual must know μ_{t+1}^{AG} and μ_{t+1}^i as a function of μ_t^{AG} and μ_t^i and aggregate shocks Z_{t+1}. Here we show that this can be achieved by specifying individuals' beliefs for the laws of motion of four quantities:

A1 K_{t+1},

A2 p_{t+1}^H,

A3 q_{t+1}, and

A4 $[\frac{\beta^k \Lambda_{t+k}}{\Lambda_t}(Q_{C,t+1} - Q_{H,t+1})]$, where $Q_{C,t+1} \equiv V_{C,t+1}/K_{C,t+1}$ and analogously for $Q_{H,t+1}$.

The beliefs are approximated by a linear function of the aggregate state variables as follows:

$$\varkappa_{t+1} = A^{(n)}(Z_t, Z_{t+1}) \times \widetilde{\varkappa}_t, \quad (26)$$

where $A^{(n)}(Z_t, Z_{t+1})$ is a 4×5 matrix that depends on the aggregate shocks Z_t, and $Z_{t+1}$and where

$$\begin{aligned} \varkappa_{t+1} &\equiv \left[K_{t+1}, p^H_{t+1}, q_{t+1}, [M_{t+1}(Q_{C,t+1} - Q_{H,t+1})]\right]', \\ \widetilde{\varkappa}_t &\equiv \left[K_t, p^H_t, q_t, S_t, H_t\right]'. \end{aligned}$$

We initialize the law of motion (26) with a guess for the matrix $A^{(n)}(Z_t, Z_{t+1})$, given by $A^{(0)}(Z_t, Z_{t+1})$. The initial guess is updated in an iterative procedure (described below) to insure that individuals' beliefs are consistent with the resulting equilibrium.

Given (26), individuals can form expectations of μ^{AG}_{t+1} and μ^i_{t+1} as a function of μ^{AG}_t and μ^i_t and aggregate shocks Z_{t+1}. To see this, we employ the following equilibrium relation (as shown below) linking the investment-capital ratios of the two production sectors:

$$\frac{I_{H,t}}{K_{H,t}} = \frac{I_{C,t}}{K_{C,t}} + \frac{1}{2\varphi} E_t\left[M_{t+1}(Q_{C,t+1} - Q_{H,t+1})\right]. \tag{27}$$

Moreover, note that $E_t\left[M_{t+1}(Q_{C,t+1} - Q_{H,t+1})\right]$ can be computed from (26) by integrating the 4th equation over the possible values of Z_{t+1} given $\widetilde{\varkappa}_t$ and Z_t.

Equation (27) is derived by noting that each firm solves a problem taking the form

$$V(K_t) = \max_{I_t, N_t} Z_t K_t^{\alpha} N_t^{1-\alpha} - w_t N_t - I_t - \varphi\left(\frac{I_t}{K_t} - \delta\right)^2 + E_t\left[M_{t+1} V(K_{t+1})\right],$$

where $M_{t+1} \equiv \frac{\beta \Lambda_{t+1}}{\Lambda_t}$. The first-order condition for optimal labor choice implies $N_t = \left(\frac{Z_t(1-\alpha)}{w_t}\right)^{1/\alpha} K_t$. Substituting this expression into $V(K_t)$, the optimization problem may be written

$$\begin{aligned} V(K_t) &= \max_{I_t} X_t K_t - I_t - \varphi\left(\frac{I_t}{K_t} - \delta\right)^2 K_t + E_t\left[M_{t+1} V(K_{t+1})\right] \quad (28) \\ s.t. \quad K_{t+1} &= (1-\delta) K_t + I_t \end{aligned}$$

where $X_t \equiv \alpha\left(\frac{Z_t}{w_t}(1-\alpha)\right)^{(1-\alpha)/\alpha} Z_t$ is a function of aggregate variables over which the firm has no control. We now guess and verify that $V(K_{t+1})$ takes the form

$$V(K_{t+1}) = Q_{t+1} K_{t+1}, \tag{29}$$

where Q_{t+1} depends on aggregate state variables but is not a function of the firm's capital stock K_{t+1} or investment I_t. Plugging (29) into (28) we obtain

$$V(K_t) = \max_{I_t} X_t K_t - I_t - \varphi\left(\frac{I_t}{K_t} - \delta\right)^2 K_t + E_t\left[M_{t+1} Q_{t+1}\right]\left[(1-\delta) K_t + I_t\right]. \tag{30}$$

The first-order conditions for the maximization (30) imply

$$\frac{I_t}{K_t} = \delta + \frac{E_t\left[M_{t+1}Q_{t+1}\right] - 1}{2\varphi}. \tag{31}$$

Substituting (31) into (30) we verify that $V\left(K_t\right)$ takes the form $Q_t K_t$:

$$\begin{aligned} V\left(K_t\right) &= Q_t K_t = X_t K_t - \left(\delta + \frac{E_t\left[M_{t+1}Q_{t+1}\right] - 1}{2\varphi}\right) K_t - \varphi\left(\frac{E_t\left[M_{t+1}Q_{t+1}\right] - 1}{2\varphi}\right)^2 K_t \\ &\quad + (1-\delta)\left(E_t\left[M_{t+1}Q_{t+1}\right]\right) K_t + E_t\left[M_{t+1}Q_{t+1}\right]\left(\delta + \frac{E_t\left[M_{t+1}Q_{t+1}\right] - 1}{2\varphi}\right) K_t. \end{aligned}$$

Rearranging terms, it can be shown that Q_t is a recursion:

$$Q_t = X_t + (1-\delta) + 2\varphi\left(\frac{E_t\left[M_{t+1}Q_{t+1}\right] - 1}{2\varphi}\right) + \varphi\left(\frac{E_t\left[M_{t+1}Q_{t+1}\right] - 1}{2\varphi}\right)^2. \tag{32}$$

Since Q_t is a function only of X_t and the expected discounted value of Q_{t+1}, it does not depend on the firm's own K_{t+1} or I_t. Hence we verify that $V_t\left(K_t\right) = Q_t K_t$. Although Q_t does not depend on the firm's individual K_{t+1} or I_t, in equilibrium it will be related to the firm's investment-capital ratio via:

$$Q_t = X_t + (1-\delta)\left[1 + 2\varphi\left(\frac{I_t}{K_t} - \delta\right)\right] + \varphi\left(\frac{I_t}{K_t}\right)^2 - 2\varphi\delta\left(\frac{I_t}{K_t}\right), \tag{33}$$

as can be verified by plugging (31) into (32). Note that (31) holds for each of the two representative firms, thus we obtain (27) above, where Q_t is now distinguished across firms using subscripts, i.e., $Q_{C,t}$ and $Q_{H,t}$.

With (33), it is straightforward to show how individuals can form expectations of μ_{t+1}^{AG} and μ_{t+1}^{i} as a function of μ_t^{AG} and μ_t^i and aggregate shocks Z_{t+1}. Given a grid of values for K_t and S_t individuals can solve for $K_{C,t}$ and $K_{H,t}$ from $K_{C,t} = K_t S_t$ and $K_{H,t} = K_t\left(1 - S_t\right)$. Combining this with beliefs about K_{t+1} from (26), individuals can solve for $I_t \equiv I_{C,t} + I_{H,t}$ from $K_{t+1} = (1-\delta) K_t + I_t$. Given I_t and beliefs about $\left[\frac{\beta^k \Lambda_{t+k}}{\Lambda_t}\left(Q_{C,t+1} - Q_{H,t+1}\right)\right]$ from (26), individuals can solve for $I_{C,t}$ and $I_{H,t}$ from (27). Given $I_{H,t}$ and the accumulation equation $K_{H,t+1} = (1-\delta) K_{H,t} + I_{H,t}$, individuals can solve for $K_{H,t+1}$. Given $I_{C,t}$ individuals can solve for $K_{C,t+1}$ using the accumulation equation $K_{C,t+1} = (1-\delta) K_{C,t} + I_{C,t}$. Using $K_{H,t+1}$ and $K_{C,t+1}$, individuals can solve for S_{t+1}. Given a grid of values for H_t, H_{t+1} can be computed from $H_{t+1} = \left(1 - \delta_H\right) H_t + Y_{H,t}$, where $Y_{H,t} = Z_{H,t} K_{H,t}^{\nu} N_{H,t}^{1-\nu}$ is obtained from knowledge of $Z_{H,t}$, $K_{H,t}$ (observable today) and by combining (17) and (19) to obtain the decomposition of N_t into $N_{C,t}$ and $N_{H,t}$. Equation (26) can be used directly to obtain beliefs about q_{t+1} and p_{t+1}^H.

To solve the dynamic programming problem individuals also need to know the equity values $V_{C,t}$ and $V_{H,t}$. But these come from knowledge of Q_t (using (33)) and $K_{j,t}$ via $V_{j,t} = Q_{j,t}K_{j,t}$ for $j = C, H$. Values for dividends in each sector are computed from

$$D_{j,t} = Y_{j,t} - I_{j,t} - w_t N_{j,t} - \phi_K \left(\frac{I_{j,t}}{K_{j,t}} \right) K_{j,t},$$

and from $w_t = (1-\alpha) Z_{j,t} K_{j,t}^{\alpha} N_{j,t}^{-\alpha} = (1-\nu) Z_{j,t} K_{j,t}^{\nu} N_{j,t}^{-\nu}$ and by again combining (17) and (19) to obtain the decomposition of N_t into $N_{C,t}$ and $N_{H,t}$. Finally, the evolution of the aggregate technology shocks Z_{t+1} is given by the first-order Markov chain described above; hence agents can compute the possible values of Z_{t+1} as a function of Z_t.

Values for $\mu_{t+1}^i = (Z_{t+1}^i, W_{t+1}^i, H_{t+1}^i)$ are given from all of the above in combination with the first order Markov process for idiosyncratic income $\log\left(Z_{a,t}^i\right) = \log\left(Z_{a-1,t-1}^i\right) + \epsilon_{a,t}^i$. Note that H_{t+1}^i is a choice variable, while $W_{t+1}^i = \theta_t^i(V_{C,t+1} + V_{H,t+1} + D_{C,t+1} + D_{H,t+1}) + B_{t+1}^i$ requires knowing $V_{j,t+1} = Q_{j,t+1}K_{t+1}$ and $D_{j,t+1}$, $j = C, H$ conditional on Z_{t+1}.These in turn depend on $I_{j,t+1}$, $j = C, H$ and may be computed in the manner described above by rolling forward one period both the equation for beliefs (26) and accumulation equations for $K_{C,t+1}$, and $K_{H,t+1}$.

The individual's problem, as approximated above, may be summarized as follows (where we drop age subscripts when no confusion arises):

$$V_{a,t}\left(\mu_t^{AG}, \mu_t^i\right) = \max_{H_{t+1}^i, \theta_{t+1}^i, B_{t+1}^i} U(C_t^i, H_t^i) + \beta \pi_i E_t[V_{a+1,t+1}\left(\mu_{t+1}^{AG}, \mu_{t+1}^i\right)] \quad s.t. \tag{34}$$

$$C_t^i + B_{t+1}^i q_t + \theta_{t+1}^i (V_{C,t} + V_{H,t}) + p_t^h H_{t+1}^i + F_t^i = W_t^i + Y_t^i + p_t^h (1-\delta_H) H_t^i$$

$$W_t^i = \theta_t^i (V_{C,t} + V_{H,t} + D_{C,t} + D_{H,t}) + B_t^i$$

$$W_t^i + Y_t^i + p_t^h (1-\delta_H) H_t^i - C_t^i - F_t^i \geq \varpi p_t^h H_{t+1}^i$$

$$\mu_{t+1}^{AG} = \Gamma^{(n)}(\mu_t^{AG}, Z_{t+1}),$$

where Y_t^i is the after-tax income (wage or retirement) of individual i. The above problem is solved subject to (5), (6), (7), and (8) if the individual of working age, and subject to the analogous versions of (5), (6), (7), and (8) (using pension income in place of wage income), if the individual is retired. $\Gamma^{(n)}$ is the system of forecasting equations that is obtained by stacking all the beliefs from (26) and accumulation equations into a single system. This dynamic programming problem is quite complex numerically because of a large number of state variables but is otherwise straightforward. Its implementation is described below.

Next we simulate the economy for a large number of individuals using the policy functions from the dynamic programming problem. The continuum of individuals born each period is approximated by a number large enough to insure that the mean and volatility of aggregate variables is not affected by idiosyncratic shocks. We check this by simulating the model for successively larger numbers of individuals in each age cohort and checking whether the mean and volatility of aggregate variables changes. We have solved the model for several different numbers of agents. For numbers ranging from a total of 2,400 to 40,000 agents in the population we found no significant differences in the aggregate allocations.

An additional numerical complication is that two markets (the housing and bond market) must clear each period. This makes p_t^H and q_t convenient state variables: the individual's policy functions are a response to a menu of prices p_t^H and q_t, Given values for $Y_{H,t}$, $H_{a+1,t+1}^i$, $H_{a,t}^i$, $B_{a,t}^i$ and B_t^F form the simulation, and given the menu of prices p_t^H and q_t and the beliefs (26), we then choose values for p_{t+1}^H and q_{t+1} that clear markets in $t+1$. The initial allocations of wealth and housing are set arbitrarily to insure that prices in the initial period of the simulation, p_1^H and q_1, clear markets. However, these values are not used since each simulation includes an initial burn-in period of 150 years that we discard for the final results.

Using data from the simulation, we calculate (A1)-(A4) as linear functions of $\widetilde{\varkappa}_t$ and an initial guess $A^{(0)}$. In particular, for every Z_t and Z_{t+1} combination we regress (A1)-(A4) on K_t, S_t, H_t, p_t^H, and q_t. This is used to calculate a new $A^{(n)} = A^{(1)}$ which is used to re-solve for the entire equilibrium. We continue repeating this procedure, updating the sequence $\{A^{(n)}\}$, $n = 0, 1, 2, ...$ until (1) the coefficients in $A^{(n)}$ between successive iterations is arbitrarily small, (2) the regressions have high R^2 statistics, and (3) the equilibrium is invariant to the inclusion of additional state variables such as additional lags and/or higher order moments of the cross-sectional wealth and housing distribution.

The R^2 statistics for the four equations (A1)-(A4) are (.999, .999, .989, .998), respectively. The lowest R^2 is for the bond price equation. We found that successively increasing the number of agents (beyond 2400) successively increases the R^2 in the bond price equation, without affecting the equilibrium allocations or prices. However, we could not readily increase the number of agents beyond 40,000 because attempts to do so exceeded the available memory on a workstation computer. Our interpretation of this finding is that the equilibrium is unlikely to be affected by an approximation using more agents, even though doing so could result in an improvement in the R^2 of the bond equation. For this reason, and because of the already high computational burden required to solve the model, we stopped at the slightly lower level of accuracy for the bond forecasting regression as compared to the other forecasting regressions.

Numerical Solution to Individual's Dynamic Programming Problem

We now describe how the individual's dynamic programming problem is solved.

First we choose grids for the continuous variables in the state space. That is we pick a set of values for W^i, H^i, K, H, S, p^H, and q. Because of the large number of state variables, it is necessary to limit the number of grid points for some of the state variables given memory/storage limitations. We found that having a larger number of grid points for the individual state variables was far more important than for the aggregate state variables, in terms of the effect it had on the resulting allocations. Thus we use a small number of grid points for the aggregate state variables but compensate by judiciously choosing the grid point locations after an extensive trial and error experimentation designed to use only those points that lie in the immediate region where the state variables ultimately reside in the computed equilibria. As such, a larger number of grid points for the aggregate state variables was found to produce very similar results to those reported using only a small number of points. We pick 25 points for W^i, 12 points for H^i, three points for K, H, S, p^H, and four points for q. The grid for W^i starts at the borrowing constraint and ends far above the maximum wealth reached in simulation. This grid is very dense around typical values of financial wealth and is sparser for high values. The housing grid is constructed in the same way.

Given the grids for the state variables, we solve the individual's problem by value function iteration, starting for the oldest (age A) individual and solving backwards. The oldest individual's value function for the period after death is zero for all levels of wealth and housing (alternately it could correspond to an exogenously specified bequest motive). Hence the value function in the final period of life is given by $V_A = \max_{H^i_{t+1}, \theta^i_{t+1}, B^i_{t+1}} U(C^i_A, H^i_A)$ subject to the constraints above for (34). Given V_A (calculated for every point on the state space), we then use this function to solve the problem for a younger individual (aged $A - 1$). We continue iterating backwards until we have solved the youngest individual's (age 1) problem. We use piecewise cubic splines (Fortran methods PCHIM and CHFEV) to interpolate points on the value function. Any points that violate a constraint are assigned a large negative value.

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Figure 1: Price-Rent Ratios in the Data

The figure compares three measures of the price-rent ratio. The first measure ("Flow of Funds") is the ratio of residential real estate wealth of the household sector from the Flow of Funds to aggregate housing services consumption from NIPA. The second measure ("Freddie") is the ratio of the Freddie Mac Conventional Mortgage Home Price Index for purchases to the Bureau of Labor Statistics's price index of shelter (which measures rent of renters and imputed rent of owners). The third series ("Case-Shiller") is the ratio of the Case-Shiller national house price index to the Bureau of Labor Statistics's price index of shelter. All indices are normalized to a value of 100 in 2000.Q4. The data are quarterly from 1970.Q1 until 2008.Q4. The REITs series starts in 1972.Q4 and the Case-Shiller series in 1987.Q1.

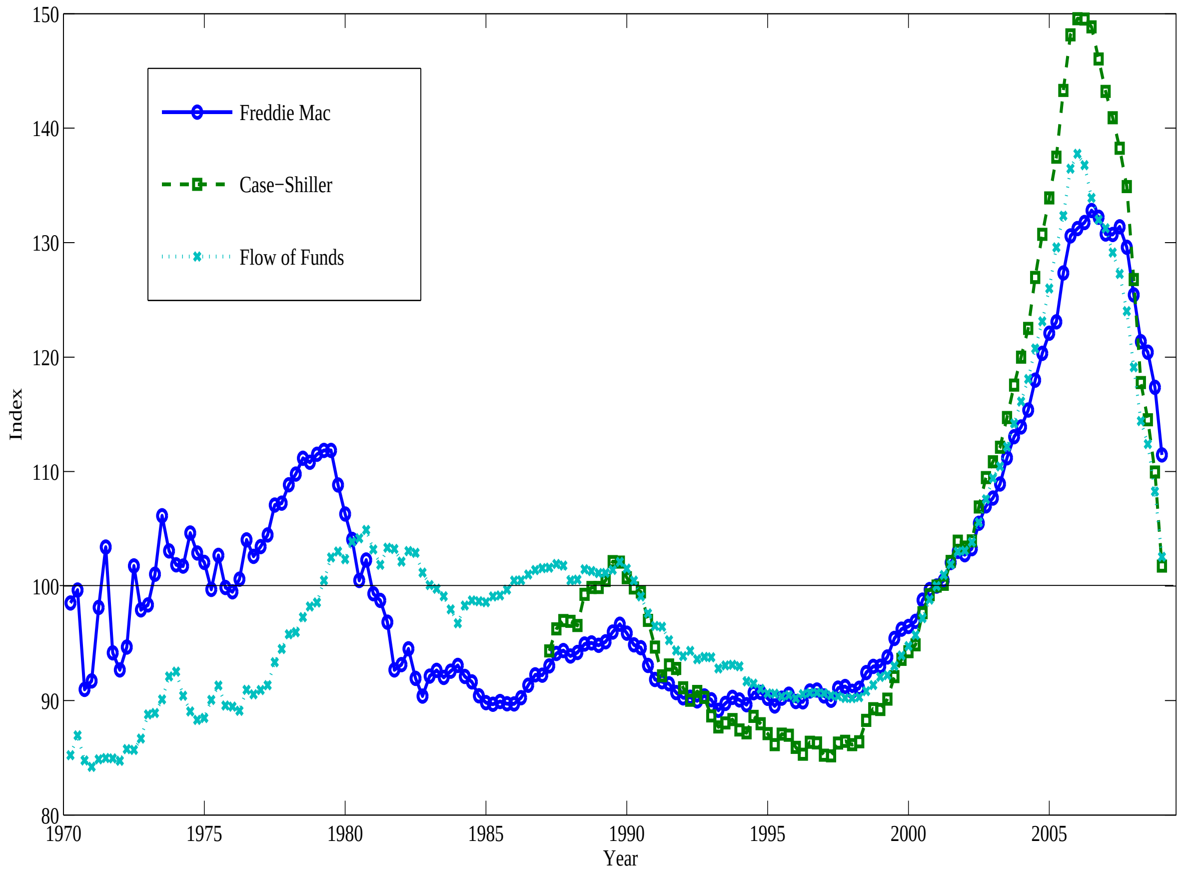

Figure 2: Initial Fees and Charges

The solid line plots the initial fees an charges on all mortgages. They are expressed as a percentage of the value of the loan, and averaged across all mortgage contracts. The data are from the Federal Housing Financing Board's Monthly Interest Rate Survey. The data are monthly from January 1973 until January 2009.

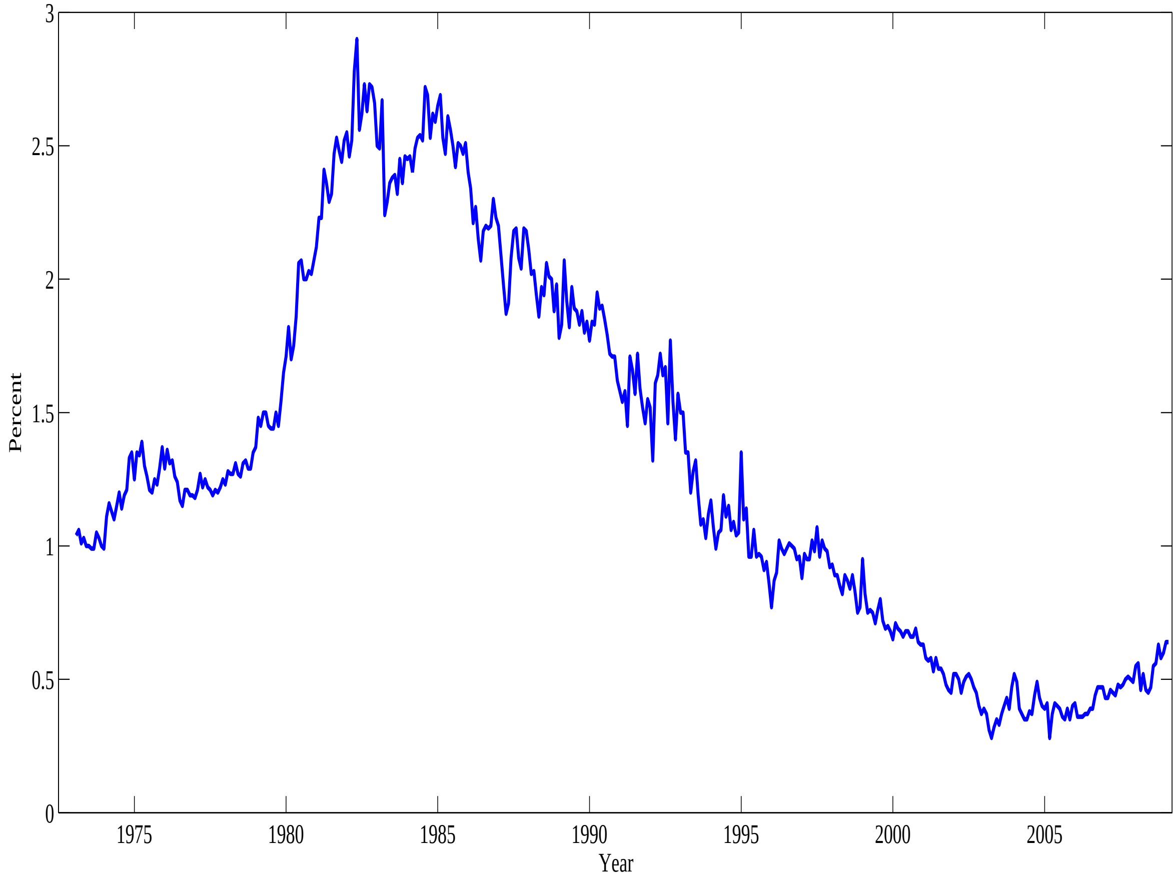

Figure 3: Fixed-rate Mortgage Rate and Ten-Year Constant Maturity Treasury Rate

The solid line plots the 30-year Fixed-Rate Mortgage rate (FRM); the dashed line plots the ten-year Constant Maturity Treasury Yield (CMT). The FRM data are from Freddie Mac's Primary Mortgage Market Survey. They are average contract rates on conventional conforming 30-year fixed-rate mortgages. The CMT yield data are from the St.-Louis Federal reserve Bank (FRED). The data are monthly from April 1971.4 until February 2009.

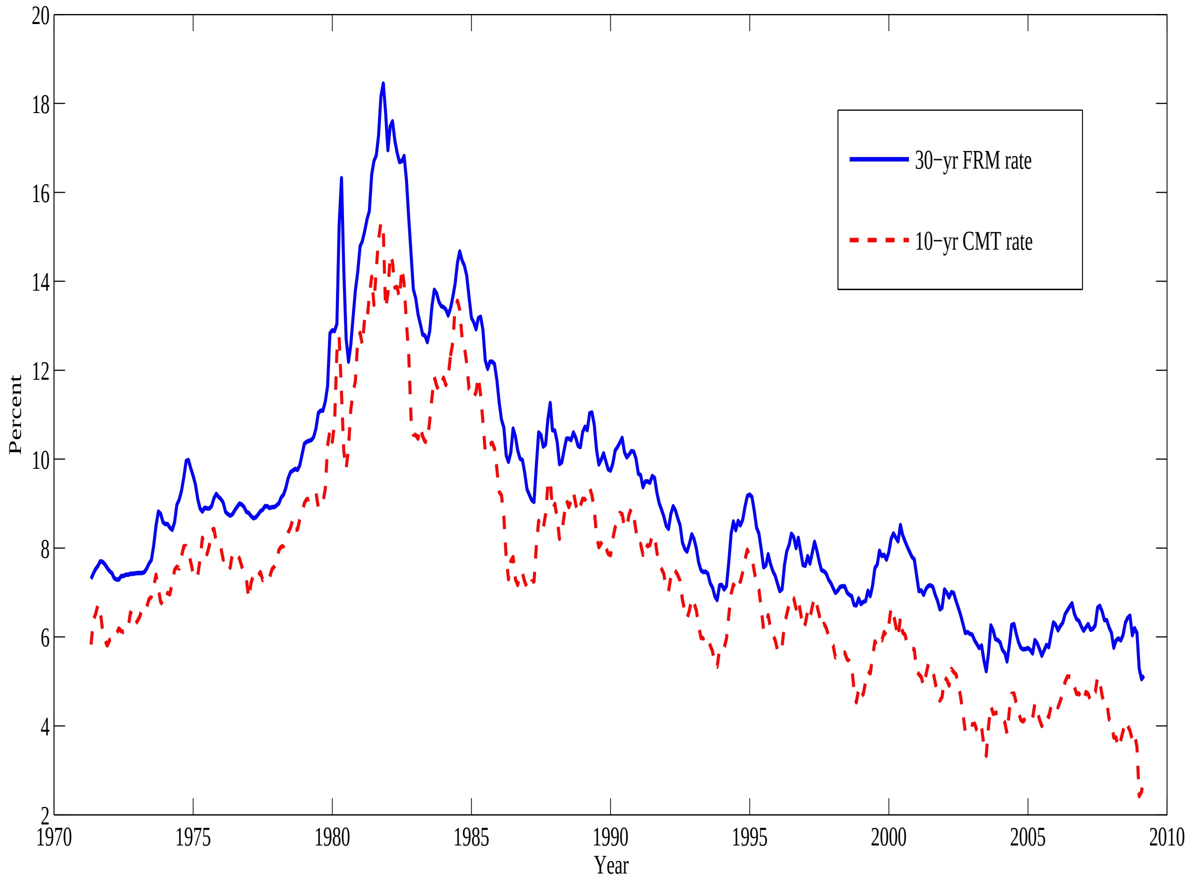

Figure 4: Foreign Holdings of US Treasuries

The solid line, measured against the right axis, plots foreign holdings of long-term U.S. Treasury securities (T-notes, and T-bonds). It excludes (short-term) T-bills. The bars, measured against the left axis, plot those same holdings as a percent of total marketable U.S. Treasuries. Marketable U.S. Treasuries are available from the Office of Public Debt, and are measured as total marketable held by the public less T-bills. The foreign holdings data from the Treasury International Capital System of the U.S. Department of the Treasury. The foreign holdings data are available for December 1974, 1978, 1984, 1989, 1994, 1997, March 2000, annually for June 2002 through June 2008, and for January 2009.

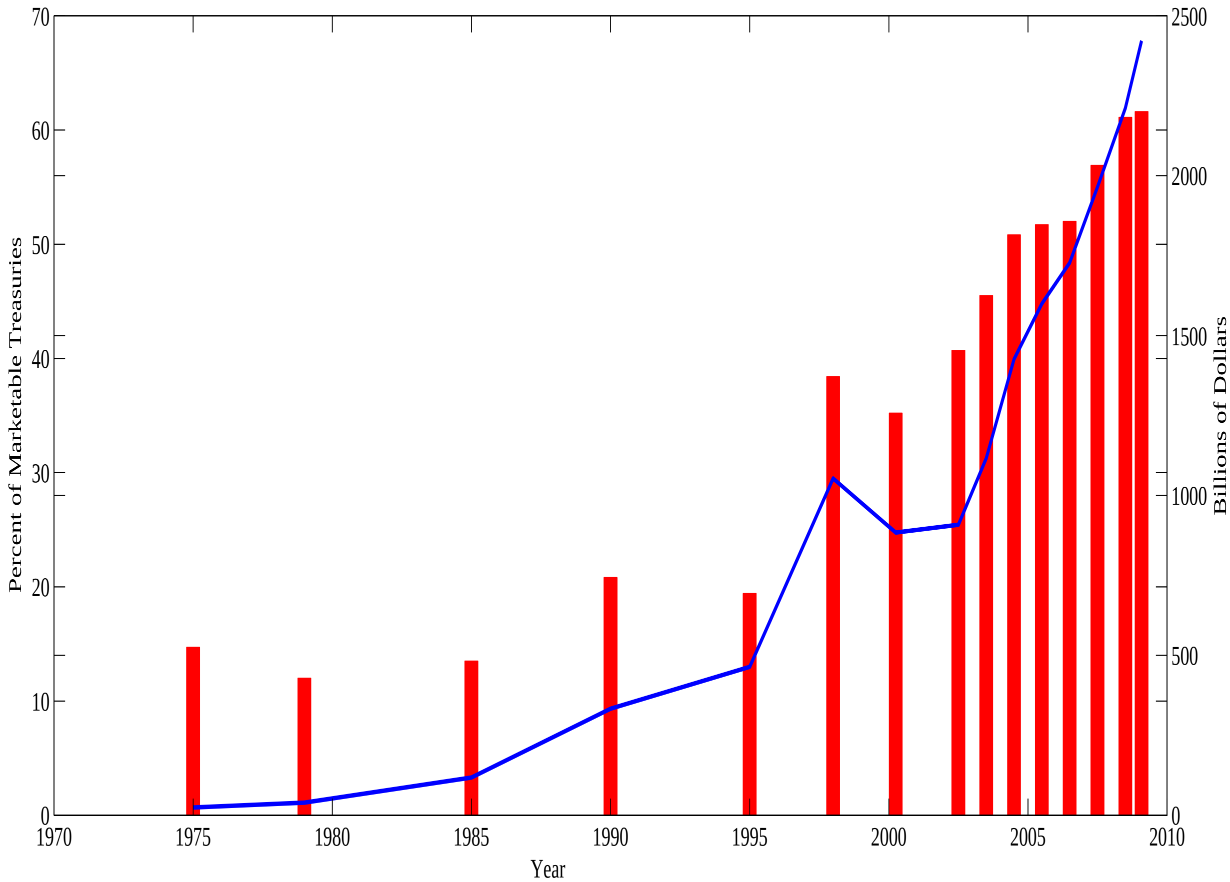

Figure 5: Foreign Holdings of U.S. Treasuries and U.S. Agency Debt Relative to U.S. GDP

The figure plots foreign holdings of U.S. Treasury securities (T-bills, T-notes, and T-bonds) and the sum of U.S. treasuries and U.S. Agency debt (e.g., debt issued by Freddie Mac and Fannie Mae), relative to GDP. The first two series report only long-term debt holdings, while the other two series add in short-term debt holdings. Since no short-term debt holdings are available before 2002, we assume that total holdings grow at the same rate as long-term holdings before 2002. Data are from the Treasury International Capital System of the U.S. Department of the Treasury. The foreign holdings data are available for December 1974, 1978, 1984, 1989, 1994, 1997, March 2000, and annual for June 2002 through June 2008. Nominal GDP is from the National Income and Product Accounts, Table 1.1.5, line 1.

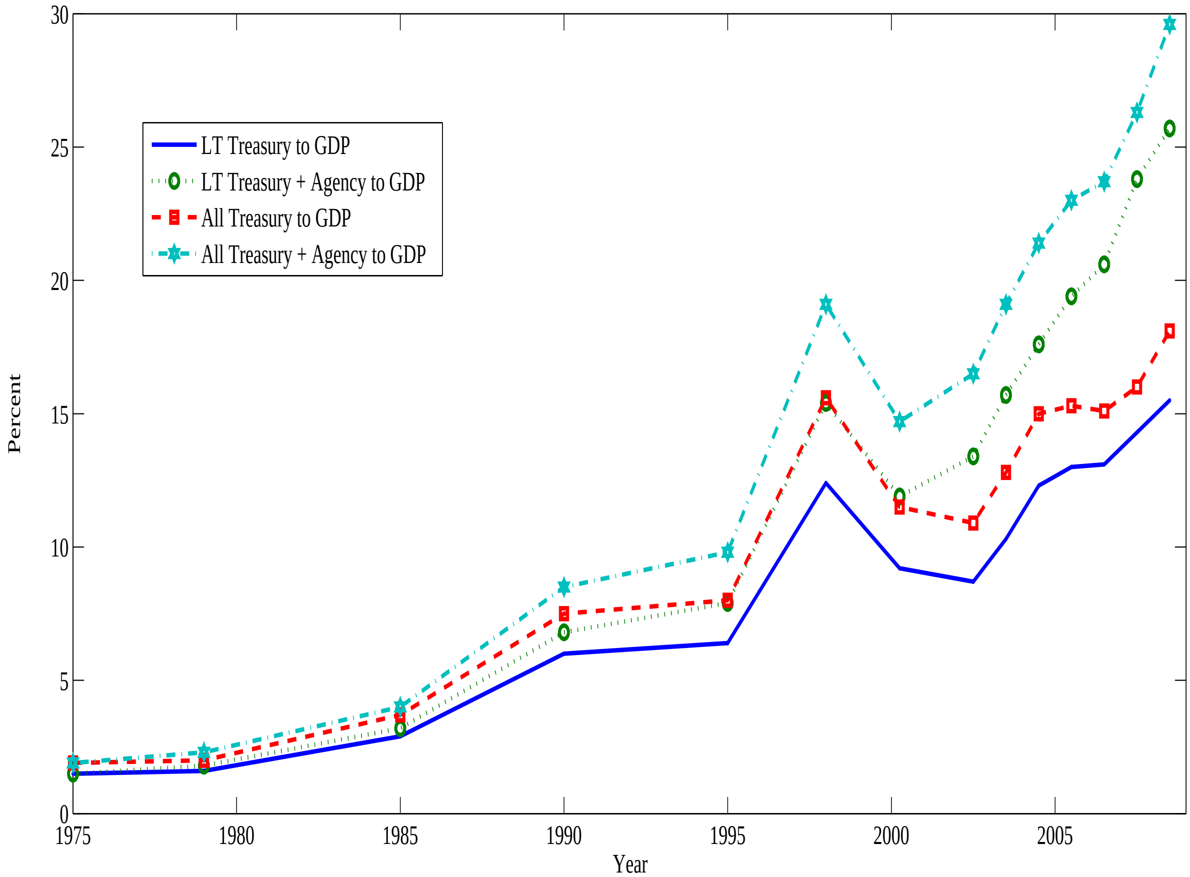

Figure 6: Wealth by Age and Income in Model and Data

The figure plots net financial wealth ("Wealth") by age in the left columns and housing wealth ("Housing") by age in the right columns. The top panels are for the Data, the middle panels for Model 1, and the bottom panels for Model 2. We use all 9 waves of the Survey of Consumer Finance (1983-2007, every 3 years). We construct housing wealth as the sum of primary housing and other property. We construct net financial wealth as the sum of all other assets (bank accounts, bonds, IRA, stocks, mutual funds, other financial wealth, private business wealth, and cars) minus all liabilities (credit card debt, home loans, mortgage on primary home, mortgage on other properties, and other debt). We express wealth on a per capital basis by taking into account the household size, using the Oxford equivalence scale for income. For each age between 22 and 81, we construct average net financial wealth and housing wealth using the SCF weights. To make information in the different waves comparable to each other and to the model, we divide housing wealth and net financial wealth in a given wave by average net worth (the sum of housing wealth and net financial wealth) across all respondents for that wave. We do the same in the model. The Low Earner label refers to those in the bottom 25% of the income distribution, where income is wage plus private business income. The Medium Earner group refers to the 25-75 percentile of the income distribution, and the High Earner is the top 25%. The model computations are obtained from a 1,000 year simulation. The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%.

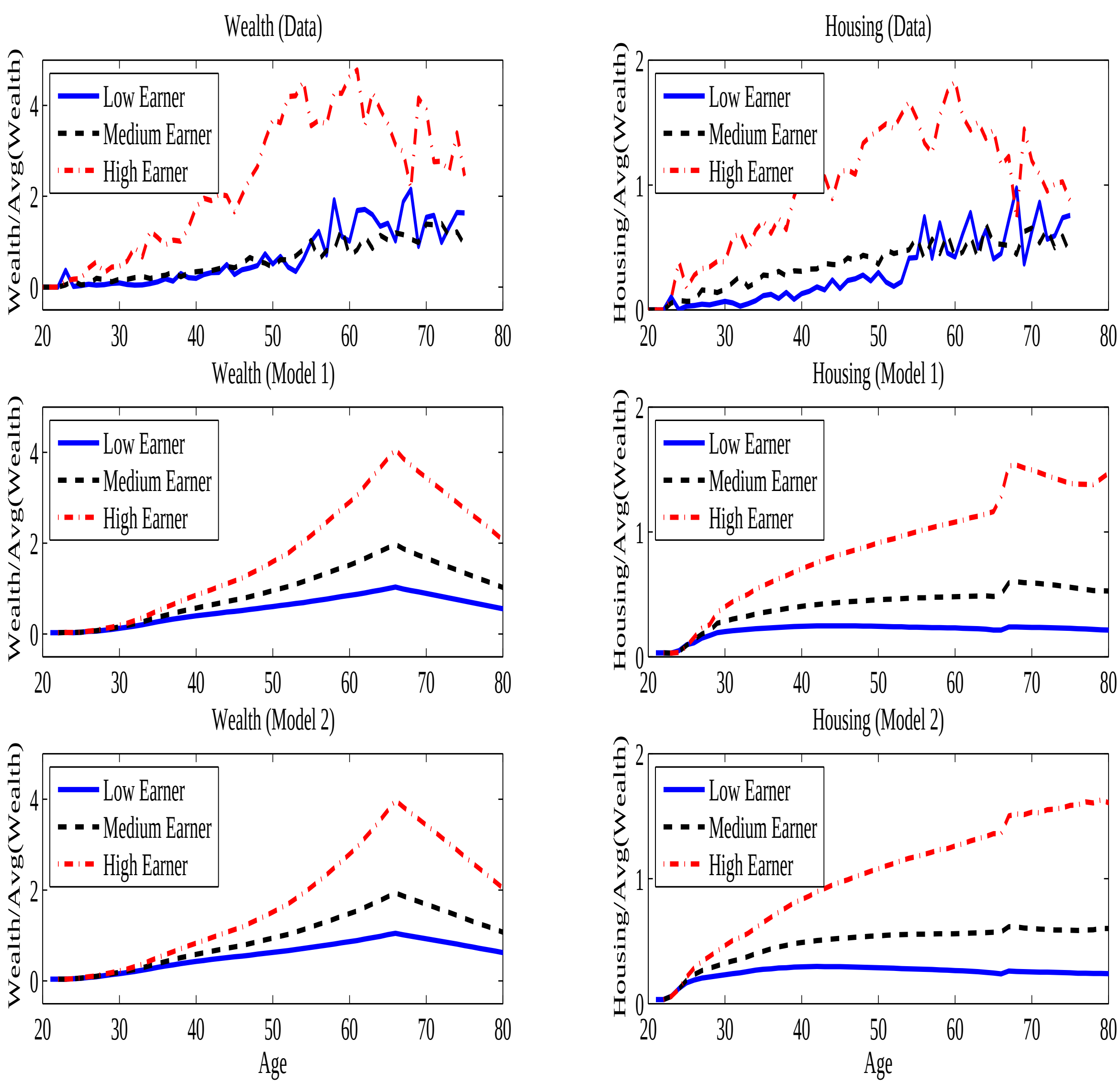

Figure 7: Transition Dynamics in Model: Price-Rent Ratio and Price

The figure plots the house price p^H, plotted against the left axis, and the price-rent ratio $p^H/\mathcal{R}$, plotted against the right axis for a transition generated from the model. The path begins in the year 2000 in the stochastic steady state of Model 1, the model with tight borrowing constraints and high transaction costs. In 2001, the world undergoes an unanticipated change to Model 3, the model with looser borrowing constraints, lower transaction costs, and foreign holdings of U.S. bonds equal to 18% of GDP. The figure traces the first 6 years of the transition from the stochastic steady state of Model 1 to the stochastic steady state of Model 3. Along the transition path, agents use the policy functions from Model 3 evaluated at state variables that begin at the stochastic steady state values of Model 1, and gradually adjust to their stochastic steady state values of Model 3. Along the transition path, foreign holdings of U.S. bonds increase linearly from 0% in 2000 to 18% of GDP by 2006, and remain constant thereafter. In 2007, the world unexpectedly changes to Model 4. Model 4 is the same as Model 1 but with foreign holdings of U.S. bonds equal to 18% of GDP, as in Model 3 ("Reversal of FML in 2007"). The transition path is drawn for a particular sequence of aggregate productivity shocks in the housing and non-housing sectors, as explained in the text.

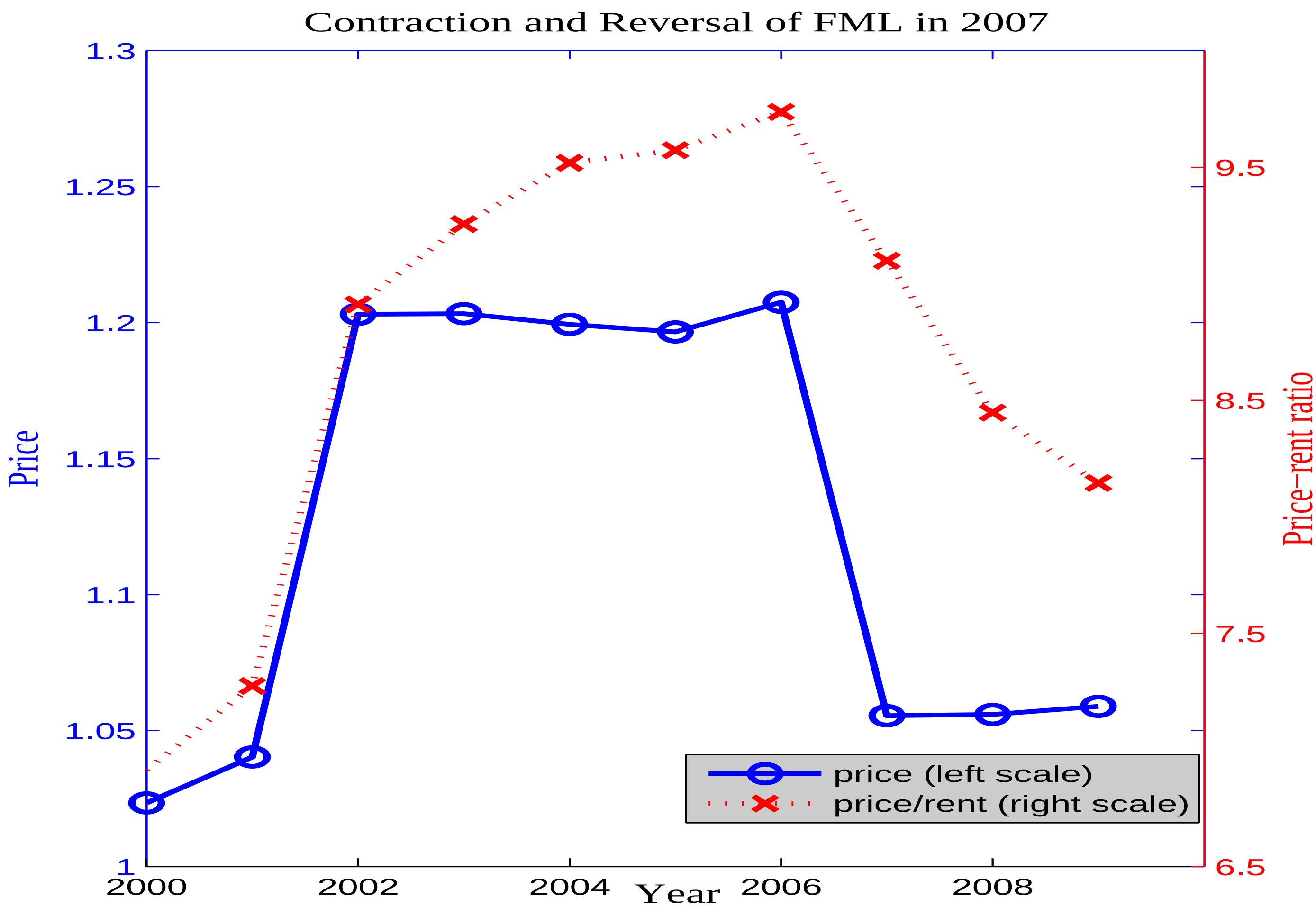

Figure 8: The Macroeconomic Effects of Financial Market Liberalization

The figure plots transitional dynamics between Model 1, the model with tight borrowing constraints and high transaction costs and Model 2 with looser borrowing constraints and lower transaction costs. The lines trace the first 50 years of the transition from the dynamic steady state of Model 1 to the dynamic steady state of Model 2. All quantities are expressed relative to the corresponding quantities from Model 1. In particular, we start in the (dynamic) steady state of Model 1 and evaluate the policy functions at values for the state variables that are typical for Model 1 (obtained by averaging over a 1,000-period simulation of Model 1). Households learn at time 1 that the parameters of the economy are now those from Model 2. They make decisions based on the policy functions of Model 2. These decisions gradually change the values of the state variables and move the economy towards the steady state of Model 2. The plots are averages over 40 simulations. The first panel reports aggregate consumption, GDP, and investment. The second panel reports consumption by age group. The last panel reports consumption for net borrowers and net lenders. The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%.

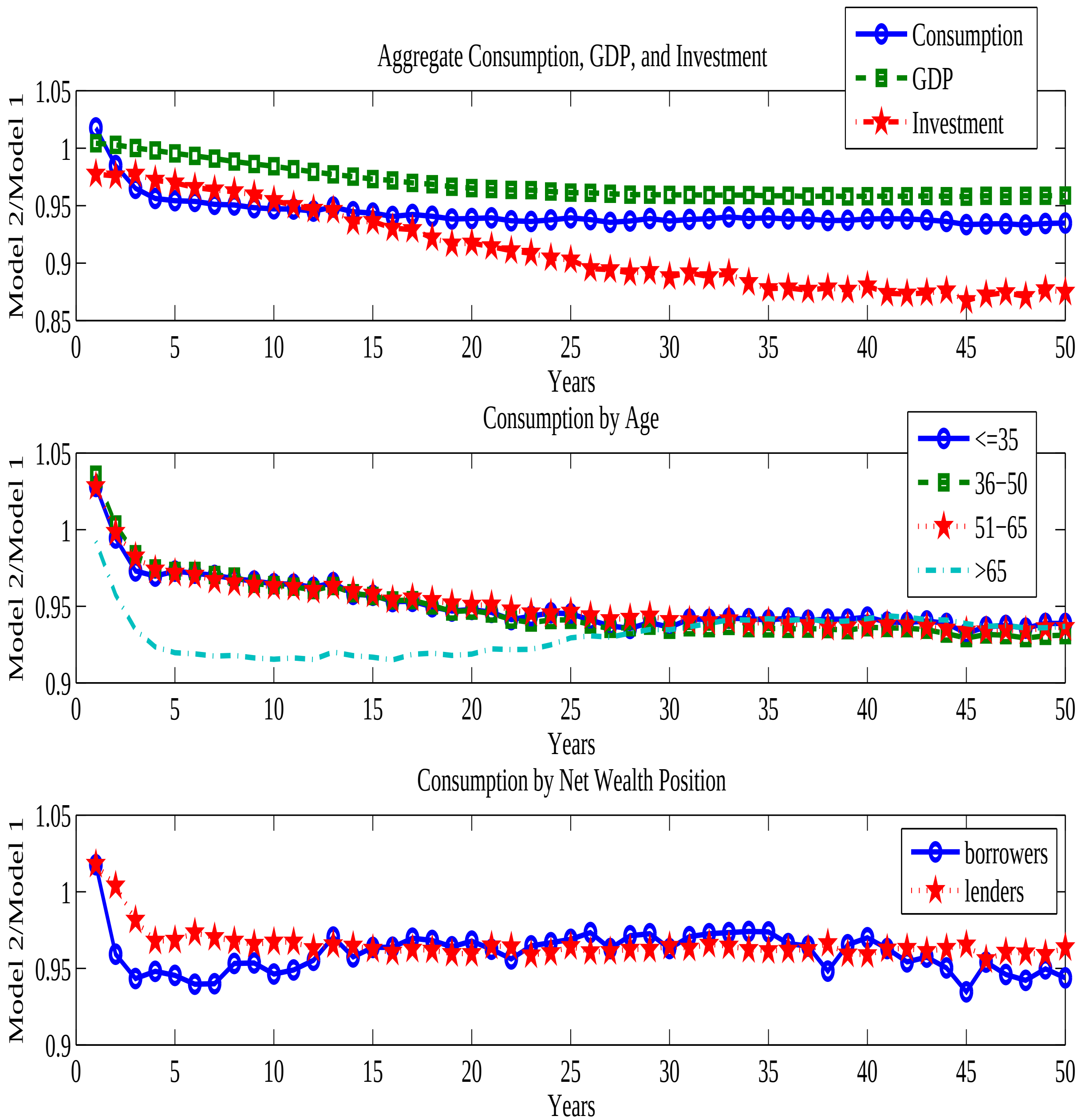

Figure 9: Wealth Inequality in Model and Data

The figure plots the Gini coefficient of total wealth (left panel), financial wealth (middle panel), and housing wealth (right panel). In each panel, the Gini in the data is measured against the left axis, while the Gini in the model is measured against the right axis. The data are shown for the years 2001, 2004, and 2007, indicated by the solid line with dots. For the model, we report the steady state Gini values in Models 1, 2 (star), and 3 (square). The right axes are chosen so that the Model 1 Gini coincides with the value in Model 1. The data are from three waves of the Survey of Consumer Finance. We construct housing wealth as the sum of primary housing and other property. We construct financial wealth as the sum of all other assets (bank accounts, bonds, IRA, stocks, mutual funds, other financial wealth, private business wealth, and cars) minus all liabilities (credit card debt, home loans, mortgage on primary home, mortgage on other properties, and other debt). We express wealth on a per capital basis by taking into account the household size, using the Oxford equivalence scale for income. We use the SCF weights to calculate the Gini coefficients. The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%. Finally, "Model 3" is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 18% of GDP.

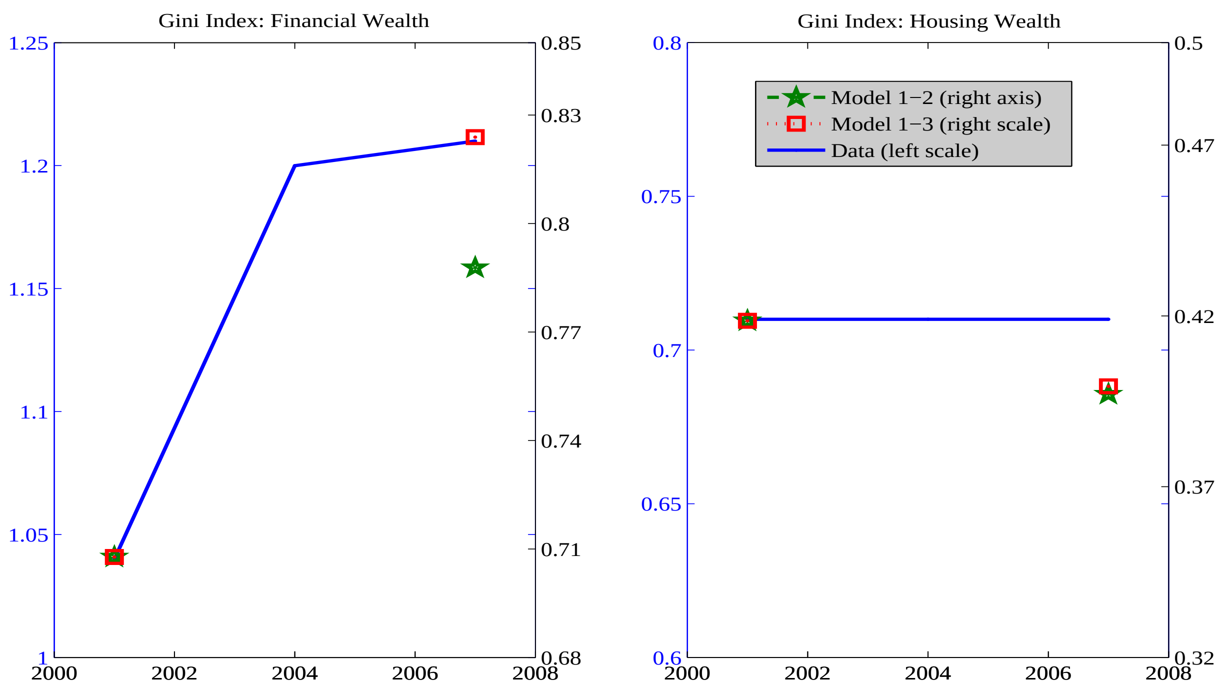

Table 1: Calibration

This table reports the parameter values of our model. The baseline "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%. Finally, "Model 3" is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 18% of GDP.

	Parameter	Description	Baseline, Model 1	Model 2	Model 3
			Production		
1	$\{\phi_C(\cdot), \phi_H(\cdot)\}$	adj. cost	$\left\{\varphi\left(\frac{I}{K}-\delta\right)^2, \varphi\left(\frac{I}{K}-\delta\right)^2\right\}$		
2	δ	deprec., K_C, K_H	12% p.a.		
3	δ_H	depreciation, H	2.5% p.a.		
4	α	capital share, Y_C	0.36		
5	ν	capital share, Y_H	0.30		
			Preferences		
6	σ	risk aversion	8		
7	β	time disc factor	0.923		
8	ε	elast of sub, C, H	1		
9	χ	weight on C	0.70		
			Demographics and Income		
10	G_a	age earnings profile	SCF		
11	$\pi_{a+1\|a}$	survival prob	mortality tables		
12	σ_E	st. dev ind earnings, E	0.0768		
13	σ_R	st. dev ind earnings, R	0.1298		
			Transactions Costs		
14	$\overline{F}$	participation cost, K	$\approx 1\% \ \overline{C}^i$		
15	ψ_0	fixed trans cost, H	$\approx 3.2\% \ \overline{C}^i$	$\approx 2.2\% \ \overline{C}^i$	$\approx 2.2\% \ \overline{C}^i$
16	ψ_1	variable trans cost, H	$\approx 5.5\% \ p_t^H H^i$	$\approx 3.5\% \ p_t^H H^i$	$\approx 3.5\% \ p_t^H H^i$
17	ϖ	collateral constr	25%	1%	1%
			Foreign Supply		
18	B^F	foreign capital	0	0	18% $\overline{Y}$

Table 2: Real Business Cycle Moments

Panel A denotes business cycle statistics in annual post-war U.S. data (1953-2008). The data combine information from NIPA Tables 1.1.5, 3.9.5, and 2.3.5. Output ($Y = Y_C + p^H Y_H + C_H$) is gross domestic product minus net exports minus government expenditures. Total consumption (C_T) is total private sector consumption (housing and non-housing). Housing consumption ($C_H = \mathcal{R} * H$) is consumption of housing services. Non-housing consumption (C) is total private sector consumption minus housing services. Housing investment ($p^H Y_H$) is residential investment. Non-housing investment (I) is the sum of private sector non-residential structures, equipment and software, and changes in inventory. Total investment is denoted I_T (residential and non-housing). For each series in the data, we first deflate by the disposable personal income deflator, We then construct the trend with a Hodrick-Prescott (1997) filter with parameter $\lambda = 100$. Finally, we construct detrended data as the log difference between the raw data and the HP trend, multiplied by 100. The standard deviation (first column), correlation with GDP (second column), and the first-order autocorrelation are all based on these detrended series. The autocorrelation AC is a one-year correlation in data and model. The share of GDP (fourth column) is based on the raw data. Panel B denotes the same statistics for the Model 1 with normal transaction costs and collateral constraints. Panel C reports on Model 2 with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%.

	Panel A: Data (1953-2008)			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	2.78	1.00	0.46	1.00
C_T	1.78	0.91	0.62	0.80
C	1.89	0.91	0.60	0.68
C_H	1.64	0.62	0.74	0.12
I_T	8.01	0.93	0.36	0.20
I	8.66	0.80	0.37	0.14
$p^H Y_H$	12.77	0.71	0.49	0.06

	Panel B: Model 1			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	2.77	1.00	0.14	1.00
C_T	2.14	0.97	0.17	0.72
C	1.88	0.95	0.11	0.45
C_H	2.95	0.87	0.31	0.27
I_T	4.73	0.96	0.12	0.28
I	4.37	0.89	0.09	0.23
$p^H Y_H$	14.87	0.51	0.13	0.05

	Panel C: Model 2			
	st.dev.	corr. w. GDP	AC	share of gdp
Y	2.71	1.00	0.12	1.00
C_T	1.85	0.99	0.14	0.73
C	1.79	0.94	0.12	0.49
C_H	2.30	0.92	0.12	0.25
I_T	5.21	0.99	0.09	0.27
I	5.19	0.81	0.08	0.21
$p^H Y_H$	13.83	0.61	0.15	0.06

Table 3: Correlations House Prices and Real Activity

The table reports the correlations between house prices p^H and house price-rent ratios $p^H/\mathcal{R}$ with output $(Y = Y_C + p^H Y_H + C_H)$ and with residential investment $(p^H Y_H)$. The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%. Finally, "Model 3" is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 18% of GDP. In the data, the housing price and price-rent ratio are measured three different ways. In the first row (Data 1), the housing price is the aggregate value of residential real estate wealth in the fourth quarter of the year (Flow of Funds). The price-rent ratio divides this housing wealth by the consumption of housing services summed over the four quarters of the year (NIPA). In Data 2, the housing price is the repeat-sale Freddie Mac Conventional Mortgage House Price index for purchases only (Freddie Mac). The price-rent ratio divided this price by the rental price index for shelter (BLS). It assumes a price rent ratio in 1970, equal to the one in Data 1. In Data 3, the housing price is the repeat-sale Case-Shiller National House Price index. The price-rent ratio divided this price by the rental price index for shelter (BLS). It assumes a price rent ratio in 1987, equal to the one in Data 1. The price and price-rent ratio values in a given year are the fourth quarter values. The annual price index, GDP, and residential investment are first deflated by the disposable personal income price deflator and then expressed as log deviations from their Hodrick-Prescott trend.

Correlations	(Y, p^H)	$(p^H Y_H, p^H)$	$(Y, p^H/\mathcal{R})$	$(p^H Y_H, p^H/\mathcal{R})$
Data 1 (1953-2008)	0.23	0.43	0.23	0.31
Data 1 (1973-2008)	0.33	0.50	0.27	0.39
Data 2 (1973-2008)	0.33	0.52	0.29	0.46
Data 3 (1987-2008)	0.36	0.75	0.10	0.62
Model 1	0.95	0.28	0.17	0.02
Model 2	0.91	0.28	0.62	0.08
Model 3	0.87	0.39	0.60	0.18

Table 4: Housing Wealth Relative to Total Wealth

The first column reports average housing wealth of the young (head of household is aged 35 or less) divided by average total wealth (i.e., net worth) of the young. The second column reports average housing wealth of the old divided by average net worth of the old. The third column reports average housing wealth of the young plus average housing wealth of the old divided by average net worth of the young plus average net worth of the old. The fourth (fifth) [sixth]column reports average housing wealth of the low (medium) [high]earners divided by average net worth of the low (medium) [high] earners. Low (medium) [high] earners are those in the bottom 25% (middle 50%) [top 25%] of the income distribution, relative to the cross-sectional income distribution at each age. The data are from the Survey of Consumer Finance for 1998-2007. The last two rows report the model. In the model, housing wealth is $P_H * H$ and total wealth is $W + P_H * H$. The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%. Finally, "Model 3" is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 15% of GDP.

	young	old	all	low earn	medium earn	high earn
1998	0.67	0.44	0.46	0.43	0.63	0.40
2001	0.67	0.43	0.44	0.44	0.58	0.40
2004	1.14	0.53	0.55	0.49	0.70	0.51
2007	0.92	0.52	0.54	0.51	0.71	0.50
Model 1	1.50	0.48	0.52	0.44	0.49	0.56
Model 2	1.83	0.52	0.56	0.49	0.54	0.60
Model 3	1.78	0.54	0.59	0.50	0.56	0.64

Table 5: Return Moments

The table reports the mean and standard deviation of the return on physical capital, on a levered claim to physical capital, and on housing, as well as their Sharpe ratios. The Sharpe ratios are defined as the average excess return, i.e., in excess of the riskfree rate, divided by the standard deviation of the excess return. It also reports the mean and standard deviation of the riskfree rate. The last column is the price-rent ratio. The leverage ratio (debt divided by equity) we use in the model is 2/3: $R_E = R_f + (1 + B/E)(R_K - R_f)$. The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%. Finally, "Model 3" is the same as Model 2 except with a positive demand for bonds from foreigners, equal to 18% of GDP. In the data, the housing return and price-rent ratio are measured three different ways. In the first row (Data 1), the housing return is the aggregate value of residential real estate wealth in the fourth quarter of the year (Flow of Funds) plus the consumption of housing services summed over the four quarters of the year (NIPA) divided by the value of residential real estate in the fourth quarter of the preceding year. We subtract CPI inflation to express the return in real terms and population growth in order to correct for the growth in housing quantities due to population growth. In Data 2, the housing return uses the repeat-sale Freddie Mac Conventional Mortgage House Price index for purchases only (Freddie Mac) and the rental price index for shelter (BLS). It assumes a price rent ratio in 1970, equal to the one in Data 1. We subtract realized CPI inflation from realized housing returns to form monthly real housing returns. We construct annual real housing returns by compounding monthly real housing returns over the year. The levered physical capital return in the data is measured as the CRSP value-weighted stock return. We subtract realized annual CPI inflation from realized annual stock returns between 1953 and 2008 to form real annual stock returns. The risk-free rate is measured as the yield on a one-year government bond at the start of the year minus the realized inflation rate over the course of the year. The data are from the Fama-Bliss data set and available from 1953 until 2008.

	$E[R_K]$	$Std[R_K]$	$E[R_E]$	$Std[R_E]$	$E[R_H]$	$Std[R_H]$	$E[R_f]$	$Std[R_f]$	$SR[R_E]$	$SR[R_H]$	$p^H/\mathcal{R}$
Data 1 (53-08)			7.86	19.11	9.89	4.91	1.62	2.49	0.34	1.49	14.72
Data 1 (72-08)			6.60	19.43	9.78	5.87	1.66	3.01	0.27	1.22	15.25
Data 2 (72-08)			6.60	19.43	9.11	4.32	1.66	3.01	0.27	1.36	13.68
Model 1	4.02	6.49	5.62	11.40	13.02	6.20	1.63	3.50	0.31	1.52	7.56
Model 2	5.71	7.88	7.15	13.86	10.42	6.71	3.56	4.31	0.23	0.80	9.33
Model 3	4.66	8.72	7.82	15.41	9.90	7.84	0.00	4.92	0.44	1.01	9.90

Table 6: Predictability

This table reports the the coefficients, t-stats, and R^2 of real return and real dividend growth predictability regressions. The return regression specification is: $\frac{1}{k}\sum_{j=1}^{k} r^i_{t+j} = \alpha + \kappa^r pd^i_t + \varepsilon_{t+k}$, where k is the horizon in years, r^i is the log housing return (left panel) or log stock return (right panel), and pd^i_t is the log price-rent ratio (left panel) or price-dividend ratio on equity (right panel). The dividend growth predictability specification is similar: $\frac{1}{k}\sum_{j=1}^{k} \Delta d^i_{t+j} = \alpha + \kappa^d pd^i_t + \varepsilon_{t+k}$, where Δd^i is the log rental growth rate (left panel) or log dividend growth rate on equity (right panel). In the model, we use the return on physical capital for the real return on equity. The model objects are obtained from a 1150-year simulation, where the first 150 periods are discarded as burn-in. In the data we use the CRSP value-weighted stock return, annual data for 1953-2008. The housing return in the data is based on the annual Flow of Funds data for 1953-2008. We subtract CPI inflation to obtain the real returns and real dividend or rental growth rates.

	Housing - Model 1							Equity - Model 1					
k	κ^r	t-stat	R^2	κ^d	t-stat	R^2	k	κ^r	t-stat	R^2	κ^d	t-stat	R^2
1	-0.26	-16.72	13.46	-0.06	-5.75	1.83	1	-0.14	-18.18	25.46	0.48	19.45	29.22
2	-0.20	-20.60	24.75	-0.04	-4.44	1.67	2	-0.09	-20.03	33.96	0.30	20.84	37.36
3	-0.17	-24.50	35.86	-0.02	-3.30	1.21	3	-0.06	-20.75	36.37	0.22	22.71	41.84
5	-0.13	-30.55	54.36	-0.01	-1.70	0.48	5	-0.04	-23.59	38.48	0.13	24.32	44.13
10	-0.09	-34.72	71.45	-0.00	-0.31	0.02	10	-0.02	-24.67	47.25	0.07	27.03	50.76
20	-0.05	-29.51	75.83	0.00	0.79	0.19	20	-0.01	-27.51	53.91	0.04	34.14	58.06
30	-0.03	-29.52	75.43	0.00	1.18	0.40	30	-0.01	-24.52	57.90	0.02	34.67	65.26
	Housing - Model 2							Equity - Model 2					
k	κ^r	t-stat	R^2	κ^d	t-stat	R^2	k	κ^r	t-stat	R^2	κ^d	t-stat	R^2
1	-0.89	-17.34	27.30	-0.30	-11.50	12.27	1	-0.22	-20.97	32.30	0.49	20.14	32.82
2	-0.59	-20.75	38.95	-0.20	-12.99	18.40	2	-0.14	-23.40	39.73	0.29	22.79	40.00
3	-0.42	-23.52	45.75	-0.14	-13.51	20.45	3	-0.10	-27.59	44.78	0.22	25.96	45.89
5	-0.27	-24.28	55.35	-0.09	-12.51	22.52	5	-0.07	-26.81	50.63	0.14	26.86	51.29
10	-0.14	-32.02	69.72	-0.05	-13.18	27.00	10	-0.03	-29.45	53.83	0.07	35.93	59.05
20	-0.07	-40.67	80.44	-0.02	-13.50	30.12	20	-0.02	-36.93	63.20	0.04	47.97	70.81
30	-0.05	-42.07	80.97	-0.02	-13.01	29.40	30	-0.01	-40.83	65.70	0.02	51.48	79.00
	Housing - Model 3							Equity - Model 3					
k	κ^r	t-stat	R^2	κ^d	t-stat	R^2	k	κ^r	t-stat	R^2	κ^d	t-stat	R^2
1	-0.85	-18.83	28.73	-0.20	-9.34	8.08	1	-0.15	-20.23	33.65	0.55	21.48	33.67
2	-0.56	-25.16	40.54	-0.15	-11.91	13.09	2	-0.10	-25.91	43.99	0.37	30.75	46.52
3	-0.42	-27.15	48.45	-0.12	-13.09	17.69	3	-0.07	-26.89	50.20	0.27	31.11	52.13
5	-0.26	-35.30	56.73	-0.08	-14.92	21.39	5	-0.04	-29.29	48.79	0.16	34.91	53.85
10	-0.14	-44.73	73.41	-0.04	-16.78	27.95	10	-0.02	-29.60	54.73	0.08	33.43	59.48
20	-0.07	-61.08	81.56	-0.02	-17.55	28.34	20	-0.01	-33.41	58.90	0.04	38.75	67.79
30	-0.05	-60.43	82.78	-0.01	-17.15	28.26	30	-0.01	-37.95	61.39	0.03	46.80	72.67
	Housing - Data (FoF, annual 1953-2008)							Equity - Data (CRSP, annual 1953-2008)					
k	κ^r	t-stat	R^2	κ^d	t-stat	R^2	k	κ^r	t-stat	R^2	κ^d	t-stat	R^2
1	-0.12	-2.2	5.3	0.00	-0.1	0.0	1	-0.14	-2.4	9.3	-0.07	-2.9	4.6
2	-0.12	-3.0	8.1	0.00	0.1	0.0	2	-0.12	-2.4	13.3	-0.03	-1.9	3.5
3	-0.11	-4.3	9.4	0.01	1.0	0.4	3	-0.09	-3.1	14.4	-0.01	-0.6	0.4
5	-0.09	-5.4	11.7	0.03	2.4	4.0	5	-0.07	-4.2	16.0	0.01	0.7	0.7

Table 7: Excess Return Predictability

This table reports the the coefficients, t-stats, and R^2 of *excess* return predictability regressions. The return regression specification is: $\frac{1}{k}\sum_{j=1}^{k} r_{t+j}^{i,e} = \alpha + \kappa^{r,e} pd_t^i + \varepsilon_{t+k}$, where k is the horizon in years, $r^{i,e}$ is the log real housing return in excess of a real short-term bond yield (left panel) or the log real stock return in excess of a real short-term bond yield (right panel), and pd_t^i is the log price-rent ratio (left panel) or price-dividend ratio on equity (right panel). In the model, we use the return on physical capital for the real return on equity and the return on the one-year bond as the real bond yield. The model objects are obtained from a 1150-year simulation, where the first 150 periods are discarded as burn-in. In the data we use the CRSP value-weighted stock return minus CPI inflation, annual data for 1953-2008. The housing return in the data is based on the annual Flow of Funds data for 1953-2008. We subtract CPI inflation to obtain the real return. The real bond yield is the 1-year Fama-Bliss yield in excess of CPI inflation.

	Housing - Model 1				Equity - Model 1		
k	$\kappa^{r,e}$	t-stat	R^2	k	$\kappa^{r,e}$	t-stat	R^2
1	−0.16	−5.76	2.63	1	−0.09	−7.90	6.03
2	−0.12	−5.61	4.27	2	−0.06	−6.75	6.25
3	−0.10	−5.72	5.98	3	−0.04	−5.64	5.02
5	−0.08	−5.97	9.16	5	−0.02	−3.74	2.38
10	−0.06	−6.33	14.94	10	−0.01	−1.96	1.00
20	−0.04	−6.77	20.52	20	−0.00	−0.67	0.15
30	−0.02	−6.90	21.99	30	−0.00	−0.86	0.29
	Housing - Model 2				Equity - Model 2		
k	$\kappa^{r,e}$	t-stat	R^2	k	$\kappa^{r,e}$	t-stat	R^2
1	−0.53	−6.23	4.39	1	−0.16	−9.52	8.96
2	−0.34	−5.87	5.13	2	−0.09	−8.48	8.56
3	−0.24	−5.30	4.81	3	−0.07	−8.30	8.41
5	−0.15	−4.26	4.27	5	−0.05	−7.30	8.14
10	−0.08	−3.75	3.83	10	−0.02	−6.12	5.92
20	−0.05	−3.95	5.24	20	−0.01	−6.05	5.96
30	−0.03	−3.69	5.45	30	−0.01	−5.35	4.32
	Housing - Model 3				Equity - Model 3		
k	$\kappa^{r,e}$	t-stat	R^2	k	$\kappa^{r,e}$	t-stat	R^2
1	−0.50	−6.39	4.54	1	−0.10	−8.51	7.98
2	−0.31	−6.20	4.76	2	−0.06	−8.44	8.48
3	−0.23	−5.75	4.77	3	−0.05	−7.56	8.64
5	−0.14	−5.12	3.85	5	−0.03	−6.14	5.37
10	−0.09	−5.99	4.87	10	−0.02	−5.92	4.63
20	−0.05	−6.23	5.45	20	−0.01	−6.87	3.96
30	−0.03	−6.05	5.39	30	−0.01	−6.03	2.40
Housing - Data (FoF, annual 1953-2008)				Equity - Data (CRSP, annual 1953-2008)			
k	$\kappa^{r,e}$	t-stat	R^2	k	$\kappa^{r,e}$	t-stat	R^2
1	-0.15	-1.8	7.8	1	-0.16	-2.4	11.7
2	-0.15	-2.0	11.4	2	-0.11	-2.4	12.9
3	-0.15	-2.7	14.0	3	-0.08	-3.3	13.1
5	-0.16	-4.6	20.8	5	-0.06	-3.4	14.6

Table 8: Risk Sharing

This table reports the cross-sectional standard deviation of the consumption share $C^i_{T,a,t}/C_{T,t}$, as well as the cross-sectional standard deviation of the individual-level inter-temporal marginal rate of substitution (IMRS). The third panel reports the ratio of consumption for a given group relative to consumption for all households. The first column pools households of all ages, the next four columns look at various age groups. The third panel also splits total consumption into consumption by net borrowers and net lenders in the last two columns. Consumption across age groups sums to 100 and so does consumption of borrowers and lenders. The last panel reports the Gini coefficient of consumption and the variance of log consumption; both are multiplied by 100. We simulate the model for $N = 2400$ households and for $T = 1150$ periods (the first 150 years are burn-in and discarded). We calculate cross-sectional means and standard deviations of individual consumption share or consumption growth within each age group for each period, and then average over periods. The "Model 1" is the model with normal moving costs and collateral constraints, "Model 2" reports on the model with lower transaction costs and looser collateral constraints. In particular, fixed transaction costs go from 3.2% of average consumption to 2.2%, variable costs go from 5.5% to 3.5% of home value, and the down-payment goes from 25% to 1%. Finally, "Model 3" is the model with foreign holdings of bonds to the extent of 19% of GDP.

	Cross-sectional St. Dev. Consumption Share				
	all	≤ 35	36-50	51-65	>65
Model 1	79.63	49.44	55.74	70.72	81.56
Model 2	77.30	47.86	54.08	68.38	76.82
Model 3	78.33	49.01	55.33	69.69	79.66
	Cross-sectional St. Dev. IMRS				
	all	≤ 35	36-50	51-65	>65
Model 1	60.35	64.88	57.90	66.43	33.08
Model 2	55.14	62.96	54.89	55.80	28.35
Model 3	62.50	68.75	60.63	65.42	35.71
	Inequality Measures				
			Gini cons.	Var of log cons.	
Model 1			37.63	45.05	
Model 2			36.42	42.37	
Model 3			36.73	42.81	